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Numerosity and density judgments: Biases for area but not for volume

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Human observers can rapidly judge the number of items in a scene. This ability is underpinned by specific mechanisms encoding number or density. We investigated whether judgments of number and density are biased by a change in volume, as they are by a change in area. Stimuli were constructed using nonoverlapping black and white luminance-defined dots. An eight-mirror Wheatstone stereoscope was used to present the dots as though in a volume. Using a temporal two-alternative forced-choice (2AFC) task and the Method of Constant Stimuli (MOCS), we measured the precision and bias (PSE shift) of numerosity and density judgments, separately, for stimuli differing in area or volume. For two-dimensional (2-D) stimuli, consistent with previous literature, perceived density was biased as area increased. However, perceived number was not. For three-dimensional (3-D) stimuli, despite a vivid impression of the dots filling a cylindrical volume, there was no bias in perceived density or number as volume increased. A control experiment showed that all of our observers could easily perceive disparity in our stimuli. Our findings reveal that number and density judgments that are biased by area are not similarly biased by volume changes.

Introduction

For small sets of items, typically comprising four or less, our ability to estimate number is very fast and highly accurate. As the number of items increases beyond this range, response latency increases, and accuracy decreases. This performance dichotomy between small and large numbers is often theorized to reflect distinct processes: subitizing for the former and counting for the latter (Kaufman & Lord, 1949; Piazza, Mechelli, Butterworth, & Price, 2002; Piazza, Pinel, Le Bihan, & Dehaene, 2007; Sawamura, Shima, & Tanji, 2002; Simon & Vaishnavi, 1996). However, under conditions where a counting strategy is ineffective, such as for short presentation times or for large numbers, judgments of number can still be made, suggesting that under these circumstances, another process is in play (Allik & Tuulmets, 1991; Allik, Tuulmets, & Vos, 1991; He, Zhang, Zhou, & Chen, 2009; Ross, 2003). A considerable amount of research has been focused on elucidating the mechanisms underpinning this skill. A fundamental question here is whether this ability is underpinned by the calculation of number (Burr & Ross, 2008b; Ross & Burr, 2010) or density (Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Durgin & Huk, 1997; Tibber, Greenwood, & Dakin,

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2012). Indeed, much of the recent work in this field has focused on determining if in fact those are separable dimensions or whether one arises out of the other (e.g., Anobile, Cicchini, & Burr, 2014; Arrighi, Togoli, & Burr, 2014; Burr & Ross, 2008a; Dakin et al., 2011; Durgin, 2008). The current study is not aimed at resolving that debate; instead we ask a fundamental question that is important for understanding either process: Is the calculation of number or density made with respect to area only, or is the volume also important in such calculations? This question is especially pertinent in light of recent reports that our sense of density and of number is biased by the area subtended by a given stimulus (Dakin et al., 2011; Tibber et al., 2012). In this communication we ask whether such biases extend to three-dimensional (3-D) volumes.

Within this field of research, density is commonly defined within a plane as the number of elements, or dots, per degree^2 (Burr & Ross, 2008b; Dakin et al., 2011; Durgin & Proffitt, 1996), or by occupancy indices that describe the average space between elements (Allik & Tuulmets, 1991). The definition of number is more intuitive, but the existing literature is similarly focused on estimates within an area. Constraining the problem to a two-dimensional (2-D) approach is a sensible first step, both because of the flat panel displays usually employed in this field of research and also because it is typical in vision to describe a stimulus in terms of the visual area subtended on the retina, itself a planar receptor. However, even on a flat display, the elements displayed on a monitor are not all on the horoptor,that is, they are at different depths relative to the observer (Howard & Rogers, 1995). Further, in natural visual scenes the items in a given set are frequently positioned at different depths relative to the observer. Ross and Burr's (Burr & Ross, 2008a) example of the stockman counting his cattle (or their legs and dividing by four) illustrates the ecological relevance of this point. The herded cattle occupy a physical volume, meaning that they are necessarily spread out in depth. In fact, in the physical sciences, such as chemistry, it is critical to consider the density of interacting elements within a volume, not by area (Mills, Cvitas, Hosman, Kallay, & Kuchitsu, 1993). The current question is therefore whether by extension volume changes, like area, can bias judgments of numerosity or density. Density aftereffects show strong dichoptic transfer, a finding taken to imply that density encoding is a primarily binocular process (Durgin, 2001; Durgin & Proffitt, 1996). This establishes a prerequisite for stereo selectivity without directly testing for it. More recent work involving judgments of numerosity in motion transparency showed biases in number due to depth ordering: Motion directions perceived as behind were judged to be more numerous (Schütz, 2012). A static variant of the stimuli showed no such bias, although stereoscopic disparity of this static stimulus was not manipulated, nor was perceived density measured. Therefore, the current questions regarding the role of stereoscopic cues in judgments of perceived numerosity and density, for static stimuli, remain unanswered.

Stereovision offers more than simply helping to judge the distance to, and between, nearby objects (Howard & Rogers, 1995; Regan, 2000). The addition of stereoscopic cues has been shown to modify our ability to recognize and discriminate many different classes of visual stimuli, including: simple lines (Fendick & Westheimer, 1983; McKee & Taylor, 2010; Westheimer & Pettet, 1990), curved contours and planes (Bell, Dickinson, Badcock, & Kingdom, 2013; Bell, Kanji, & Kingdom, 2013; Welchman, Deubelius, Conrad, Bulthoff, & Kourtzi, 2005), shapes (Bell, Dickinson, & Badcock, 2008; Lim Lee & Saunders, 2011), faces (Burke, Taubert, & Higman, 2007), other objects (Bennett & Vuong, 2006; Burke, 2005), and finally, motion-defined patterns (Greenwood & Edwards, 2006; Snowden & Rossiter, 1999). This is not to say that stereoscopic cues always modify visual processing (cf. Bell, Kanji, et al., 2013; Gheorghiu, Kingdom, Thai, & Sampasivam, 2009). However, one can conclude that stereoscopic information, where it is relevant and potentially useful, is frequently utilised in processing and representing a range of visual information. Given the breadth of its influence in visual processing, a question that emerges is whether stereoscopic cues to depth modify our estimates of the number or density of items in a visual scene.

