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Extending Removed Sets Revision to partially preordered belief bases

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Most of belief revision operations have been proposed for totally preordrered information. However, in case of partial ignorance, pieces of information are partially preordered and few effective approaches of revision have been proposed. The paper presents a new framework for revising partially preordered information, called Partially Preordered Removed Sets Revision (PPRSR). The notion of removed set, initially defined in the context of the revision of non ordered or totally preordered information is extended to partial preorders. The removed sets are efficiently computed thanks to a suitable encoding of the revision problem into logic programming with answer set semantics. This framework captures the possibilistic revision of partially preordered information and allows for implementing it with ASP. Finally, it shows how PPRSR can be applied to a real application of the VENUS european project.

1. Introduction

Belief revision has been extensively studied in the domain of knowledge representation for artificial intelligence, mainly for totally preordered information. A characterization of belief revision has been provided by Alchourron, Gärdenfors, Makinson (AGM) with a set of postulates that any revision operation should satisfy [22]. Katsuno and Mendelzon (KM) reformulated AGM's postulates and provided a representation theorem that characterizes revision operations based on total preorders [30]. Belief revision has been discussed within different frameworks (probabillity theory, Sphon's conditional functions, Grove's system of spheres, ...) and several revision operations have been proposed like the possibilistic revision [20,21] or adjustment revision [46], the linear-based revision [38], the natural revision [12], the lexicographic-based revision [1,37,34], the revision with memory [2,40], and more recently several works on iterared revision [10,28,11,19]. As pointed out by Delgrande et al. [18], the different approaches proposed in the literature can be classified according to three different points of view. Given a plausibility ordering on interpretations describing the background knowledge and a new piece of information, belief revision as defeasible inference (BRDI) amounts to find the most plausible interpretation satisfying the input information, belief revision as incorporation of evidence (BRIE), amounts to change the plausibility ordering in presence of a new piece of information and belief revision of background knowledge (BRBK) means revising the background knowledge by a generic information.

In order to provide effective revision operations, strategies have to be chosen. When a new piece of information is inconsistent with the initial beliefs, the revision problem is related to the problem of consistency restoration. This problem has been addressed according to three main families of approaches: construction of preferred consistent (or maximal consistent)

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subsets of formulae [42,1,15], forgetting some variables responsible of the inconsistency [36,33,9], inconsistency minimization [41,14,25,48].

Some approaches have been implemented [47,17], among them, Removed Sets Revision which has been initially proposed in [48] for revising a set of propositional formulae. Contrary to consistency maximization, this approach stems from removing a minimal number of formulae, called removed set, to restore consistency. The Removed Sets Revision (RSR) and then a prioritized form of Removed Sets Revision, called Prioritized Removed Sets Revision (PRSR) [8] have been encoded into answer set programming and allowed for solving a practical revision problem coming from a real application in the framework of geographical information system.

However in some applications, an agent has not always a total preorder between situations at his disposal, but is only able to define a partial preorder between situations, particularly in case of partial ignorance and incomplete information. In such cases, an epistemic state can be represented by either a partial preorder on interpretations or a partially preordered belief base.

The revision of partially preordered information has been less investigated in the literature, however Lagrue and coworkers [6] pointed out that the KM's postulates are not appropriate for partial preorders and proposed a suitable definition of faithful assignment, called P-faithful assignment, a new set of postulates and a representation theorem. Some revision operations initially defined for total preorders, such as revision with memory and possibilistic revision have been successfully extended to partial preorders [4].

This paper proposes a new framework for revising partially preordered information and provides an efficient implementation thanks to answer set programming. The main contributions of this paper are the following:

- It extends the Removed Sets Revision to partially preordered information, called Partially Preordered Removed Sets Revision (PPRSR). The paper shows how the notion of removed set, roughly speaking, the subsets of formulae to remove to restore consistency, initially defined in the context of non ordered [48] or totally ordered [8] information is extended to the case of the revision of partially preordered information.
- It provides an implementation of PPRSR with ASP. The revision problem is translated into a logic program with answer set semantics and a one-to-one correspondence between removed sets and preferred answer sets is shown. The computation of answer sets is performed with any ASP solver.
- It shows that the possibilistic revision of partially preordered information can be captured within the PPRSR framework allowing for an efficient implementation with ASP.

The rest of this paper is organized as follows. Section 2 fixes the notations and gives a refresher on the Removed Sets Revision (RSR), on answer set programming and on partial preorders. Section 3 presents the Partially Preordered Removed Set Revision (PPRSR) and shows how it captures the possibilistic revision. Section 4 details the encoding of PPRSR into logic programming with answer set semantics and the computation of answer sets thanks to ASP solvers. It then shows the one-to-one correspondence between removed sets and preferred answer sets. Section 5 illustrates how PPRSR can be applied in the context of the VENUS project before concluding.

2. Background and notations

2.1. Notations

In this paper we use propositional calculus, denoted by $\mathcal{L}_{\mathcal{PC}}$, as knowledge representation language with usual connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$. Let X be a set of propositional formulae, we denote by $Cons(X)$ the set of logical consequences of X. We denote by $\mathcal{W}$ the set of interpretations of $\mathcal{L}_{\mathcal{PC}}$ and by $Mod(\psi)$ the set of models of a formula ψ, that is $Mod(\psi) = \{\omega \in \mathcal{W}, \omega \models \psi\}$ where $\models$ denotes the inference relation used for drawing conclusions.

2.2. Removed Sets Revision

We briefly recall the Removed Sets Revision approach. Removed Sets Revision [48] deals with the revision of a set of propositional formulae by a set of propositional formulae.[1] Let K and A be finite sets of clauses. Removed Sets Revision (RSR) focuses on the minimal subsets of clauses to remove from K, called *removed sets*, in order to restore the consistency of $K \cup A$. More formally,

Definition 1. Let K and A be two consistent sets of clauses such that $K \cup A$ is inconsistent. R a subset of clauses of K, is a removed set of $K \cup A$ iff

(i) $(K \backslash R) \cup A$ is consistent;
(ii) $\forall R' \subseteq K$, if $(K \backslash R') \cup A$ is consistent then $|R| \leqslant |R'|$[2].

Let denote by $\mathcal{R}(K \cup A)$ the collection of removed sets of $K \cup A$, RSR is defined as follows:

[1] The initial approach considers propositional formulae in their equivalent conjunctive normal form (CNF).
[2] $|R|$ denotes the number of clauses of R.

Definition 2. Let K and A be two consistent sets of clauses,

$$K \circ_{RSR} A =_{def} \bigvee_{R \in \mathcal{R}(K \cup A)} Cons((K \setminus R) \cup A).$$

According to a semantic point of view, $|\mathcal{NS}_K(\omega)|$ denotes the number of clauses of K falsified by an interpretation ω and a total preorder on interpretations is defined by:

Definition 3. $\omega_i \leqslant_K \omega_j$ iff $|\mathcal{NS}_K(\omega_i)| \leqslant |\mathcal{NS}_K(\omega_j)|$.

Removed Sets Revision can be semantically defined by:

Definition 4. Let K and A be two consistent sets of clauses,

$$Mod(K \circ_{RSR_{sem}} A) = min(Mod(A), \leqslant_K).$$

It minimizes the number of clauses falsified by the models of A and $Mod(K \circ_{RSR} A) = Mod(K \circ_{RSR_{sem}} A)$. In case of prioritized belief bases, RSR has been extended to Prioritized Removed Sets Revision (PRSR) [8].

2.3. Partial preorders

A partial preorder, denoted by $\preceq$ on a set A is a reflexive and transitive binary relation. Let x and y be two members of A, the equality is defined by $x = y$ iff $x \preceq y$ and $y \preceq x$. The corresponding strict partial preorder, denoted by $\prec$, is such that, $x \prec y$ iff $x \preceq y$ holds but $x = y$ does not hold. We denote by $\sim$ the incomparability relation $x \sim y$ iff $x \preceq y$ does not hold nor $y \preceq x$. The set of minimal elements of A with respect to $\prec$, denoted by $Min(A, \prec)$, is defined as: $Min(A, \prec) = \{x \in A, \nexists y \in A : y \prec x\}$.

An epistemic state allows for encoding the agent's beliefs but also for encoding its relative confidence in alternative possible states of the world. Epistemic states can be represented by several means: preorders on interpretations [16,34], conditionals [13,16], epistemic entrenchments [45,37], prioritized belief bases [2,3], ... When focusing on representations by means of preorders, epistemic states are generally represented by total preorders on interpretations, however, as mentioned in the introduction, in case of partial ignorance, the agent is unable to compare all situations between them and a partial preorder seems to be more suitable to represent epistemic states.

Katsuno and Mendelzon [30] proposed a set of postulates and a representation theorem that characterize revision operations based on partial preorders. However, the proposed approach is not satisfactory since only one class of partial preorders can be revised [6]. In particular, the concept of faithfull assignment defined in [30] is unable to represent all partial preorders on interpretations. Let Ψ be an epistemic state represented by a partial preorder on interpretations, denoted by $\preceq_\Psi$ and let $Bel(\Psi)$ be its corresponding belief set. An alternative definition of faithfull assignment, called P-faithfull assignment, is proposed in [6] as follows.

Definition 5. Let $Bel(\Psi) = min(\mathcal{W}, \prec_\Psi), \preceq_\Psi$ is a P-faifhful assignment if

1. if ω, $\omega' \models Bel(\Psi)$ then $\omega \prec_\Psi \omega'$ does not hold,
2. if $\omega' \not\models Bel(\Psi)$, then there exists ω such that $\omega \models Bel(\Psi)$ and $\omega \prec_\Psi \omega'$,
3. if $\Psi = \Phi$ then $\preceq_\Psi = \preceq_\Phi$.