In the current study we sought to determine the importance of volume in the calculation of number or density. In our first experiment we found, as expected, that comparative judgments of density are strongly biased by area. However, we did not observe this bias in the perception of number under the current experimental configuration. Thresholds were similar for both judgments, and as predicted, were approximately half of those obtained on an equivalent task involving peripheral judgments. In Experiment 2 we investigated whether analogous biases exist as a function of increasing volume. Expanding the volume containing the dot elements had no effect on perceived density or number. This was not due to any change in task precision (thresholds), or to any difficulties in resolving stereoscopic cues to depth in our display. Our findings are consistent with a recent model of number and density estimation (Dakin et al., 2011) and serve as an important extension of the current research literature. We demonstrate that an ecologically motivated rationale for expecting an extension of the area-driven estimation biases to volume manipulation does not apply and discuss why.

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General methods

Participants

Six observers participated in this study; AM was an author, while the other five were naive as to the experimental aims of this study. All participants had normal or corrected-to-normal visual acuity. Each observer's stereoacuity was tested using the Stereo Fly and Stereo Butterfly Tests (Stereo Optical Co., Inc., Chicago, IL). All observers had a stereoacuity better than 40 s of arc, as assessed on these instruments. We also measured the stereoacuity of each observer for the dot elements in our experimental display, in a control task (See Experiment 2 control data in Figure 4). Again the threshold for each observer was better than 40 s of arc. All participation was voluntary and unpaid. The protocol was approved by the Australian National University Human Ethics Committee and thus, accords with the conventions set out in the Declaration of Helsinki.

Apparatus

Stimuli were created using the Matlab (version 2011b) programming environment. Images were subsequently displayed using routines from the Psychophysics Toolbox (Brainard, 1997) onto a Dell CRT monitor. Screen resolution was 1024 × 768, with a refresh rate of 85 Hz and a mean luminance of 50.4 cd/m^2. The display was gamma corrected by modifying the gamma function of the NVidia Graphics (Santa Clara, CA) card control panel, and linearity was measured and verified using a Minolta CS-100 Chroma Meter (Minolta, Chiyoda, Tokyo, Japan). The stimuli were presented to the observer at a viewing distance of 1.15 m, where each pixel subtended 1′ of visual angle, or 0.017°.

In stereo conditions, stimuli were viewed through a custom built eight-mirror Wheatstone stereoscope (Wheatstone, 1838). The mean luminance of the stimuli as measured through the stereoscope was 34 cd/m^2. The viewing distance through the stereoscope was 1.15 m, resulting in each pixel subtending 1′ of visual angle. Prior to testing, each observer performed a series of judgments in a control program, whereby they adjusted the horizontal distance between a pair of fixation crosses presented separately to the left and right eye until binocular fusion was achieved. This measurement was then used in the actual experiments. The fixation crosses were presented with this separation, and at the center of each pattern, during the stereo trials to aid binocular fusion (see inset of Figure 4).

Stimuli

The dot arrays were constructed of hard-edged square dots that subtended 3′ × 3.′ The luminance polarity (black or white) of individual dots in a given display was balanced to ensure an equal number of each (see Figure 1). The position of each element was randomly assigned within a circular aperture, the size of which was determined by each experimental condition (2°, 2.8°, or 4° in radius). To avoid overlaps, which can bias perceived density (Allik & Tuulmets, 1991; Dakin et al., 2011) and to minimize the possibility of false correspondences between the stereo halves, we precluded any two dots from appearing within 10′ of one another (edge to edge). In stereo conditions the pattern was replicated in each eye, plus or minus any binocular disparity cues applied to individual elements.

Procedure

Our method was analogous to that described in Dakin et al. (2011) with the exception that we presented our pairs of stimuli in a temporal two-alternative forced-choice (2AFC), rather than side by side in a simultaneous presentation. We did this primarily because the mirror stereoscope requires that the useable area of the display be reduced by half (with half going to each eye), making a spatial 2AFC with large radius patterns impossible. This methodological change also allowed us to assess the importance of stimulus configuration on task precision (see Experiment 1 results). Each pattern was shown for 500 ms with a 500 ms ISI. Observers reported which of the two patterns appeared more numerous, or more dense, in separate noninterleaved blocks. No feedback was provided. It is important at the outset to realize that the density and number estimates were done with very different stimuli. In the density case, with unequal size of test and standard, the number of dots was very different between the two and had to be ignored; in the number case, the two stimuli had very different densities, so density had to be ignored. The reader can appreciate this point by comparing the stimuli in each row of Figure 1.

The method of constant stimuli (MOCS) was used to control stimulus presentation. The reference pattern always contained 128 elements while the number of elements in/density of the test pattern varied across trials in order to obtain a full psychometric function. A cumulative Gaussian function was fit to the raw response data in each condition in order to obtain a measure of the point of subjective equality (PSE) and task precision/threshold (slope). In Experiment 1, across blocked conditions the size of the test and reference patterns were: 2°, 2.8°, or 4° in radius,

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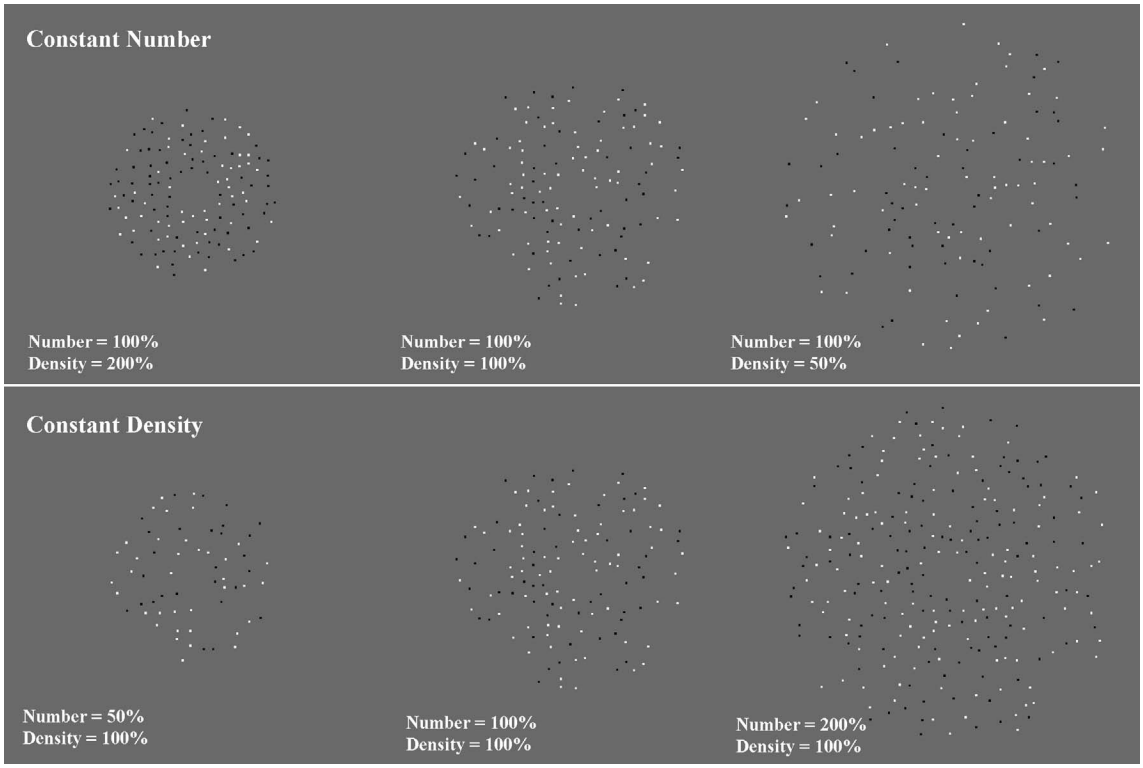

Figure 1. Examples of the 2-D stimuli used in Experiment 1. In all panels the number of dots and density are listed as percentages relative to the center panels in each row, which is a pattern with 2.8° radius and 128 elements, giving a density of 5.2 dots/degree^2. Top row: Constant number. Across the three patterns the number of elements is held constant at 128 while the radius changes from 2° to 2.8° to 4°, meaning that the density of the pattern varies from 10.4 dots/degree^2 to 5.2 dots/degree^2 to 2.5 dots/degree^2. Bottom row: Constant density. Across the three patterns the density of the patterns is held constant at 5.2 dots/degree^2 while the radius changes from 2° to 2.8° to 4°, meaning that the number of elements in the pattern varies from 64 to 128 to 256.

resulting in separate 3 (ref radii) × 3 (test radii) within subjects designs for both the number and density experiments, separately. In Experiment 2, across blocked conditions the volume of the test was varied through a manipulation of: 0′, 5′, 10′, or 20′ of stereo disparity while the ref pattern could have 0′ or 10′ of disparity. These correspond to depths of 0 cm, 3.2 cm, 6.4 cm, and 12.8 cm, derived in Appendix A. This resulted in a 2 (ref depth) × 4 (test depth) within-subjects design for both number and density experiments, separately. In all experiments, the order of the conditions was randomly interleaved for each participant and each participant was randomly assigned to do number or density judgments first.

Experiments

Experiment 1: Mismatching area biases estimates of density but not number

Previous reports have shown that our judgment of density and number are biased by the size of the area containing the elements (Dakin et al., 2011). In this paper we aimed to test for and compare biases in perceived density and number in 2-D and 3-D displays using a single stimulus design and method. For 2-D displays, Dakin et al. have developed a sound method for measuring biases in density and number. We adopted this method in Experiment 1 (2-D) and expanded its use to a 3-D display in Experiment 2. Employing this method has the added benefit of allowing us a direct comparison with their 2-D data. On that note, Experiment 1 also tested the importance of presentation configuration on task precision, and consequently, the importance of task precision on bias magnitude. Previous reports of biases in both density (larger bias) and number (smaller bias) as a function of area were obtained using a brief (250 ms) simultaneous presentation of two peripherally viewed patterns. Performance thresholds, or just-noticeable differences (JNDs) were 40% (Dakin et al., 2011), which are double those reported elsewhere, at approximately 20% (Anobile et al., 2014; Burr & Ross, 2008b). Note: Anobile et al. also simultaneously presented two patterns in the periphery; however, in their study the numerosity of the reference pattern was fixed within a

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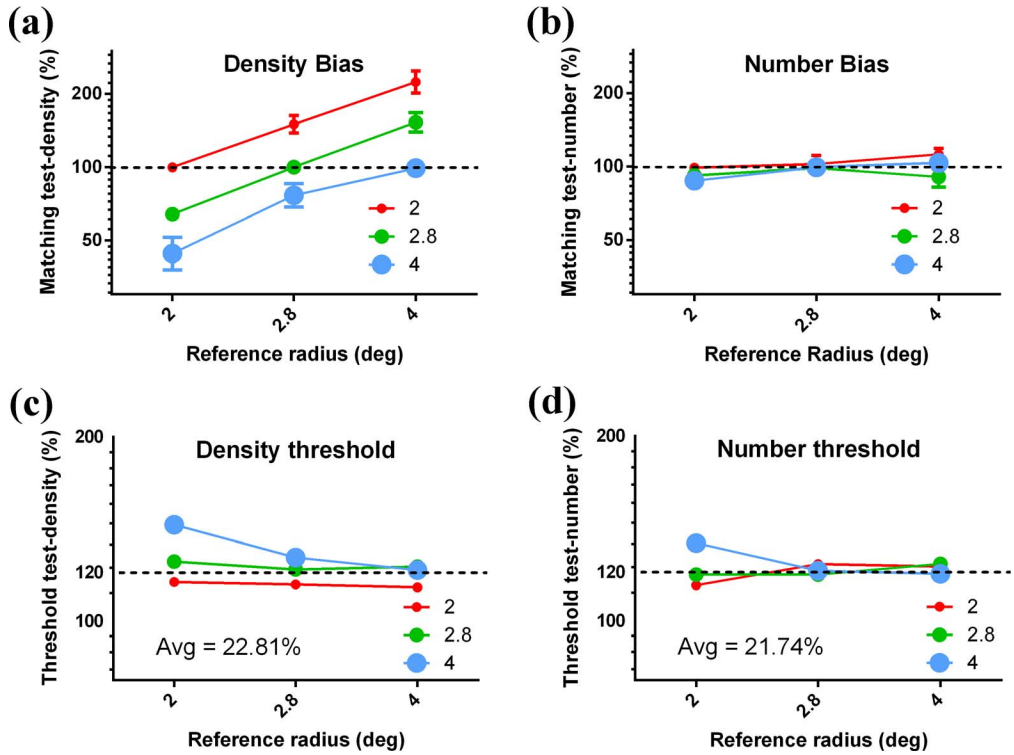

Figure 2. Biases (a–b) and thresholds (c–d) data for our 2-D displays. In all four panels the horizontal axes describe the radius of the reference pattern, and the colored lines describe the radius of the test pattern. (a–b) The vertical axes describe in percentile terms, the physical density (a) or number (b) of the test pattern relative to the ref, at the PSE. A value of 100% indicates veridical performance: Test and ref are matched without bias by the observer. Error bars represent ± 1 *SEM*, across observers. (c–d) Here the vertical axes describe the proportion of extra dots/density required to raise performance from 50% to 82%, as per Dakin et al. (2011). The dashed line represents a threshold of 20%. Data are entirely descriptive, showing the mean only. The average threshold for all conditions in each lower panel is given in Figure.

session and the reference was always presented to the right of fixation; so a peripheral judgment was *not necessarily* required. If the high thresholds reported by Dakin et al. are a consequence of making peripheral judgments, as was argued, then thresholds should be reduced by changing to a temporal 2AFC, involving foveally viewed patterns which are presented for longer, as we have done here. This change in stimulus configuration can be thought to reduce the uncertainty in task judgments. The model proposed by Dakin et al. included Gaussian noise contributions to take such uncertainty into account, and we subsequently consider what effect this could have on this replication.