Moreover, [6] gives a set of postulates an operation $\circ$ has to satisfy and a representation theorem such that $Mod(Bel(\Psi \circ \mu)) = min(Mod(\mu), \preceq_\Psi)$. An alternative syntactic but equivalent representation of an epistemic state, Ψ is a partially preordered belief base, denoted by $(\Sigma, \preceq_\Sigma)$, where Σ is a set of propositional formulae, and $\preceq_\Sigma$ is a partial preorder on the formulae of Σ. Several ways of defining a partial preorder on subsets of formulae belonging to Σ, called comparators, from a partial preorder on a set of formulae Σ have been proposed: inclusion-based [29], possibilistic [5], lexicographic [49] comparators. They are such that the preferred formulae are kept in the belief base. In our approach, according to the Removed Sets strategy, we adopt a dual point of view in the sense that we want to prefer the subsets of formulae to remove. For example, we rephrase the possibilistic comparator (or weak comparator) used in [5], already defined in [35] and reused by Halpern [26] as follows. Y is preferred to X if for each element of Y, there exists at least one element of X which is preferred to it, more formally:

Definition 6. Let $\preceq_\Sigma$ be a partial preorder on Σ, $Y \subseteq \Sigma$ and $X \subseteq \Sigma$.

Y is preferred to X, denoted by $Y \trianglelefteq_w X$ iff $\forall\, y \in Y, \exists x \in X$ such that $x \preceq_\Sigma y$.

We now briefly recall the extension of the semantic possibilistic revision to partial preorders [4]. The semantics of possibilistic logic stems from the notion of possibility distribution [20], denoted by π, which is a function from $\mathcal{W}$, to $[0,1]$. $\pi(\omega)$ evaluates to what extent ω is compatible, or consistent with the available knowledge. $\pi(\omega) = 0$ means that ω is impossible, while $\pi(\omega) = 1$ means that ω is totally possible. $\pi(\omega') \preceq \pi(\omega)$ means that ω is more plausible than ω'. A possibility distribution is said to be normalized or consistent if there exists an interpretation ω such that $\pi(\omega) = 1$.

An epistemic state Ψ is represented by $(\mathcal{W}, \preceq_\Psi)$ where the partial preorder on interpretations $\preceq_\Psi$ is associated to a possibility distribution as follows: $\forall \omega, \omega' \in \mathcal{W}, \omega \preceq_\Psi \omega'$ iff $\pi(\omega') \preceq \pi(\omega)$. The possibilistic revision of Ψ by a propositional

formula μ leads to the epistemic state $\Psi \circ_\pi \mu$, represented by $(\mathcal{W}, \preceq_{\Psi \circ_\pi \mu})$ which considers all the counter-models of μ as impossible and preserves the relative ordering between the models of μ. More formally,

Definition 7. $\Psi \circ_\pi \mu$ corresponds to the following partial preorder:

1. if $\omega, \omega' \in Mod(\mu)$ then $\omega \preceq_{\Psi \circ_\pi \mu} \omega'$ iff $\omega \preceq_\Psi \omega'$,
2. if $\omega, \omega' \notin Mod(\mu)$ then $\omega =_{\Psi \circ_\pi \mu} \omega'$,
3. if $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega \prec_{\Psi \circ_\pi \mu} \omega'$.

2.4. Answer sets

A *normal logic program* is a set of rules of the form $c \leftarrow a_1, \ldots, a_n, not\, b_1, \ldots, not\, b_m$ where c, $a_i (1 \leqslant i \leqslant n)$, $b_j (1 \leqslant j \leqslant m)$ are propositional atoms and the symbol *not* stands for *negation as failure*. For a rule r like above, we introduce $head(r) = c$ and $body(r) = \{a_1, \ldots, a_n, b_1, \ldots, b_m\}$. Furthermore, let $body^+(r) = \{a_1, \ldots, a_n\}$ denotes the set of positive body atoms and $body^-(r) = \{b_1, \ldots, b_m\}$ the set of negative body atoms, and $body(r) = body^+(r) \cup body^-(r)$. Let r be a rule, r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative body atoms in the body of r.

A set of atoms X is *closed under* a basic program P iff for any rule $r \in P$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program P is denoted by $CN(P)$. The *reduct* or Gelfond–Lifschitz transformation [24], P^X of a program P relatively to a set X of atoms is defined by $P^X = \{r^+ | r \in P \text{ and } body^-(r) \cap X = \emptyset\}$. A set of atoms X is an *answer set* of P iff $CN(P^X) = X$.

3. Partially Preordered Removed Sets Revision (PPRSR)

Let Ψ be an epistemic state for partially preordered information and $Bel(\Psi)$ its corresponding belief set. Ψ can be interpreted according either a syntactic or a semantic point of view. Ψ is syntactically represented by $(\Sigma, \preceq_\Sigma)$ where Σ is a finite set of arbitrary formulae and $\preceq_\Sigma$ is a partial preorder on Σ. Moreover, Ψ can also be semantically represented as a partial preorder on interpretations such that the models of $Bel(\Psi)$ are minimal with respect to this partial preorder.

3.1. The PPRSR framework

Revising partially preordered belief bases involves the definition of a partial preorder on subsets of formulae, called comparators [5,49]. We first present a general framework, Partially Preordered Removed Sets Revision (PPRSR) without specifying a particular comparator. We start with the syntactic approach then with the semantic one for revising an epistemic state Ψ by a formula μ.

PPRSR Syntactic Approach. The original Removed Sets Revision was proposed for revising belief bases consisting of propositional formulae in conjunctive normal form, we now extend Removed Sets Revision to arbitrary formulae. Obviously, the Removed Sets Revision of arbitrary formulae does not lead to the same results than the Removed Sets Revision of formulae in CNF. However, only considering formulae in CNF, we loose the syntactic structure of the initial beliefs. Moreover, in some applications, it does not make sense to only remove part of formulae to restore consistency. In [32] the approach concentrated to the construction of maximal consistent subbases from the initial belief base while our strategy is to focus on subsets of formulae to remove to restore consistency. More precisely, our strategy is to determine the preferred subsets (with respect to the initial preorder on formulae) to remove, called removed sets.

According to the syntactic point of view, the revision of the epistemic state Ψ by a formula μ leads to an epistemic state denoted by $\Psi \circ_{\trianglelefteq_C} \mu$ and represented by a new partially preordered belief base $(\Sigma \circ_{\trianglelefteq_C} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu})$ where the partial preorder $\preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}$ is such that μ is preferred to any formula of Σ and the relative ordering between the formulae of Σ is preserved. More formally,

Definition 8. Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ, the revision of Ψ by a formula μ leads to the revised epistemic state denoted by $\Psi \circ_{\trianglelefteq_C} \mu$ represented by a partially preordered belief base $\left(\Sigma \circ_{\trianglelefteq_C} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}\right)$ where

- $\Sigma \circ_{\trianglelefteq_C} \mu = \Sigma \cup \{\mu\}$,
- $\preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}$: $(i) \forall \psi \in \Sigma, \mu \prec_{\Sigma \circ_{\trianglelefteq_C} \mu} \psi$ and $(ii) \forall \psi,\ \phi \in \Sigma, \psi \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu} \phi$ iff $\psi \preceq_\Sigma \phi$.

Since $\Sigma \cup \{\mu\}$ may be inconsistent, we have to provide the consistent belief set, denoted by $Bel(\Psi \circ_{\trianglelefteq_C} \mu)$, corresponding to the revised epistemic state. In order to syntactically compute $Bel(\Psi \circ_{\trianglelefteq_C} \mu)$ we focus on the preferred subsets of formulae, with respect to the initial partial preorder, to remove from Σ, in order to restore consistency. We first define the potential removed sets as follows:

Definition 9. Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ. Let μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. R, a subset of formulae of Σ, is a potential removed set of $\Sigma \cup \{\mu\}$ iff $(\Sigma \backslash R) \cup \{\mu\}$ is consistent.

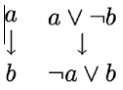

Fig. 1. Partial preorder on formulae of Σ.

Example 1. Let $\Sigma = \{a, b, a \vee \neg b, \neg a \vee b\}$ and $\preceq_\Sigma$ be a given partial preorder illustrated by Fig. 1 where $b \leftarrow a$ means that $b \prec_\Sigma a$. We revise Σ by $\mu = \neg a \vee \neg b$. $\Sigma \cup \{\mu\}$ is inconsistent. The potential removed sets are $R_0 = \{a, a \vee \neg b\}$, $R_1 = \{a, a \vee \neg b, \neg a \vee b\}$, $R_2 = \{a, b, a \vee \neg b\}$, $R_3 = \{a, b, a \vee \neg b, \neg a \vee b\}$, $R_4 = \{b, \neg a \vee b\}$, $R_5 = \{b, a \vee \neg b, \neg a \vee b\}$, $R_6 = \{a, b, \neg a \vee b\}$, $R_7 = \{a, b\}$.

Let $\mathcal{R}(\Sigma \cup \{\mu\})$ be the set of potential removed sets. Among them, we want to prefer the potential removed sets which allow us to remove the formulae that are not preferred according to $\preceq_\Sigma$. This leads to define a partial preorder on subsets of formulae of Σ, called comparator [5,49], denoted by $\trianglelefteq_C$. We now generalize the notion of removed sets to subsets of partially preordered formulae. We denote by $\mathcal{R}_C(\Sigma \cup \{\mu\})$ the set of removed sets of $\Sigma \cup \{\mu\}$.

Definition 10. Let $(\Sigma, \preceq_\Sigma)$ be a syntactic representation of Ψ. Let μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. $R \subseteq \Sigma$ is a removed set of $\Sigma \cup \{\mu\}$ iff

1. R is a potential removed set.
2. $\nexists R' \in \mathcal{R}(\Sigma \cup \{\mu\})$ such that $R' \subset R$.
3. $\nexists R' \in \mathcal{R}(\Sigma \cup \{\mu\})$ such that $R' \triangleleft_C R$.

Example 2. In the examples, we will use the weak comparator, denoted by $\trianglelefteq_w$ and defined in Section 2.3. We have $R_0 \trianglelefteq_w R_1$ because $a \preceq_\Sigma a$ and $\neg a \vee b \preceq_\Sigma a \vee \neg b$. The partial preorder on the potential removed sets is illustrated in Fig. 2.