Results

Biases

Figure 2 shows results averaged across three observers on tasks requiring the observer to discriminate density (Figure 2a) or number (Figure 2b). The data is plotted in a manner analogous to Dakin et al. (2011). The horizontal axis describes the radius of the ref pattern (in degrees), the colored lines show data for the different-sized test patterns (radius in degrees). The vertical axes describes in percentile terms, the physical density (Figure 2a) or number (Figure 2b) of the test pattern relative to the ref, at the PSE. A value of 100% indicates veridical performance: Ref and test are matched without bias by the observer. Importantly, for both judgments, when the two patterns were the same size, performance was veridical, as indicated by the three colored lines passing through the dashed line at 100% (no bias) in the appropriate conditions. A size mismatch, however, induced a bias in perceived density (Figure 2a) but not perceived number (Figure 2b). A bias in these figures is indicated by a data point not falling on the dashed line at 100%.

The nature of the bias for density can be described as follows: Consider the red data points in Figure 2a, from left to right. Here the test pattern with a small 2° radius is being matched to an increasingly large ref pattern. Leftmost, when test and ref are the same size (2°), we see veridical performance (test density is 100% of ref density, at the PSE). Moving to the rightmost point, the ref pattern radius has now doubled to 4°, increasing its area by a factor of four while decreasing its density by a factor of four. It always contains 128 dots. The data

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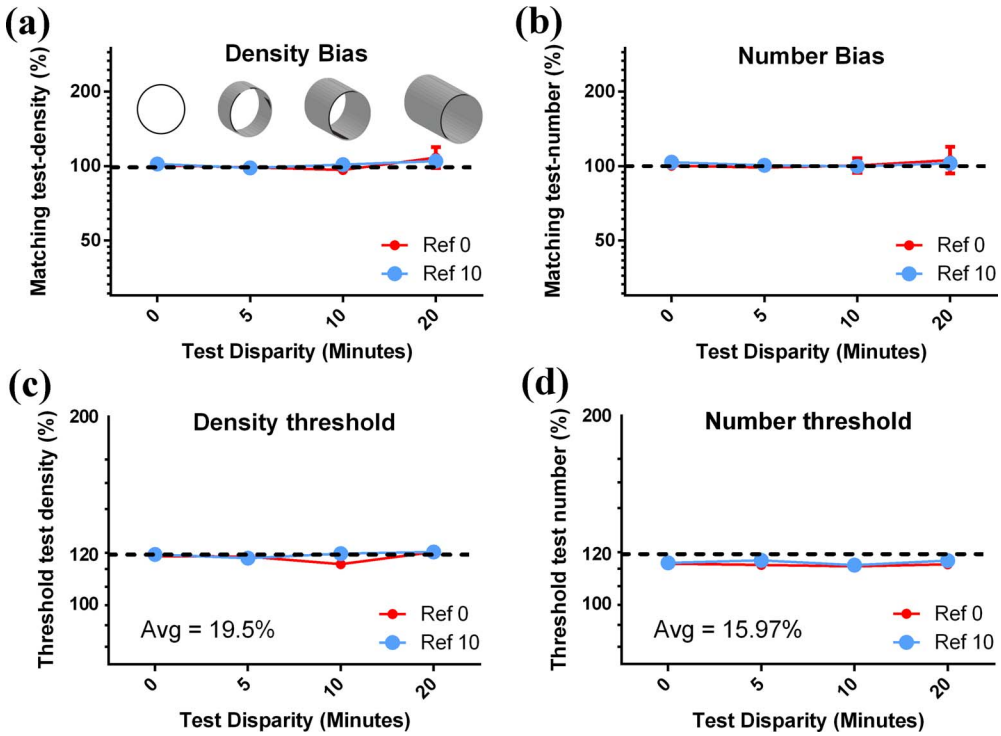

Figure 3. Biases (a–b) and thresholds (c–d) data for our 3-D displays. In all four panels the horizontal axes describe the disparity range of elements within the test pattern. The colored lines describe the disparity range of elements within the reference pattern. (a–b) The vertical axes describe in percentile terms, the physical density (a) or number (b) of the test pattern relative to the ref, at the PSE. A value of 100% indicates veridical performance: Test and ref are matched, two dimensionally, without bias by the observer. Error bars represent ± 1 *SEM*, across observers. (c–d) Here the vertical axes describe the proportion of extra dots/density required to raise performance from 50% to 82%, as per Dakin et al. (2011). The dashed line represents a threshold of 20%. Data are entirely descriptive, showing the mean only. The average threshold for all conditions in each panel is given in Figure.

show that the ref and test are perceptually matched when the physical density of the smaller test is 200% greater than the ref. This indicates that the perceived density of the larger pattern is relatively higher than that of the smaller one, leaving ambiguous which stimulus' perception is shifted from the veridical. A complimentary and opposite pattern of results is shown in blue for a large radius test pattern, adding further support to this conclusion. The effect of size mismatch on matching density is supported by a 3 × 3 RM ANOVA, which showed significant change across test, $F(2, 8) = 24.75$, $p < 0.01$ $\eta^2 = 0.92$, and ref, $F(2, 8) = 28.95$, $p < 0.01$ $\eta^2 = 0.93$, radii change. There was also a significant interaction, $F(4, 8) = 10.58$, $p < 0.01$ $\eta^2 = 0.84$. The reader can appreciate this bias by inspecting the bottom row of Figure 1. Here all three patterns have the same physical density, but the reader likely perceives the larger pattern on the right as being much more dense, as our observers did. Indeed in the original experiment we sought to replicate, the measured density effect was much larger than the number effect when compared in terms of percentages.

Figure 2b shows the analogous data for number discrimination under the same conditions. Performance in each condition hovers around 100%, or veridical, indicating no bias in the judgment of number, even in cases involving size mismatch. The accompanying 3 × 3 RM ANOVA revealed no significant main effects for test, $F(2, 8) = 4.5$, $p = 0.094$ $\eta^2 = 0.69$, or ref, $F(2, 8) = 1.4$, $p = 0.34$ $\eta^2 = 0.41$, and no significant interaction was observed, $F(4, 8) = 2.03$, $p = 0.18$ $\eta^2 = 0.5$. The reader can appreciate this result by inspecting the top row of Figure 1. Here all three patterns have the same physical number: Our observers perceived these patterns as equally numerous.