Remark. We could refine the notion of removed set with an extra preference according to a strategy P (cardinality or minimality). $\mathcal{R}_{C,P}(\Sigma \cup \{\mu\})$ denotes the set of removed sets of $\Sigma \cup \{\mu\}$ according to the strategy P. In this case, a preferred removed set according to a strategy P is a removed set R such that $\nexists R' \in \mathcal{R}_C(\Sigma \cup \{\mu\})$ such that $R' <_P R$. According to the cardinality, $R_Y \leqslant_{CARD} R_X$ iff $|R_Y| \leqslant |R_X|$ with $|X|$ the cardinality of the set X. According to the minimality, $R_Y \leqslant_{MIN} R_X$ iff $|R_Y \cap MIN| \leqslant |R_X \cap MIN|$ with $MIN = \{x | x \in \Sigma, \nexists y \in \Sigma, y \prec_\Sigma x\}$.

Example 3. We can apply strategies: $\mathcal{R}_{w,CARD}(\Sigma \cup \{\mu\}) = \{R_0, R_7\}$ and $\mathcal{R}_{w,MIN}(\Sigma \cup \{\mu\}) = \{R_0\}$.

Stemming from the above notion of removed set, the belief set corresponding to the revised epistemic state, denoted by $Bel(\Psi \circ_{\trianglelefteq_C} \mu)$, is defined as follows:

Definition 11. Let $(\Sigma, \preceq_\Sigma)$ be the syntactic representation of Ψ and let μ be a formula, the belief set corresponding to the revised epistemic state $\Psi \circ_{\trianglelefteq_C} \mu$ is $Bel(\Psi \circ_{\trianglelefteq_C} \mu) = \bigvee_{R \in R_C(\Sigma \cup \{\mu\})} Cons((\Sigma \setminus R) \cup \{\mu\})$.

Example 4. According to the Example 1, Ψ is syntactically represented by $(\Sigma, \preceq_\Sigma)$ and revising by μ using the weak comparator gives:

$$\Sigma \circ_{\trianglelefteq_w} \mu = \{a, b, a \vee \neg b, \neg a \vee b, \neg a \vee \neg b\},$$

$Bel(\Psi \circ_{\trianglelefteq_w} \mu) = Cons(\{b, \neg a \vee b, \neg a \vee \neg b\}) \vee Cons(\{a \vee \neg b, \neg a \vee b, \neg a \vee \neg b\})$ and $\preceq_{\Sigma \circ_{\trianglelefteq_w} \mu}$ as shown by Fig. 3.

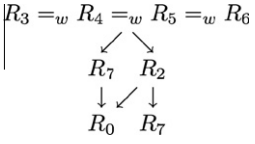

Fig. 2. Partial preorder on the potential removed sets.

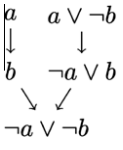

Fig. 3. $\preceq_{\Sigma \circ_{\trianglelefteq_w} \mu}$.

PPRSR semantic approach According to a semantic point of view, the epistemic state Ψ can be equivalently represented by a partial preorder on interpretations such that $Mod(Bel(\Psi))$ is minimal in this preorder. We now construct a partial preorder on interpretations applying the comparator to the set of formulae of Σ falsified by the interpretations. Let ω be an interpretation, $F_\Sigma(\omega)$ denotes the set of formulae of Σ falsified by ω. More formally,

Definition 12. Let Ψ be an epistemic state syntactically represented by $(\Sigma, \preceq_\Sigma)$, the partial preorder $\preceq^C_\Psi$ on interpretations is such that:

$$\forall \omega,\ \omega' \in \mathcal{W}, \quad \omega \preceq^C_\Psi \omega' \quad \text{iff} \quad F_\Sigma(\omega) \trianglelefteq_C F_\Sigma(\omega').$$

Using this definition, the semantic representation of Ψ is $(\mathcal{W}, \preceq^C_\Psi)$ and is such that $Mod(Bel(\Psi)) = min(\mathcal{W}, \preceq^C_\Psi)$. We are now able to define the semantic counterpart of PPRSR as follows:

Definition 13. Let $(\mathcal{W}, \preceq^C_\Psi)$ be a semantic representation of Ψ, the revision of Ψ by a formula μ leads to the semantically revised epistemic state $\Psi \circ^{sem}_{\trianglelefteq_C} \mu$ such that

$$Mod\Big(Bel(\Psi \circ^{sem}_{\trianglelefteq_C} \mu)\Big) = min\big(Mod(\mu), \preceq^C_\Psi\big).$$

The revised epistemic state is semantically represented by $\left(\mathcal{W}, \preceq^C_{\Psi \circ^{sem}_{\trianglelefteq_C} \mu}\right)$, where $\preceq^C_{\Psi \circ^{sem}_{\trianglelefteq_C} \mu}$ is a new partial preorder on interpretations, as illustrated in Fig. 4a). This partial preorder is defined from the sets of formulas belonging to $\Sigma \cup \{\mu\}$ falsified by the interpretations. More formally:

Definition 14. Let $\left(\Sigma \circ_{\trianglelefteq_C} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_C} \mu}\right)$ be the representation of the syntactically revised epistemic state. The partial preorder on interpretations $\preceq^C_{\Psi \circ^{sem}_{\trianglelefteq_C} \mu}$ is such that

$$\forall \omega,\ \omega' \in \mathcal{W}, \quad \omega \preceq^C_{\Psi \circ^{sem}_{\trianglelefteq_C} \mu} \omega' \quad \text{iff} \quad F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega) \trianglelefteq_C F_{\Sigma \circ_{\trianglelefteq_C} \mu}(\omega').$$

PPRSR agrees with the notion of P-faithful assignment and the following proposition holds.

Proposition 1. *Let Ψ be an epistemic state and $\preceq^C_\Psi$ be a partial preorder on $\mathcal{W}$ associated to Ψ. Then, $\preceq^C_\Psi$ is a P-faithful assignment.*

The proof of the Proposition 1 (provided in Appendix A) follows from the construction of partial preorder $\preceq^C_\Psi$.

According to the Proposition 1, the revision operation $\circ^{sem}_{\trianglelefteq_C}$ satisfies the postulates $P_1 - P_7$ proposed in [6] that extend the KM-postulates to the revision of partially preordered belief bases.

3.2. PPRSR in the case of the weak comparator

We now focus on the weak comparator $\trianglelefteq_w$ defined in Section 2.3. We show that the semantic counterpart captures the extension of possibilistic revision to partial preorders. Moreover, we refine the semantic counterpart given in Definition 13, in order to provide an equivalence between the PPRSR syntactic and semantic approaches.

When we select the weak comparator the PPRSR framework can capture the possibilistic revision, illustrated in Fig. 4b, recalled in Section 2.3 and the following proposition holds.

Proposition 2. *Let $\circ_\pi$ be the possibilistic revision operator.*

$$\forall \omega,\ \omega' \in \mathcal{W},\ \omega \preceq^w_{\Psi \circ^{sem}_{\trianglelefteq_w} \mu} \omega' \text{ iff } \omega \preceq_{\Psi \circ_\pi \mu} \omega'.$$

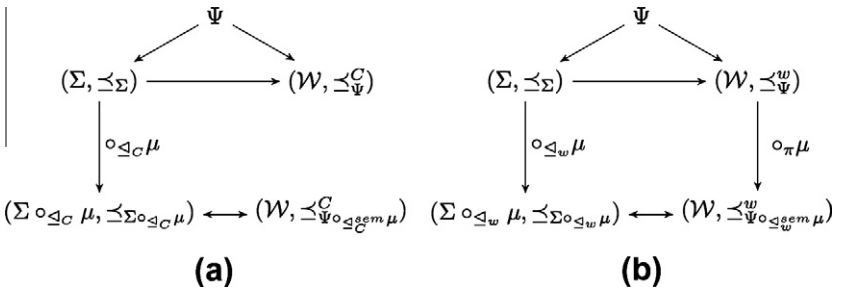

Fig. 4. The syntactic and semantic representation of the revised epistemic state.

Table 1
The sets of formulae falsified by the interpretations.

ω_i	a	b	$F_{\Sigma(\omega_i)}$	$F_{\Sigma_{\circ_{\preceq_\omega}}\mu(\omega_i)}$
ω_0	$\neg a$	$\neg b$	$\{a,b\}$	$\{a,b\}$
ω_1	$\neg a$	b	$\{a, a \vee \neg b\}$	$\{a, a \vee \neg b\}$
ω_2	a	$\neg b$	$\{b, \neg a \vee b\}$	$\{b, \neg a \vee b\}$
ω_3	a	b	θ	$\{\neg a \vee \neg b\}$

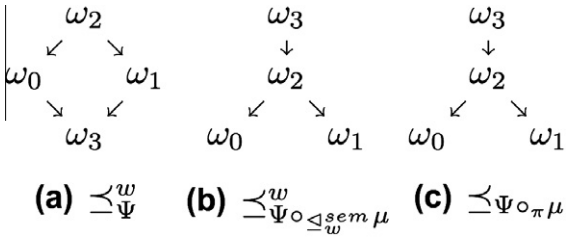

Fig. 5. Partial preorders between interpretations.

Example 5. Let $(\Sigma, \preceq_\Sigma)$ be the syntactic representation of Ψ from the Example 1 with $\Sigma = \{a, b, a \vee \neg b, \neg a \vee b\}$. Using the Definition 12 with the weak comparator, we construct a partial preorder on interpretations. The sets of formulae of Σ falsified by the interpretations is illustrated in Table 1 and the partial preorder $\preceq^w_\Psi$ is given by the Fig. 5a. Therefore $(\mathcal{W}, \preceq^w_\Psi)$ is the semantic representation of Ψ and is such that $Mod(Bel(\Psi)) = min(\mathcal{W}, \preceq^w_\Psi) = \{\omega_3\}$.