Thresholds

Threshold data for density and number are shown in Figure 2c and d, respectively. The horizontal axes and colored lines are as before, but now, the vertical axes describe the proportion of extra dots/density required to raise performance from 50% to 82%, as per Dakin et al. (2011). Those authors reported higher discrimination thresholds (~40%) on their tasks than reported elsewhere (Anobile et al., 2014) for number/density discriminations (~20%), though performance can vary widely under a range of conditions (cf. Morgan,

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Raphael, Tibber, & Dakin, 2014). Here, in all conditions, and for both density (Figure 2c) and number judgments (Figure 2d), thresholds average approximately 20%. This accords with the view that the higher thresholds seen by Dakin et al. are due to parafoveal presentation of their stimuli and highlights the decrease in performance when making density or number judgments on peripherally viewed stimuli.

These findings of Experiment 1 are consistent with the findings of Dakin et al. (2011), as we discuss later. With regards to density discrimination, we concur that an area increment disrupts our estimation of density by scaling it. With regard to number discrimination, we do not find strong evidence of such a bias, although the nonsignificant trends in this data are consistent with Dakin et al.'s findings. The number bias found elsewhere was abolished here under conditions where we measured higher task precision by using an arguably stronger visual stimulus. Interestingly, for density discrimination, we find near identical biases to those reported by Dakin et al., despite observers having twice the precision (half the thresholds) on our temporal 2AFC task. This 2-D bias and threshold data now provide a direct comparison point for our main research question: Is our sense of density or number similarly disrupted by a change in volume?

Experiment 2: Mismatching volume does not bias estimates of density or number

The addition of stereoscopic cues has been shown to mediate the processing of a range of visual features and objects (cf. Bell et al., 2008; Bell, Kanji et al., 2013; Bennett & Vuong, 2006; Bulthoff, Edelman, & Tarr, 1995; Burke, 2005; Burke et al., 2007; Fendick & Westheimer, 1983; Lim Lee & Saunders, 2011; McKee & Taylor, 2010; Westheimer & Pettet, 1990). What of the mechanisms for processing number or density: Are they also sensitive to stereoscopic cues to depth? This is plausible, given that density coding mechanisms are primarily binocularly driven (Durgin, 2001). Since estimates of density and number are biased by changes in area (Dakin et al., 2011; Tibber et al., 2012), a logical extension of this is to ask if these biases occur across changes in volume.

We used an eight-mirror Wheatstone stereoscope (Wheatstone, 1838) to create the perception that the elements of the pattern were randomly dispersed throughout a cylindrical volume. We then conducted an experiment analogous to our first, but now we fixed the area of both patterns by fixing the radius at 2.8° and varied the volume of the test and ref patterns, independently, across conditions. In each condition the disparity range of the elements in the test was set to: 0′, 5′, 10′, or 20′, resulting in approximate volumes of 0 cm^3, 318 cm^3, 636 cm^3, and 1272 cm^3 (calculated in Appendix A). Each element was randomly distributed within this range, including zero depth, using a rectangular distribution (i.e., there was no modal disparity). The disparity value listed describes the total range; maximum values for each sign of disparity (crossed or uncrossed) are half of that. The disparity of the ref pattern was either set to 0′ or 10′ using the same dispersion method described above. The reader can appreciate the 3-D stimuli for themselves by free fusing the example in the upper part of Figure 4. A control experiment (described below) shows that all five of our

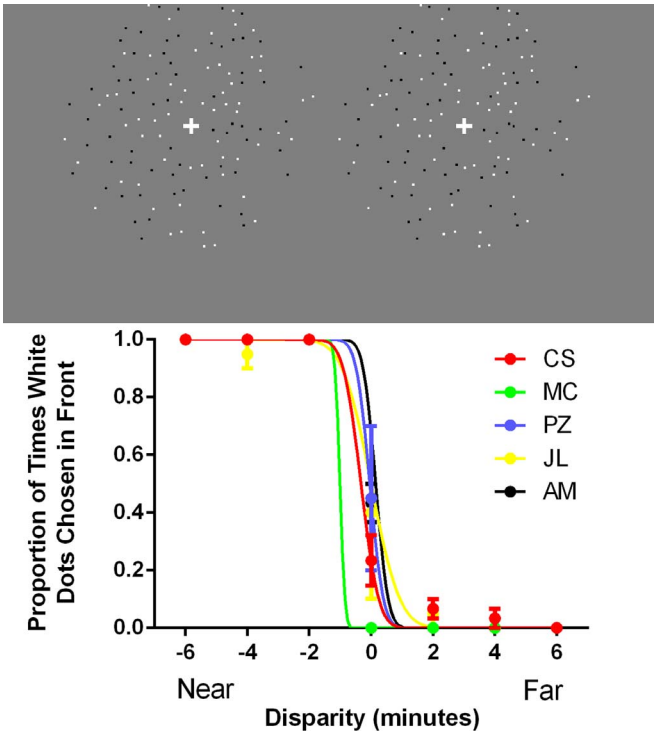

Figure 4. Stereo demonstration. If the reader can free fuse this example, the patterns in the top half of the figure, when fused, demonstrate the appearance of a pattern for which the elements are spread out across a volume. The cross serves to aid stereo fusion. As in the experiment proper, the black and white elements here are randomly distributed throughout the volume. This differs from the control below, where black and white elements were assigned opposite signs of disparity. Data: The data points describe discrimination functions for five observers on a task in which they were required to judge whether the white elements of the pattern were in front (nearer) or behind (further away) than the black elements. The horizontal axis describes the physical disparity difference between black and white elements; the vertical axis the proportion correct. The raw response data have been fit with a Cumulative Gaussian function. *Note*: Data and accompanying fits for individual observers have been nudged horizontally to avoid occlusions.

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observers could readily resolve binocular disparity in our display.

Results

Biases

Figure 3 shows results averaged across observers on tasks requiring the observer to match density (Figure 3a) or number (Figure 3b). Since density was the only judgment that showed a bias with area in our Experiment 1, we tested two additional naive observers on this task in Experiment 2, making a total of five. The horizontal axes describe the range of disparities across which the elements of the test pattern were allocated. The colored lines show data for two sets of conditions: red, where the ref pattern had zero disparity (no volume), and blue, where the elements in the ref were spread throughout a volume with a total disparity of 10 arc minutes (volume = 636 cm^3). The vertical axes show the physical 2-D density (Figure 3a) or number (Figure 3b) of the test as a percentage of the ref pattern, at the PSE.