Let $(\Sigma\circ_{\trianglelefteq_w}\mu, \preceq_{\Sigma\circ_{\trianglelefteq_w}\mu})$ be the syntactic representation of the epistemic state Ψ revised by μ. Using the Definition 12 with the weak comparator, we construct a new partial preorder on the interpretations. The sets of formulae of $\Sigma\circ_{\trianglelefteq_c}\mu$ falsified by the interpretations are illustrated in Table 1 and the partial preorder $\preceq_{\Psi\circ_{\trianglelefteq^{sem}_w}}$ is given by the Fig. 5b. Therefore $\left(\mathcal{W}, \preceq^w_{\Psi\circ_{\trianglelefteq^{sem}_w}\mu}\right)$ is the semantic representation of Ψ revised by μ and with the Proposition 3 is such that $Mod(Bel(\Psi\circ_{\trianglelefteq_w}\mu)) = min(Mod(\mu), \preceq^w_\Psi) = \{\omega_0, \omega_1\}$.

If we apply, the semantic possibilistic revision of $(\mathcal{W}, \preceq^w_\Psi)$ by μ which preserves the relative ordering between the models of μ and considers all the counter-models of μ as impossible, we obtain the partial preorder $\preceq_{\Psi\circ_\pi\mu}$ illustrated in Fig. 5c. Therefore $\left(\mathcal{W}, \preceq^w_{\Psi\circ_{\trianglelefteq^{sem}_w}\mu}\right) = (\mathcal{W}, \preceq_{\Psi\circ_\pi\mu})$.

As illustrated in the following example, the semantic counterpart defined in the general case does not provide the equivalence between syntactic and semantic approaches.

Example 6. We consider the following counter-example. Let Φ be an epistemic state and $(\Gamma, \preceq_\Gamma)$ be the syntactic representation of Φ such that $\Gamma = \{a, b, \neg c\}$ and $\preceq_\Gamma$ is illustrated by the Fig. 6a. We revise Γ by $\mu = a \to b \wedge c$. We obtain the following syntactic representation of the revised epistemic state: $\Gamma\circ_{\trianglelefteq_w}\mu = \{a, b, \neg c, a \to b \wedge c\}$ and $\preceq_{\Gamma\circ_{\trianglelefteq_w}\mu}$ is illustrated by the Fig. 6b.

From a syntactic point of view, we obtain two removed sets $R_0 = \{a\}$ and $R_1 = \{\neg c\}$ and the belief set corresponding to the revised epistemic state is $Bel(\Phi\circ_{\trianglelefteq_w}\mu) = Cons(\{b, \neg c, a \to b \wedge c\}) \bigvee Cons(\{a, b, a \to b \wedge c\})$. From a semantic point of view, the

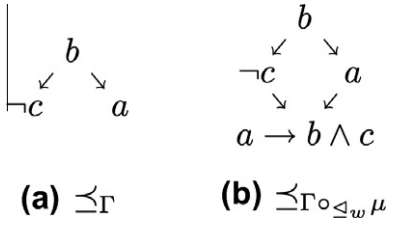

Fig. 6. Partial preorder on formulae.

Table 2
The sets of formulae falsified by the interpretations.

ω_i	a	b	c	$F_\Gamma(\omega_i)$	$F_{\Gamma_{\circ_{\alpha\circ}}\mu}(\omega_i)$
ω_0	$\neg a$	$\neg b$	$\neg c$	$\{a,b\}$	$\{a,b\}$
ω_1	$\neg a$	$\neg b$	c	$\{a,b,\neg c\}$	$\{a,b,\neg c\}$
ω_2	$\neg a$	b	$\neg c$	$\{a\}$	$\{a\}$
ω_3	$\neg a$	b	c	$a,\neg c$	$a,\neg c$
ω_4	a	$\neg b$	$\neg c$	$\{b\}$	$\{b,a\rightarrow b\wedge c\}$
ω_5	a	$\neg b$	c	$\{b,\neg c\}$	$\{b,\neg c,a\rightarrow b\wedge c\}$
ω_6	a	b	$\neg c$	θ	$a\rightarrow b\wedge c$
ω_7	a	b	c	$\{\neg c\}$	$\{\neg c\}$

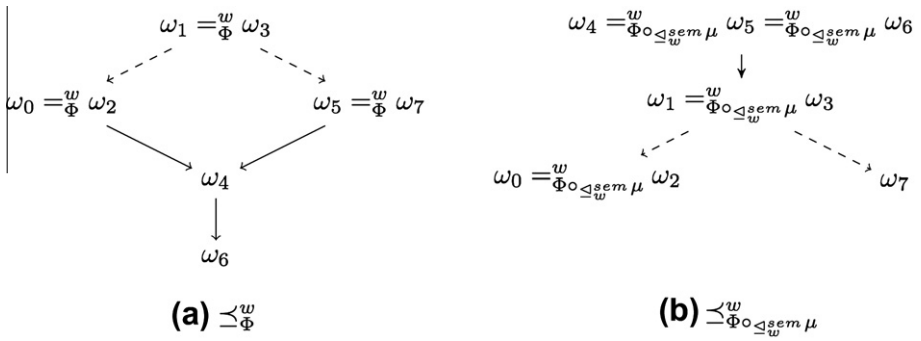

Fig. 7. Partial preorders between interpretations.

sets of formulae falsified by the interpretations are illustrated in Table 2. We construct the partial preorder $\preceq^w_\Phi$ according to the Definition 12 and the partial preorder $\preceq^w_{\Phi\circ_{\trianglelefteq^{sem}_w\mu}}$ according to Definition 14 as it is illustrated in Fig. 7.

From the partial preorder $\preceq^w_{\Phi\circ_{\trianglelefteq^{sem}_w\mu}}$ and from the Definition 13, we have $Mod(Bel(\Phi\circ_{\trianglelefteq^{sem}_w}\mu)) = \{\omega_0,\omega_2,\omega_7\}$. But the models of $Bel(\Phi\circ_{\trianglelefteq_w}\mu)$ are ω_2 and ω_7. So, the syntactic and the semantic approaches are not equivalent.

In order to obtain the equivalence between syntactic and semantic approaches, we refine the semantic counterpart. When defining the removed sets within the syntactic approach, we consider minimal (with respect to inclusion) subsets of formulae, this minimality has to be taken into account in the semantic approach too.

Definition 15. Let $(\mathcal{W},\preceq^w_\Psi)$ be a semantic representation of Ψ, the revision of Ψ by a formula μ leads to the semantically revised epistemic state $\Psi\circ_{\trianglelefteq^{sem}_w}\mu$ such that

$$Mod(Bel(\Psi\circ_{\trianglelefteq^{sem}_w}\mu)) = \{\omega|\omega\in min(Mod(\mu),\preceq^w_\Psi) \quad \text{and} \quad \nexists\omega', F_{\Sigma\circ_{\trianglelefteq_w}\mu}(\omega')\subset F_{\Sigma\circ_{\trianglelefteq_w}\mu}(\omega)\}.$$

The equivalence between the semantic and the syntactic PPRSR is given by the following proposition.

Proposition 3. *Let $\Psi\circ_{\trianglelefteq_w}\mu$ be the syntactically revised epistemic state by a formula μ and let $\Psi\circ_{\trianglelefteq^{sem}_w}\mu$ be the semantically revised epistemic state by a formula μ.*

$$Mod(Bel(\Psi\circ_{\trianglelefteq_w}\mu)) = Mod(Bel(\Psi\circ_{\trianglelefteq^{sem}_w}\mu)).$$

Sketch of proof of Proposition 3 (provided in Appendix A): follows from the definition of $Mod(Bel(\Psi\circ_{\trianglelefteq^{sem}_w}\mu))$.

Example 7. We first consider the Exemple 1, in this context, we have $Mod(Bel(\Psi\circ_{\trianglelefteq_w}\mu)) = \{\omega_0,\omega_1\}$ and $Mod(Bel(\Psi\circ_{\trianglelefteq^{sem}_w}\mu)) = \{\omega_0,\omega_1\}$. The Definition 15 has no effect on $Mod(Bel(\Psi\circ_{\trianglelefteq^{sem}_w}\mu))$ because the sets of formulae of $\Sigma\cup\{\mu\}$ falsified by ω_0 and ω_1 are minimal with respect to inclusion.

We now consider the Exemple 6, we have $Mod(Bel(\Phi\circ_{\trianglelefteq_w}\mu)) = \{\omega_2,\omega_7\}$ and $Mod(Bel(\Phi\circ_{\trianglelefteq^{sem}_w}\mu)) = \{\omega_0,\omega_2,\omega_7\}$. If we apply the Definition 15, $Mod(Bel(\Phi\circ_{\trianglelefteq^{sem}_w}\mu)) = \{\omega_2,\omega_7\}$ because $F_{\Gamma\circ_{\trianglelefteq_w}\mu}(\omega_2)\subset F_{\Gamma\circ_{\trianglelefteq_w}\mu}(\omega_0)$. We have $Mod(Bel(\Phi\circ_{\trianglelefteq_w}\mu)) = Mod(Bel(\Phi\circ_{\trianglelefteq^{sem}_w}\mu))$.

4. Encoding PPRSR in answer set programming

As mentioned in the previous section, the original RSR approach was proposed for revising belief bases consisting of formulas in CNF. RSR was formalized in terms of answer set programming by the construction of a logic program with the same

spirit of [39] and the smodels system [44] was adapted in order to compute the preferred answer sets corresponding to the removed sets [8]. More recently, the extension of RSR to fusion, Removed Sets Fusion (RSF) has been proposed for merging belief bases consisting of arbitrary propositional formulae, and a suitable encoding in answer programming enables the computation of removed sets with any ASP solver equipped with the minimize statement [27]. We now extend this method to the revision of partially preordered information.

We first translate our revision problem into a logic program with answer sets semantics, denoted by $\Pi_{\Sigma\cup\{\mu\}}$. The set of answer sets is denoted by $S(\Pi_{\Sigma\cup\{\mu\}})$. We then define a partial preorder between answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and we show a one-to-one correspondence between removed sets of $\Sigma\cup\{\mu\}$ and preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$.