For density (Figure 3a) and number (Figure 3b) data, the data remains veridical (matching percentages remain at 100%) across all conditions, including volume-matched (ref 0′: test 0′; and ref 10′: test 10′) and all volume-mismatched conditions. Note: veridical here is taken from the perspective of the null, whereby changing volume does not alter the perceived density or number of elements within the pattern. We consider the data for density first (left panel). The red data (left to right) show that increasing the volume containing the test pattern elements does not change, or specifically, does not reduce perceived density relative to the 2-D ref. What about cases involving a volume mismatch? Note the three rightmost blue data points. Here the ref has a disparity of 10′ (volume = 636 cm^3), giving a volumetric density of 0.2 dots/cm^3. Under volumetric density the test labeled 5′, 10′, and 20′ have double (5′ = 0.4 dots/cm^3), the same (10′ = 0.2 dots/cm^3), or half (0.1 dots/cm^3) the density of the ref, respectively. The large magnitude of our volume mismatches should be sufficient to identify a bias by volume, if it occurs. Remember, in the 2-D analogue in Experiment 1, the same magnitude mismatch in area produced density biases of approximately 50% (green data points in Figure 2).

Under purely 2-D defined density, the three conditions are, of course, all physically matched at 5.2 dots/deg^2 and therefore veridical performance is expected. Under a 3-D, or volumetric definition, the test and ref densities differ by an octave in either direction, and nonveridical performance (not 100%) is expected, if volume is important for density calculations. In fact, performance remains veridical across all three conditions, which is consistent with the patterns being matched in density by area, and *unaffected* by the physical mismatch in their density by volume (i.e., 2-D not 3-D). Consistent with this interpretation the 2 × 4 RM ANOVA showed no change in density matching across the ref, $F(1, 12) = 0.18$, $p = 0.899$, $\eta^2 = 0.005$, and test, $F(3, 12) = 1.1$, $p = 0.39$, $\eta^2 = 0.21$, disparities. There was no significant interaction, $F(3, 12) = 0.48$, $p = 0.7$, $\eta^2 = 0.11$.

For number (Figure 3b), the data are effectively identical. In short, there is no bias or change in the perceived number of the test as a function of the difference in volume between test and ref. Performance remains veridical across all conditions. Again the 2 × 4 RM ANOVA was consistent with this interpretation: ref, $F(1, 6) = 0.04$, $p = 0.95$, $\eta^2 = 0.002$; test, $F(3, 6) = 0.158$, $p = 0.92$, $\eta^2 = 0.07$, and there was no significant interaction, $F(3, 6) = 1.59$, $p = 0.29$, $\eta^2 = 0.43$.

Thresholds

Figure 3 shows thresholds for density (Figure 3c) and number (Figure 3d) averaged across observers (five for density, three for number). The horizontal axes and colored lines are as before, but now, the vertical axes describes the proportion of extra dots/density required to raise performance from 50% to 82%, as per Dakin et al. (2011). Of interest here is how the discrimination of density or number in 3-D displays compares with those for 2-D displays. The thresholds are near identical to those reported for our 2-D stimuli in Figure 2c and d (see avg in each panel), indicating that discrimination of density or number within 3-D displays is no more difficult for the observer—including cases where the test and ref are both in 3-D. In fact, task precision is perhaps even slightly higher in 3-D. Again performance is in line with previous reports of thresholds, using comparable stimuli and experimental configurations, being approximately 20%–25% (Burr & Ross, 2008b).

One obvious rebuttal of our data in Experiment 2 is that our observers simply could not resolve the disparity cues present in our display, and thus perceived no depth. We note here that, (a) all our observers reported a vivid impression of the elements of the pattern filling a cylindrical volume, and (b) all observers undertook the Titmus Stereofly test prior to testing and had stereoacuities better than 40 arc seconds. However, it seems sensible to test the stereo sensitivity of all our observers in the specific apparatus used for testing. We did so. The apparatus and stimuli were unchanged. In this control, the disparity of the white and black elements were put in opposition such that when one set had an uncrossed disparity, the other set had crossed. The magnitude of the disparity was determined by a MOCS procedure. In a single-interval 2AFC task, the observer was instructed to judge whether the white elements were in front or behind the

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black elements. A cumulative Gaussian was fit to the raw response data in order to estimate sensitivity to disparity in the dot elements of the display.

The raw data and fits are shown for each observer in Figure 4. All five observers in Experiment 2 were readily able to resolve disparity in our stimuli, with thresholds for all observers estimated to be less than 30 arc seconds. These estimates accord with the estimates for each observer using the Titmus test. Importantly, the control data show that our observers could resolve the depth in our stimulus display. Finally, no observers reported diplopia or difficulties with binocular fusion in any of our conditions, even for our 20′ disparity test, which is a max of 10′ disparity for an element in either signed direction.

In summary, Experiment 2 found no evidence that the calculation of density or of number is influenced by a change in the volume containing the elements in the display. At least for density, this represents a clear departure from the biases observed as area changes. Finally, these findings cannot be explained by any of our observers having difficulties resolving stereoscopic cues in our display.

General discussion

In this study we have shown:

- Judgments of density are strongly biased by area, as expected. Judgments of number, however, were not measurably biased by area—a deviation from the findings of previous work where the number effect was comparatively weaker than density but clearly measurable.
- Task precision for number and density judgments is affected by the stimulus configuration: foveal versus parafoveal, presentation time, and use of hard-edged dots.
- There is no evidence that judgments of number or density are biased by a large change in volume. This result has important implications for understanding the specific mechanisms processing number or density.

With regards to judgments of density within a 2-D aperture, we found near identical biases to those reported previously (Dakin et al., 2011; Tibber et al., 2012), this despite our foveal presentation method increasing task precision by a factor of two relative to Dakin et al.'s (2011) parafoveal presentation. This indicates that the magnitude of the observed density bias is not measurably affected by task precision. There are implications for the different relative susceptibilities of number and density judgments to noise within the task that we discuss later. This density bias appears to be independent from retinal position: Foveal and parafoveal presentations induced identical biases. At the same time, we did not find a bias in number across area, as other studies have (Dakin et al., 2011; Tibber et al., 2012). It is likely that the lack of a number bias by area in our study is related to the increased task precision. In support of this argument for number estimation, others have reported that number biases are subject to experimenter practice (Tokita & Ishiguchi, 2010), implying that empirically measured number biases can show context dependent changes.