Let Σ be a set of partially preordered formulae and μ a formula such that $\Sigma\cup\{\mu\}$ is inconsistent. The set of all positive literals of $\Pi_{\Sigma\cup\{\mu\}}$ is denoted by V^+ and the set of all negative literals of $\Pi_{\Sigma\cup\{\mu\}}$ is denoted by V^-. The set of all rule atoms representing formulae is defined by $R^+ = \{r_f | f \in \Sigma\}$ and $F_O(r_f)$ represents the formula of Σ corresponding to r_f in $\Pi_{\Sigma\cup\{\mu\}}$, namely $\forall r_f \in R^+, F_O(r_f) = f$. This translation requires the introduction of intermediary atoms representing subformulae of f. We denote by ρ_f^j the intermediary atom representing f^j which is a subformula of $f \in \Sigma$. To each answer set S of $\Pi_{\Sigma\cup\{\mu\}}$, an interpretation of $\Sigma\cup\{\mu\}$ is associated. Each interpretation of $\Sigma\cup\{\mu\}$ corresponds to several potential removed sets denoted by $F_O(R^+\cap S)$.

1. In the first step, we introduce rules in order to build a one-to-one correspondence between answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and interpretations of V^+. For each atom, $a \in V^+$ two rules are introduced: $a \leftarrow not\ a'$ and $a' \leftarrow not\ a$ where $a' \in V^-$ is the negative atom corresponding to a.
2. In the second step, we introduce rules in order to exclude the answer sets S corresponding to interpretations which are not models of $(\Sigma\backslash F)\cup\{\mu\}$ with $F = \{f | r_f \in S\}$. According to the syntax of f, the following rules are introduced:
   - If $f =_{def} a$, the rule $r_f \leftarrow not\ a$ is introduced;
   - If $f =_{def} \neg f^1$, the rule $r_f \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $f =_{def} f^1 \vee \cdots \vee f^m$, the rule $r_f \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $f =_{def} f^1 \wedge \cdots \wedge f^m$, it is though necessary to introduce several rules to the program. These rules are introduced: $\forall 1 \leqslant j \leqslant m, r_f \leftarrow \rho_{f^j}$.
3. The third step rules out answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ which correspond to interpretations which are not models of μ. According to the syntax of μ, the following rules are introduced:
   - If $\mu =_{def} a$, the rule $false \leftarrow not\ a$ is introduced;
   - If $\mu =_{def} \neg f^1$, the rule $false \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $\mu =_{def} f^1 \vee \cdots \vee f^m$, the rule $false \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $\mu =_{def} f^1 \wedge \cdots \wedge f^m$, the rules $\forall 1 \leqslant j \leqslant m, false \leftarrow \rho_{f^j}$ are introduced.

   In order to rule out *false* from the models of μ, the following rule is introduced: $contradiction \leftarrow faslse, not\ contradiction$.

Example 8. For the previous example, the logic program $\Pi_{\Sigma\cup\{\mu\}}$ is the following:

$$a \leftarrow not\ a' \quad b \leftarrow not\ b' \quad r_a \leftarrow not\ a \quad r_{a\vee\neg b} \leftarrow \rho_a, \rho_{\neg b}$$
$$a' \leftarrow not\ a \quad b' \leftarrow not\ b \quad r_b \leftarrow not\ b \quad r_{\neg a\vee b} \quad \rho_{\neg a}, \rho_b$$
$$\rho_a \leftarrow not\ a \quad \rho_{\neg a} \quad a \quad \rho_b \leftarrow not\ b \quad \rho_{\neg b} \leftarrow b$$
$$false \leftarrow \rho_{\neg a}, \rho_{\neg b} \quad contradiction \leftarrow false, not\ contradiction$$

If $f = \neg a \vee b$ belongs to a removed set, then $r_{\neg a\vee b}$ should belong to an answer set. f has to be falsified and so $\neg f$, i.e. $a \wedge \neg b$, has to be satisfied that is why the rules $r_{\neg a\vee b} \leftarrow \rho_{\neg a}$, ρ_b, $\rho_{\neg a} \leftarrow a$ and $\rho_b \leftarrow not\ b$ are introduced to $\Pi_{\Sigma\cup\{\mu\}}$.

From the logic program, we show how we obtain a one-to-one correspondence between the preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and the removed sets of $\Sigma\cup\{\mu\}$. Let S be a set of atoms, we define the interpretation over the atoms of $S\cap V^+$ as $I_S = \{a | a \in S\}\cup\{\neg a | a' \in S\}$ and the following result holds.

Proposition 4. *Let ρ a rule atom or an intermediary atom.*

$$\rho \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \quad \text{iff } I_S \not\models F_O(\rho).$$

The correspondence between answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and interpretations of $(\Sigma\backslash F_O(R^+\cap S))\cup\{\mu\}$ is given in the following proposition:

Proposition 5. *Let Σ be a set of partially preordered formulae. Let $S \subseteq V$ be a set of atoms. S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ iff S corresponds to an interpretation I_S of V^+ which satisfies $(\Sigma\backslash F_O(R^+\cap S))\cup\{\mu\}$.*

The proofs of the Propositions 4 and 5 (provided in Appendix A) are both based on the rules construction.

Example 9. The answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ are: $S_0 = \{a', b, r_a \vee \neg b, r_a\}, S_1 = \{a, b', r_{\neg a} \vee b, r_b\}$ and $S_2 = \{a', b', r_a, r_b\}$.

In order to compute the answer sets corresponding to the removed sets, we introduce new preference relations between answer sets according to a partial preorder. We define the notion of preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ according to the weak comparator denoted by $S_w(\Pi_{\Sigma\cup\{\mu\}})$.

Definition 16. Let $\preceq_\Sigma$ be a partial preorder on Σ,μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent, $S \in S(\Pi_{\Sigma\cup\{\mu\}})$. S is a preferred answer set of $\Pi_{\Sigma\cup\{\mu\}}$ iff

- $\nexists S' \in S(\Pi_{\Sigma\cup\{\mu\}})$ such that $F_O(S' \cap R^+) \subset F_O(S \cap R^+)$,
- $\nexists S' \in S(\Pi_{\Sigma\cup\{\mu\}})$ such that $F_O(S' \cap R^+) \lhd_w F_O(S \cap R^+)$.

Example 10. We have $F_O(S_0 \cap R^+) \lhd_w F_O(S_1 \cap R^+)$ and $F_O(S_2 \cap R^+) \lhd_w F_O(S_1 \cap R^+)$. So, $S_w(\Pi_{\Sigma\cup\{\mu\}}) = \{S_0, S_2\}$.

Remark. As previously, it is possible to refine the notion of preferred answer set with an extra preference according to a strategy P. Let S_X, $S_Y \in S_w(\Pi_{\Sigma\cup\{\mu\}})$.$S_Y$ is preferred to S_X according to *CARD* (resp. *MIN*) iff $|F_O(S_Y \cap R^+)| \leqslant |F_O(S_X \cap R^+)|$ (resp. $|F_O(S_Y \cap R^+) \cap MIN| \leqslant |F_O(S_X \cap R^+) \cap MIN|$).

Example 11. We have S_0 is as preferred as S_2 according to *CARD* and S_0 is preferred to S_2 according to *MIN*.

The one-to-one correspondence between preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ and the removed sets is given by the following proposition:

Proposition 6. *Let Σ be a finite set of partially preordered formulae and μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. X is a removed set of $\Sigma \cup \{\mu\}$ iff there exists a preferred answer set S of $\Pi_{\Sigma\cup\{\mu\}}$ such that $F_O(R^+ \cap S) = X$.*

Sketch of the proof of the Proposition 6 (provided in Appendix A): we show that the set of removed sets of $\Sigma \cup \{\mu\}$ equals the set of preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$.

Example 12. We have $F_O(S_0 \cap R^+) = \{a, a \vee \neg b\}$ and $F_O(S_2 \cap R^+) = \{a, b\}$ which correspond to the removed sets R_0 and R_7 found in the previous section.

Performing PPRSR The theoretical computational complexity of the decision problem "is the formula phi a weak consequence of $\Sigma \circ_w \mu$? " is not already known, however a lower bound Δ_2^p and an upper bound Π_2^p have been provided in [7]. However, some applications, as illustrated in the following section, require the revision of partially preordered belief bases. We choose the removed set approach rather than the dual one based on preferred maximal consistent subbases because the removed set approach allows for an efficient implementation with ASP. Within the framework of the VENUS project, we conducted an experimental study on an inconsistency handling method stemming from removed sets for partially preordered archaeological information and we obtained interesting results. The practical complexity was reasonable, and therefore encouraged us to investigate further in this direction, as well as to test on revision problem.

Regarding the implementation, CLASP [23] gives us the answer sets of $\Pi_{\Sigma\cup\{\mu\}}$. But our method requires to partially preorder the answer sets with the comparator $\trianglelefteq_w$ to obtain the preferred answer sets corresponding to removed sets. This step is not yet implemented in ASP. We used a java program to partially preorder the answer sets to obtain the preferred answer sets. We denote by N the number of answer sets given by CLASP. The computation of the partial preorder between them can be realized in less than $\frac{N(N-1)}{2}$ comparisons. Indeed, as it showed in [32], it is sufficient to compare the minimal formulae according to $\preceq_\Sigma$ of each answer set and so using the following proposition, we reduce the cost of the computation

Proposition 7. *Let $\preceq_\Sigma$ be a partial preorder on Σ, μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent and S, $S' \in S(\Pi_{\Sigma\cup\{\mu\}})$. $F_O(S \cap R^+) \trianglelefteq_w F_O(S' \cap R^+)$ iff $\forall y \in Min(F_O(S \cap R^+), \preceq_\Sigma), \exists x \in Min(F_O(S' \cap R^+), \preceq_\Sigma)$ such that $x \preceq_\Sigma y$ where $Min(F_O(S \cap R^+), \preceq_\Sigma) = \{x | x \in F_O(S \cap R^+),\ \nexists y \in F_O(S \cap R^+),\ y \prec_\Sigma x\}$.*

Moreover, the determination of the minimal answer sets according to this partial preorder does not increase the computational cost since CLASP and SAT both belong to the NP-complete complexity class.

5. VENUS application

The european VENUS project (Virtual ExploratioN of Underwater Sites) no (IST-034924)[3] aims at providing scientific methodologies and technological tools for the virtual exploration of deep underwater archaeology sites. In this context, technologies like photogrammetry are used for data acquisition and the knowledge about the studied objects is provided by both archaeology and photogrammetry. We constructed an application ontology in [43] from a domain ontology which describes the vocabulary on the amphorae (the studied artefacts) and from a task ontology describing the data acquisition process (Fig. 8). This ontology consists of a set of concepts, relations, attributes and constraints like domain constraints: an amphora must have only one typology and for example, this typology is either short Dressel 2-4 or long Dressel 2-4. Our knowledge base contains our ontology and

[3] http://www.venus-project.eu.

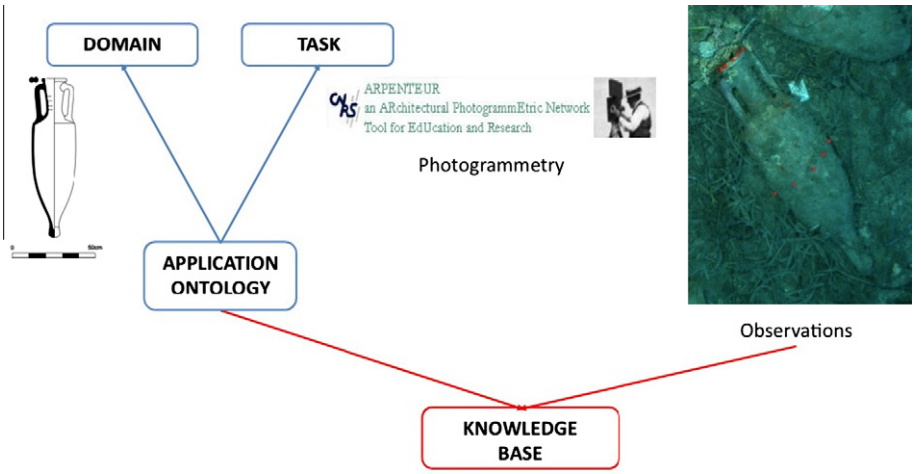

Fig. 8. Application ontology constructed in the context of the european VENUS project.

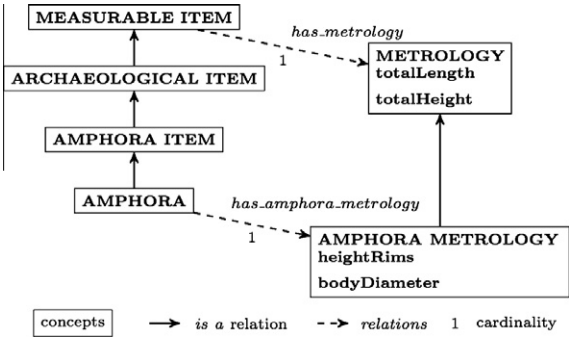

Fig. 9. Extract of the application ontology.

observations. The ontology represents the generic knowledge which is preferred to observations. The observations on the same amphora can be preordered according to the reliability of the experts who provide them. In this context, we revise the observations by more reliable observations. We only consider a small part of the ontology (Fig. 9) and some observations in order to provide a very simple example where the knowledge base is expressed in propositional logic.

We use the following propositional variables: m_i for the measurable item, ar_i for the archaeological item, a_i for the amphora item, a for the amphora, m_1, m_2 for the metrologies, d_s for the short Dressel 2-4 typology, d_l for the long Dressel 2-4 typology, h_{m_1}, h_{m_2} for has_metrology, h_1, h_2 for the total heights, l_1, l_2 for the total lengths. The propositional translation of the extract of the ontology can be resumed by the set of formulae:

$$G = \{a \rightarrow a_i \wedge (d_s \vee d_l),\ a_i \rightarrow ar_i,\ ar_i \rightarrow m_i,\ m_i \rightarrow h_{m_1} \vee h_{m_2},\ h_{m_1} \rightarrow m_1,\ h_{m_2} \rightarrow m_2,\ m_1 \rightarrow l_1 \wedge h_1,\ m_2 \rightarrow l_2 \wedge h_2,\ (d_s \vee d_l) \wedge (\neg d_s \vee \neg d_l)\}.$$

Then we have two sets of observations provided by two different experts. The observations of the first expert $\{a, d_s, l_1, h_1\}$ lead to the instance denoted by

$$I_1 = \{a,\ a_i,\ ar_i,\ d_s,\ m_1,\ m_i,\ h_{m_1},\ l_1,\ h_1\}$$

and the observations of the second one is $\{a, d_l, l_2, h_2\}$ lead to the instance denoted by:

$$I_2 = \{a,\ a_i,\ ar_i,\ d_l,\ m_2,\ m_i,\ h_{m_2},\ l_2,\ h_2\}.$$

By hypothesis, the ontology and the constraints which are also called the generic knowledge cannot be modified. Moreover, we consider that the second expert is more reliable than the first one. We revised the first observations $\Sigma = I_1 \backslash (I_1 \cap I_2)$ by

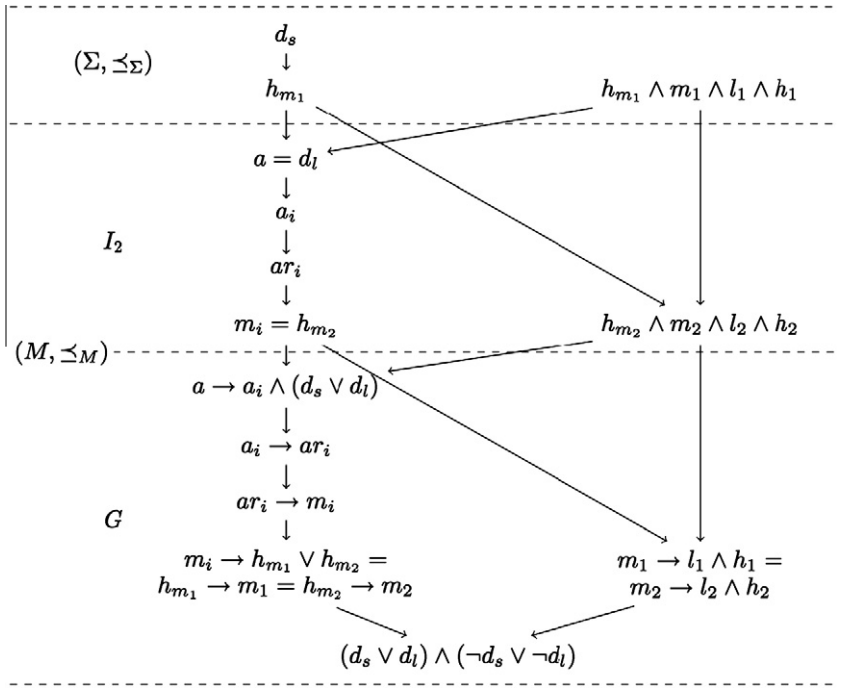

Fig. 10. $\preceq_{\Sigma \circ_{\trianglelefteq_C} M}$.

$M = G \cup I_2$ where G is the generic knowledge and I_2 is the second set of observations and the revised preorder is represented by Fig. 10 and we obtain $\Sigma \circ_{\trianglelefteq_w} M = Cons((\Sigma \setminus R \cup M)$ with $R = \{d_s\}$.

The revision presented in the Section 3 is the first step of the revision to apply in the VENUS context. Indeed, the revision could be defined as follows:

- $\Sigma \circ_{\trianglelefteq_C} M = \bigvee_{R \in \mathcal{R}_C(\Sigma \cup M)} Cons((\Sigma \setminus R) \cup M)$ with $\Sigma = I_1 \setminus (I_1 \cap I_2)$ and $M = G \cup I_2$.
- $\preceq_{\Sigma \circ_{\trianglelefteq_C} M}$:
  (i) $\forall \psi, \phi \in M, \psi \prec_{\Sigma \circ_{\trianglelefteq_C} M} \phi$ iff $\psi \preceq_M \phi$,
  (ii) $\forall \psi, \phi \in \Sigma, \psi \prec_{\Sigma \circ_{\trianglelefteq_C} M} \phi$ iff $\psi \preceq_\Sigma \phi$,
  (iii) $\forall \psi \in \Sigma, \mu \in M, \mu \prec_{\Sigma \circ_{\trianglelefteq_C} M} \psi$.

6. Conclusion

This paper presents a new framework for revising partially preordered information called Partially Preordered Removed Sets Revision (PPRSR) which extends the Removed Sets approach to partial preorders. The paper shows that PPRSR can be successfully encoded into answer set programming and proposes an implementation stemming from ASP solvers. It shows that the extension of the possibilistic revision to partial preorders can be captured within the PPRSR framework allowing for an efficient implementation with ASP. It illustrates how PPRSR can be applied within the context of the VENUS european project dealing with archaeological information. An experimental study has now to be conducted in the context of the VENUS project in order to provide a more accurate evaluation of the performance of PPRSR. We have to deeper investigate the use of ASP solver statements in order to directly define a partial preorder between answer sets. A future work will investigate the use of the lexicographic comparator for defining revision operations within the framework of PPRSR.

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Appendix A. proofs

Proposition 1. *Let Ψ be an epistemic state and $\preceq^C_\Psi$ be a partial preorder on $\mathcal{W}$ associated to Ψ. Then, $\preceq^C_\Psi$ is a P-faithful assignment.*

Proof. Let Σ be a set of formulae. The preorder on the interpretations is defined as follows: $\forall \omega, \omega' \in \mathcal{W}$, $\omega \preceq^C_\Psi \omega'$ iff $F_\Sigma(\omega) \trianglelefteq_C F_\Sigma(\omega')$. We want to show that $\preceq^C_\Psi$ is a P-faithful assignment.