In our second experiment we revealed that the calculation of number, or of density, by an observer is not swayed by the dispersion of the elements of the pattern across a cylindrical volume. This is in stark contrast to the effect of changing area, where large biases occur. This indicates that the calculation of number or density is based on the number of dots per unit area, which remained fixed across all our stereo conditions and is independent of the number of dots per unit volume, which we varied systematically. Critically, we showed that our observers could perceive a change in the disparity of the elements in the pattern, meaning that they were able to perceive depth in our displays. A previous study using transparent motion stimuli found elements assigned to the layer further from the observer to be consistently perceived as more numerous than those in the closer layer (Schütz, 2012). As this bias was not reported for static stimuli but was seen both with and without disparity, it is likely to arise following grouping performed when assigning the elements to separate depth layers. Indeed enhanced perceptual separation of transparently moving components has been found when local motion and nonmotion cues were manipulated (Meso, Durant, & Zanker, 2013). We did not expect such grouping to be applicable in the present case both because the current stimulus is static and because the volume manipulation acted on a solid volumetric unit without elements grouping into discrete layers.

Models and related neural mechanisms

The results of this study have implications for the main model of human density and number perception that captures observed biases (Dakin et al., 2011) by assuming that number *and* density can be estimated using the ratio of responses of cortical luminance-filtering mechanisms tuned to low and high spatial frequencies. The model accounts for the biases in number and density reported by Dakin et al. and therefore, also accounts for the biased density judgments under size-mismatched conditions reported in our Experiment 1. We note at the outset that there is other published work involving peripheral numerosity

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judgments that is incompatible with the framework of the Dakin et al. model (cf. Valsecchi, Toscani, & Gegenfurtner, 2013). These shortcomings arise mainly when stimulus manipulations like relative contrast changes are made. Accounting for them would require more elaborated computational approaches. However, as the principle model accounting for biases in such estimations, we look into the details of its structure and are working to better contextualize our findings. It was constructed with composite parts based on known properties of visual cortical processing. This allows us to theoretically consider the implications of the manipulations of our stimulus configurations on predicted estimation biases in the experiments reported here. The proposed correlate of density estimation for a single stimulus C is given by Equation 1.

$$C = 2^{\gamma_\sigma}\frac{R_{hi}}{R_{lo}}, \tag{1}$$

$R_{hi/lo}$ are the responses of luminance filters sensitive to high and low frequencies that are matched to the scale of the dot elements and the aperture size respectively and summed over the stimulus area. The term with the γ exponent defines the Gaussian noise term proposed by the authors to account for the uncertainty in the implementation of these filtering operations. Indeed, this step is equivalent to the noisy filters sensitive to high spatial frequency content that have been proposed elsewhere to account for stimulus dot density and relative numerosity estimates at a first approximation (Anobile et al., 2014; Morgan et al., 2014). C is used to obtain a ratio that simulates observer performance by predicting comparative density judgments between paired stimuli, a and b, given by Equation 2.

$$d_{a,b} = \frac{C_a}{C_b}, \tag{2}$$

$$d_{a,b} = \left[\frac{2^{\gamma_a}}{2^{\gamma_b}} \times \frac{R_{hi(a)}}{R_{lo(a)}} \times \frac{R_{lo(b)}}{R_{hi(b)}}\right], \tag{3}$$

Expanding Equation 2 in terms of Equation 1 to get the longer expression of Equation 3 allows us to make two inferences. First, if the Gaussian noise terms for the stimuli a and b, the first ratio, reflect a task performed at similar signal strength (i.e., $\gamma_a \approx \gamma_b$), *the density bias will be independent of the task stimulus strength variations that depend on configuration* (e.g., foveal vs. parafoveal). Second, for similarly sized dot elements of equal number being compared, the high-frequency filter outputs will be the same or close ($R_{hi(a)} \approx R_{hi(b)}$), and density estimates will therefore *only scale with the relative aperture size*. These observations are borne out in our faithful replication of the density bias despite a large change in precision resulting from the task configuration.

For number judgments, an additional scaling of the comparative density estimates of Equation 2 by a ratio of the two compared low-frequency filter outputs corresponding to the aperture scale ($R_{lo(a)}/R_{lo(b)}$) and the inclusion of more Gaussian noise terms is proposed, resulting in Equation 4.

$$n_{a,b} = \left(2^{\gamma_\sigma}\frac{R_{lo(a)}}{R_{lo(b)}}\right)^{2^{\gamma_S}} \left[\frac{2^{\gamma_a}}{2^{\gamma_b}} \times \frac{R_{hi(a)}}{R_{lo(a)}} \times \frac{R_{lo(b)}}{R_{hi(b)}}\right], \tag{4}$$

The form of Equation 4 suggests that number estimation biases, unlike density biases, are more susceptible to noise as a result of the inclusion of the terms σ and S, which serve to correct the area bias observed in density estimates with a particularly strong correction when the stimulus is stronger. As such, the bias in number estimates can be expected to depend strongly on the signal-to-noise ratio (SNR) at which the task is performed. At a given signal level (simulated by σ and S) it follows that the aperture ratio term ($R_{lo(a)}/R_{lo(b)}$) could attain a value that cancels out the effect of the reciprocal of the same ratio within the square brackets, thus eliminating the bias in number estimates with area. The changes made from previous experiments to the current experimental configuration that facilitated the stereo depth task had the effect of eliminating the number estimation bias. The longer presentation time, the use of hard-edged dots, and the foveal presentation collectively reduced the inherent noise in the stimulus as predicted in the Dakin et al. (2011) model, subsequently eliminating the bias in number estimates.

The area results are therefore consistent with those of previous studies, and we now ask how the mechanisms proposed by the modeling could be related to the volumetric manipulation. The model does not purport to make predictions about perceived density or number across changes in the volume, only across area. The low and high SF content in each monocular image is of course unaffected by the introduction of binocular disparity cues, ensuring the model in its current form remains agnostic to volume changes. Extending the logic applied to the initial model construction to 3-D, one way a volumetric dependence could be added would be through the inclusion of a depth term (e.g., $D + 1$) from the spatially summed outputs of dot disparity-sensitive filtering performed on the stimulus area, which is multiplied by the denominator of the ratio in Equation 1. Such a multiplication makes a volumetric estimation of density by combining the area of the cylinder's face and its effective depth. As such it would leave the area predictions unaltered for $D = 0$. We did not find evidence of any bias caused by stereo-based depth on density estimation and therefore conclude that density estimation shows biases fully

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explained by the planar filtering mechanism occurring within the retinotopic representation of the stimuli as proposed by Dakin et al. (2011).