1. We show that if $\omega, \omega' \in Mod(Bel(\Psi))$ then $\omega \prec^C_\Psi \omega'$ is impossible.
   Let $\omega, \omega' \in Mod(Bel(\Psi))$, then $F_\Sigma(\omega) = \emptyset$ and $F_\Sigma(\omega') = \emptyset$. So, we have $F_\Sigma(\omega) =_C F_\Sigma(\omega')$ and $\omega =^C_\Psi \omega'$.
2. We show that if $\omega' \not\models Bel(\Psi)$, then there exists $\omega \in \mathcal{W}$ such that $\omega \models Bel(\Psi)$ and $\omega \preceq^C_\Psi \omega'$.
   Let $\omega' \not\models Bel(\Psi)$ then $F_\Sigma(\omega') \neq \emptyset$. We show that there exists ω such that $\omega \models Bel(\Psi)$ and $\omega \prec^C_\Psi \omega'$.
   Ψ is an epistemic state, represented by $(\Sigma, \preceq_\Sigma)$. So there exists ω such that $\omega \models Bel(\Psi)$ and $F_\Sigma(\omega) = \emptyset$. We have $F_\Sigma(\omega') \neq \emptyset$ and $F_\Sigma(\omega) = \emptyset$, so $F_\Sigma(\omega) \trianglelefteq_C F_\Sigma(\omega')$. Therefore, $\omega \prec^C_\Psi \omega'$.
3. We show that if $\Psi = \Phi$ then $\preceq^C_\Psi = \preceq^C_\Phi$.
   Let Ψ be an epistemic state and let $\preceq^C_\Psi$ be the associated partial preorder on the interpretations. If $\Psi = \Phi$ then $\preceq^C_\Psi = \preceq^C_\Phi$. □

Proposition 2. *Let $\circ_\pi$ be the possibilistic revision operator.*

$$\forall \omega, \omega' \in \mathcal{W},\ \omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega' \text{ iff } \omega \preceq_{\Psi \circ_\pi \mu} \omega'.$$

Proof (*Possibilistic equivalence*). We can distinguish three cases:

1. We want to show that:
   If $\omega \in Mod(\mu)$ and $\omega' \in Mod(\mu)$ then $\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$ iff $\omega \preceq^w_\Psi \omega'$.

We have:

$$\begin{aligned} \omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega' &\text{ iff } F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega) \trianglelefteq_w F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \\ &\text{iff } \forall \phi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega),\ \exists \varphi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \textit{ such that } \varphi \preceq_{\Sigma \circ_{\trianglelefteq_w} \mu} \phi. \end{aligned}$$

Since $\omega \in Mod(\mu)$ and $\omega' \in Mod(\mu)$ therefore $\mu \notin F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$ et $\mu \notin F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega')$.
According to the Definition 8 of $\circ_{\trianglelefteq_w}, \forall \psi, \phi \in \Sigma : \psi \preceq_{\Sigma \circ_{\trianglelefteq_w} \mu} \phi$ iff $\psi \preceq_\Sigma \phi$
Thus:

$$\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega' \textit{ iff } \forall \phi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega), \exists \varphi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \textit{ such that } \varphi \preceq_\Sigma \phi.$$

Since $\Sigma \circ_{\trianglelefteq_w} \mu = \Sigma \cup \{\mu\}$:

$$\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega' \text{ iff } \forall \phi \in F_{\Sigma \cup \{\mu\}}(\omega),\ \exists \varphi \in F_{\Sigma \cup \{\mu\}}(\omega') \textit{ such that } \varphi \preceq_\Sigma \phi.$$

Moreover, since $\omega \in Mod(\mu)$ and $\omega' \in Mod(\mu)$ thus $\mu \notin F_{\Sigma \cup \{\mu\}}(\omega)$ and $\mu \notin F_{\Sigma \cup \{\mu\}}(\omega')$. Therefore $F_{\Sigma \cup \{\mu\}}(\omega) = F_\Sigma(\omega)$ and $F_{\Sigma \cup \{\mu\}}(\omega') = F_\Sigma(\omega')$.

$$\begin{aligned} \omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega' &\text{ iff } \forall \phi \in F_\Sigma(\omega), \exists \varphi \in F_\Sigma(\omega') \textit{ such that } \varphi \preceq_\Sigma \phi \\ &\text{iff } F_\Sigma(\omega) \trianglelefteq_w F_\Sigma(\omega') \\ &\text{iff } \omega \preceq^w_\Psi \omega'. \end{aligned}$$

2. We want to show that:
   If $\omega \notin Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega =^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$.

We have $\omega \notin Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then ω and ω' falsify the formula μ so $\mu \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$ and $\mu \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega')$.
According to the definition of the partial preorder $\preceq_{\Sigma \circ_{\trianglelefteq_w} \mu}$, we have:
$\forall \phi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega), \exists \varphi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega')$ (in this case μ) such that $\varphi \preceq_{\Sigma \circ_{\trianglelefteq_w} \mu} \phi$ and thus $\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$. By symmetry, we have $\omega' \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega$ therefore $\omega =^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$.

3. We want to show that:
   If $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$.

We have $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then ω does not falsify the formula μ and ω' falsifies the formula μ thus $\mu \notin F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$ and $\mu \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega')$.
We have: $\forall \phi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$, $\exists \varphi \in F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega')$ (in this case μ) such that $\varphi \prec_{\Sigma \circ_{\trianglelefteq_w} \mu} \phi$ and so $\omega \preceq^w_{\Psi \circ_{\trianglelefteq^{sem}_w} \mu} \omega'$. □

Proposition 3. *Let $\Psi \circ_{\trianglelefteq_w} \mu$ be the syntactically revised epistemic state by a formula μ and let $\Psi \circ_{\trianglelefteq^{sem}_w} \mu$ be the semantically revised epistemic state by a formula μ.*

$$Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu)) = Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_w} \mu)).$$

Proof. We want to show that $Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu)) = Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_w} \mu))$, according to the Definition 15, $Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_w} \mu)) = \{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi)$ and $\nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\}$. We show that

$$Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu)) = \{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi) \text{ and } \nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\}.$$

1. We show that:

$$Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu)) \subseteq \{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi) \text{ and } \nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\}.$$

$\forall \omega \in Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu))$, there exists at least one removed set $R \in \mathcal{R}_w(\Sigma \cup \{\mu\})$ such that $\omega \in Mod((\Sigma \backslash R) \cup \{\mu\})$. Let $F_\Sigma(\omega)$ be the set of formulae falsified by ω. Since $\omega \in Mod((\Sigma \backslash R) \cup \{\mu\})$ thus ω does not falsify the formulae from $(\Sigma \backslash R) \cup \{\mu\}$ but ω falsifies the formulae belonging to the removed set R. Therefore $F_\Sigma(\omega) = R$. We want to show that:

   (a) $\omega \in min(Mod(\mu), \preceq^w_\Psi)$. Suppose $\omega \notin min(Mod(\mu), \preceq^w_\Psi)$ there exists $\omega' \in Mod(\mu)$ such that $\omega' \prec^w_\Psi \omega$. By Definition 12, $\omega' \prec^w_\Psi \omega$ iff $F_\Sigma(\omega') \lhd_w F_\Sigma(\omega)$. However $F_\Sigma(\omega) = R$ thus $F_\Sigma(\omega') \lhd_w R$ which contradicts the fact that R is a removed set. Therefore $\omega \in min(Mod(\mu), \preceq^w_\Psi)$.
   (b) $\nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$. Suppose $\exists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$. Since $F_\Sigma(\omega) = R$, we have $F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset R$ which contradicts the fact that R is a removed set. Therefore $\nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)$.

2. We show that:

$$\{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi) \text{ and } \nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\} \subseteq Mod(Bel(\Psi \circ_{\trianglelefteq_w} \mu)).$$

$\forall \omega \in \{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi)$ *and* $\nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\}$, $F_\Sigma(\omega) \neq \emptyset$ because $\Sigma \cup \{\mu\}$ is inconsistent. $F_\Sigma(\omega) \subset \Sigma$ and $\omega \vDash \mu$ et $\omega \vDash (\Sigma \setminus F_\Sigma(\omega))$. Since $F_\Sigma(\omega) \cap \{\mu\} = \emptyset$ thus $\omega \vDash (\Sigma \backslash F_\Sigma(\omega)) \cup \{\mu\}$ and $(\Sigma \backslash F_\Sigma(\omega)) \cup \{\mu\}$ is consistent. We show that $F_\Sigma(\omega)$ is a removed set. We show that:

   (a) $\nexists \omega' \in Mod(\mu)$ such that $(\Sigma \setminus F_\Sigma(\omega')) \cup \{\mu\}$ is consistent et $F_\Sigma(\omega') \subset F_\Sigma(\omega)$.
   Suppose there exists $\omega' \in Mod(\mu)$ such that $(\Sigma \backslash F_\Sigma(\omega')) \cup \{\mu\}$ is consistent and $F_\Sigma(\omega') \subset F_\Sigma(\omega)$. This contradicts the starting hypothesis: $\omega \in \{\omega | \omega \in min(Mod(\mu), \preceq^w_\Psi)$ *and* $\nexists \omega', F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega') \subset F_{\Sigma \circ_{\trianglelefteq_w} \mu}(\omega)\}$.
   (b) $\nexists \omega' \in Mod(\mu)$ such that $(\Sigma \setminus F_\Sigma(\omega')) \cup \{\mu\}$ is consistent and $F_\Sigma(\omega') \lhd_w F_\Sigma(\omega)$.
   Suppose there exists $\omega' \in Mod(\mu)$ such that $(\Sigma \backslash F_\Sigma(\omega')) \cup \{\mu\}$ is consistent and $F_\Sigma(\omega') \lhd_w F_\Sigma(\omega)$. According to Definition 12, $\omega' \prec^w_\Psi \omega$ donc $\omega \notin min(Mod(\mu), \preceq^w_\Psi)$ which contradicts the starting hypothesis. □