Numerosity biases resulting in lower estimates in the foreground that were reported for transparently moving dots are relevant for the broader understanding of the current results (Schütz, 2012). The finding suggested that differences between the perceptually separated planes of elements require additional complex hierarchical global processing of the components beyond the localized detection of motion (Qian, Andersen, & Adelson, 1994). This is consistent with previous observations from transparently moving disc stimuli that component separation towards independent representations could be supported by adding cues that accentuated differences in component appearance (Meso et al., 2013). When perceptual grouping of moving stimuli occurs, this appears to facilitate the biases in relative numerosity judgments (Schütz, 2012; Watamaniuk, 1992). The direction of such biases is consistent with reported size constancy effects (Raphael, Dillenburger, & Morgan, 2013). Therefore, shifts in perceived numerosity across depth are contingent on the presence of separate perceived layers, disappearing when a volume is used instead as we have reported here.

What of the broader theoretical question: number or density? Without definitively resolving the question, which goes beyond the scope of this study, we believe that our findings make an important addition to this literature, significantly adding to our understanding of the input parameters to these calculations. In any case, recent research suggests that the question should perhaps be reframed. Anobile et al. (2014) proposed that two regimes exist, with observers sensing number directly when dot density is below 0.25 dots/deg^2 (i.e., small numbers per area) and performing a density calculation at higher densities (>0.25 dots/deg^2). We note that our lowest density in this study (in 2-D space) is 2.5 dots/deg^2, which is beyond the proposed limit of the number regime, and according to our calculations, beyond the highest density tested by Anobile et al. (maximum tested density of approximately 1 dot/deg^2). This may go some way towards explaining why our thresholds are higher than those reported by Anobile et al., but the differences in procedure and stimulus characteristics makes any direct comparison difficult. We can say, however, that according to Anobile et al., our judgments and those in the Dakin et al. (2011) study, were therefore performed under the density regime. Future studies could explore whether the subitizing range can be extended by dispersing the items across a range of depths. In other modalities such as motion, one can increase the number of transparent motion directions that can be detected simultaneously by presenting the motion signals in different depth planes (Greenwood & Edwards, 2006; Snowden & Rossiter, 1999).

In summary, we have shown a bias in perceived density as area changes, the magnitude of which is not directly affected by the precision with which the patterns can be discriminated: at least across a factor of two change in task precision. We did not measure a bias in number judgments as a function of area. These results are entirely consistent with a previous model that predicts larger SNR invariant biases for density than for number judgments. Our density results were identical to earlier work. Number judgment was found to show no biases with changing area under the higher SNR task configuration, suggesting that the underlying estimation procedure was more susceptible to noise levels. Next we showed that human observers can easily discriminate density and number for patterns containing dots that are distributed in depth. However, neither perceived density nor number is affected by the dispersion of the dots throughout a cylindrical volume. The current findings suggest that the mechanism behind the judgment biases measured is based on previously proposed multiscale luminance filter ratio computations occurring on the 2-D retinotopic representation of the stimulus within visual cortex only. Biases, therefore, do not have an ecological basis that can extend to apply analogously to include cortical volume estimates important in navigating the 3-D world.

Keywords: number, density, three-dimensional, stereo, two-dimensional

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Appendix A

Stimulus cylinder depth estimates from disparity

Depth is added to the stimuli by varying the range of binocular disparities of the dots displayed within the circular aperture to create a perceived cylindrical volume. Here we adapt a derivation of the relationship between point disparity and perceived depth used by Scarfe and Hibbard (2006) to estimate the cylindrical depth of the stimuli used in the current experiment. In a schematic of the current stimulus configuration is shown in Figure 5, a slight modification of Scarfe and Hibbard. In this figure, the point of fixation is given by **P** and points take as the left and right eyes are given by **LE** and **RE**. Using trigonometric ratios to derive the disparity from the difference in angles subtended by each eye (θ_{LE}, θ_{RE}), Scarfe and Hibbard showed that the tangent of the disparity we call s here could be well approximated by an expression including the ratio of the depth created by the disparity d and the distance from the center of the eyes to the point of fixation D, when squared. We use this as our starting point in Equation 5.

$$\tan(s) \approx \frac{Id}{D^2}. \tag{5}$$

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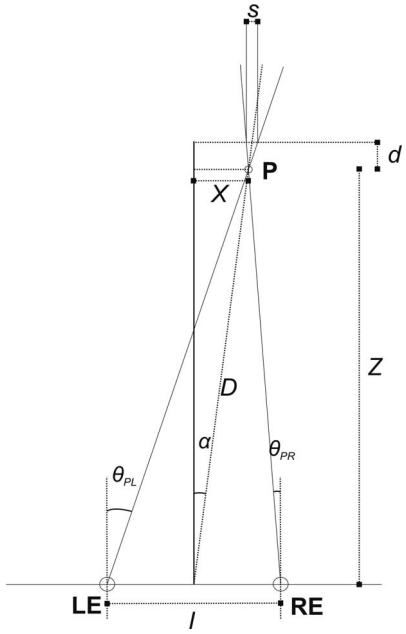

Figure 5. A schematic representation of the geometry of the depth added to a stimulus image by creating a disparity between a dot in the right eye (RE) and the left eye (LE), adapted from Scarfe and Hibbard (2006). The observer is at a distance **Z** from the point of fixation **P**, which subtends different angles from his two eyes. The depth *d* can be derived from trigonometric rules (see text for details) in terms of its relationship to the disparity between the two eyes *s*.

This remains a good approximation while D is much larger than both I, the distance between the eyes, and d. Both remain good assumptions for the psychophysical task done at 115 cm by human observers with an interocular distance of about 6 cm. We rearrange this Equation 5 to obtain an expression for the perceived depth d for a given stimulus disparity s in Equation 6. With the maximum radius from fixation of 4 cm tested (i.e., X in Figure 5), we can be sure that $D \approx Z$, or 115 cm because at that distance the maximum difference between D and Z is under 0.07 cm (from $D^2 = Z^2 + X^2$). We therefore have an expression for d in experimentally measured terms in Equation 6.

$$d \approx \frac{Z^2 \tan(s)}{I} \tag{6}$$

We use Equation 6 to derive physical depths for the disparity range of s (0′, 5′, 10′, and 20′). The interocular distance I is 6 cm and the viewing distance Z is 115 cm. The tested range of depths d is therefore 0, 3.2, 6.4, and 12.8 cm. From these, the cylindrical volume of the resulting stimuli is $\pi r^2 d$, where r is the radius of the circular aperture, which is 5.6 cm in Experiment 2. For the four tested depths, the resulting cylinders have volume 0, 318, 636, and 1272 cm^2.

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