Proposition 4. *Let ρ a rule atom or an intermediary atom.*

$$\rho \in CN(\Pi^S_{\Sigma \cup \{\mu\}}) \text{ iff } I_S \not\models F_O(\rho).$$

Proof. The proof is obvious according the rules construction. □

Proposition 5. *Let Σ be a set of partially preordered formulae. Let $S \subseteq V$ be a set of atoms. S is an answer set of $\Pi_{\Sigma \cup \{\mu\}}$ iff S corresponds to an interpretation I_S of V^+ which satisfies $(\Sigma \backslash F_O(R^+ \cap S)) \cup \{\mu\}$.*

Proof

1. We first show that if S is an answer set of $\Pi_{\Sigma \cup \{\mu\}}$ then I_S is an interpretation of V^+ that satisfies $\Sigma \backslash F_O(R^+ \cap S) \cup \{\mu\}$.
   Let S be an answer set of $\Pi_{\Sigma \cup \{\mu\}}$. We have $I_S = \{a | a \in S\} \cup \{\neg a | a' \in S\}$ that is to say if $a \in S$ then $a \in I_S$ and if $a' \in S$ then $\neg a \in I_S$.
   - We first show that I_S is an interpretation of V^+.
     - To do that, we show the mutual exclusion that is to say $\forall a \in V^+$, either $a \in S$ or $a' \in S$ because:
       * If $a \notin S$ and $a' \notin S$ then the rule $a \leftarrow not\ a'$ is applied and $a \in CN\left(\Pi^S_{\Sigma \cup \{\mu\}}\right)$, yet S is an answer set of $\Pi_{\Sigma \cup \{\mu\}}$ thus $S = CN(\Pi^S_{\Sigma \cup \{\mu\}})$ thus $a \in S$ which contradicts the hypothesis.
       * if $a \in S$ and $a' \in S$ then the only rule allowing to deduce a is $a \leftarrow not\ a'$. For that a is true, it is necessary that a' is false, thus $a' \notin S$ which contradicts the hypothesis.

– We show too that $false \notin S$ and $contradiction \notin S$. Suppose that $false \in S$, there are two cases:
  * Let $contradiction \notin S$. the rule $contradiction \leftarrow false, not\ contradiction$ is applied and $contradiction \in CN(\Pi^S_{\Sigma\cup\{\mu\}})$, yet S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ thus $S = CN(\Pi^S_{\Sigma\cup\{\mu\}})$ thus $contradiction \in S$ which contradicts the initial hypothesis.
  * Let $contradiction \in S$. The only rule allowing to deduce $contradiction$ is $contradiction \leftarrow false, not\ contradiction$. In order to have $contradiction$ true, it is necessary that $false$ is true and $contradiction$ is false thus $contradiction \notin S$ which contradicts the hypothesis.

Thus, I_S is an interpretation of V^+.

- We then show that I_S satisfies $\Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\}$ that is to say $I_S \in Mod(\Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\})$.
  – We show that I_S is a model of μ. Suppose that $I_S \notin Mod(\mu)$ then $I_S \nvDash \mu$. The rule introduced by (3), we get $false \in CN(\Pi^S_{\Sigma\cup\{\mu\}})$, yet S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ thus $S = CN(\Pi^S_{\Sigma\cup\{\mu\}})$ thus $false \in S$ which is impossible since $false \notin S$. Thus $I_S \in Mod(\mu)$.
  – Suppose that $I_S \notin Mod(\Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\})$. Then there exists a formula $f \in \Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\}$ such that $I_S \nvDash f$. Since $I_S \in Mod(\mu)$ then $f \neq \mu$ thus $f \in \Sigma \backslash F_O(R^+ \cap S)$. According to the rule introduced by the step 2 concerning f, we get $r_f \in CN(\Pi^S_{\Sigma\cup\{\mu\}})$ yet S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ thus $S = CN(\Pi^S_{\Sigma\cup\{\mu\}})$ thus $r_f \in S$. If $r_f \in S$ then $f \in F_O(R^+ \cap S)$ and thus $f \notin \Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\}$ which contradicts the hypothesis.

As a consequence I_S satisfies $\Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\}$.

2. Then, we show that, if I_S is an interpretation of V^+ which satisfies $\Sigma\backslash F_O(R^+ \cap S) \cup \{\mu\}$ then S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$. From I_S, we build a set S such that $S = \{a | a \in I_S\} \cup \{a' | \neg a \in I_S\}$ that is to say if $a \in I_S$ then $a \in S$ and if $\neg a \in I_S$ then $a' \in S$. We then add rule atoms and intermediary atoms. We show that $S = CN(\Pi^S_{\Sigma\cup\{\mu\}})$.

- We first show that $S \subseteq CN(\Pi^S_{\Sigma\cup\{\mu\}})$. Let $a \in S$ be an atom. There are two cases:
  – Let $a \in V^+$.I_S is an interpretation of $V^+, \neg a \notin V^+$ et $a' \notin S$. the rule $a \leftarrow not\ a'$ implies that $a \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$.
  – Let $a \in V^-$.The proof is similar.

Moreover, if ρ is a rule atom or an intermediary atom. We know that if $\rho \in S$ then $\rho \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$ according to the Proposition 4.

- Then we show that $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \subseteq S$. Clearly, we have $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \subseteq V \cup \{false, contradiction\}$.
  – We first show that $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap (V \setminus R^+) \subseteq S$. Suppose that there exists $a \in CN(\Pi^S_{\Sigma\cup\{\mu\}}) \cap (V \setminus R^+)$ such that $a \notin S$. There are two cases:
    * Let $a \in V^+$.$a' \in S$ because I_S is an interpretation of V^+ and we know that $S \subseteq CN(\Pi^S_{\Sigma\cup\{\mu\}})$ thus $a' \in CN(\Pi^S_{\Sigma\cup\{\mu\}})$. But the only rule allowing the deduction of a is $a \leftarrow not\ a'$ thus $a \notin CN(\Pi^S_{\Sigma\cup\{\mu\}})$which is impossible since $a \in CN(\Pi^S_{\Sigma\cup\{\mu\}}) \cap (V \setminus R^+)$. Thus $a \in S$.
    * Let $a \in V^-$. The proof is similar.

  Moreover, if ρ is a rule atom or an intermediary atom such that $\rho \in CN(\Pi^S_{\Sigma\cup\{\mu\}}) \cap (V \setminus R^+)$ and $\rho \notin S$. According to the Proposition 4, we know that $\rho \in CN(\Pi^S_{\Sigma\cup\{\mu\}}) \cap (V \setminus R^+)$ implies that $I_S \vDash F_O(\rho)$. But, if $\rho \notin S$ then $I_S \nvDash F_O(\rho)$ which is contradictory.
  – Then, we show that $CN(\Pi^S_{\Sigma\cup\{\mu\}}) \cap R^+ \subseteq S$. Suppose that there exists $a \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap R^+$ such that $a \notin S$. Then $a = r_f$ thus $r_f \in S$ and I_S is an interpretation of V^+. The only rule concerning f is introduced in 2. The body of the rule belongs to $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$. Since $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap (V \setminus R^+) \subseteq S$ then it belongs to S too. That implies that $I_S \nvDash f$. But I_S is a model of $\Sigma\backslash F_O(R^+ \cap S)$ and $r_f \notin S$ which contradicts the hypothesis.
    Moreover, if ρ is a rule atom or an intermediary atom such that $\rho \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap R^+$ and $\rho \notin S$. According to the Proposition 4, we know that $\rho \in CN(\Pi^S_{\Sigma\cup\{\mu\}})^S \cap R^+$ implies that $I_S \vDash F_O(\rho)$. But, if $\rho \notin S$ then $I_S \nvDash F_O(\rho)$ which is contradictory.
  – Finally, we show that $false \notin CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$. Suppose that $false \in CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$, then the rule introduced in 3 is such that its body belongs to $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$ and since $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap (V \setminus R^+) \subseteq S$ then it belongs to S too. But that is to say that the formula μ is not satisfied by I_S which is impossible since I_S is a model of μ. Thus $false \notin CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$ and by consequence $contradiction \notin CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$.

We have $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \subseteq V \cup \{false, contradiction\}$ and we have shown that $CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap (V \setminus R^+) \subseteq S, CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right) \cap R^+ \subseteq S$, $false \notin CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$ and $contradiction \notin CN\left(\Pi^S_{\Sigma\cup\{\mu\}}\right)$ thus

$CN(\Pi^S_{\Sigma\cup\{\mu\}}) \subseteq S$. □

Proposition 6. *Let Σ be a finite set of partially preordered formulae and μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. X is a removed set of $\Sigma \cup \{\mu\}$ iff there exists a preferred answer set S of $\Pi_{\Sigma\cup\{\mu\}}$ such that $F_O(R^+ \cap S) = X$.*

To proof this proposition, we use the following intermediary results which are a consequence of the Proposition 5:

Proposition 8. *Let $R \subseteq \Sigma$. If $(\Sigma\backslash R) \cup \{\mu\}$ is consistent then there exists a set of atoms S such that S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ and $F_O(R^+ \cap S) \subseteq R$.*

Proof. Let $R \subseteq \Sigma$ be such that $(\Sigma\backslash R) \cup \{\mu\}$ be consistent. We show that there exists a set of atoms S such that S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ and $F_O(R^+ \cap S) \subseteq R$.

$(\Sigma\backslash R) \cup \{\mu\}$ is consistent, then there exists an interpretation I_R which satisfies $(\Sigma\backslash R) \cup \{\mu\}$. From this interpretation I_R, we can build a set of atoms S such that $S = \{a | a \in I_R\} \cup \{a' | \neg a \in I_R\} \cup \{\rho | I_R \models F_O(\rho)\}$ and S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$. R contains all the formulae falsified by the interpretation I_R. But R may also contain formulae satisfied by I_R. There is no one-to-one correspondence between answer sets and potential removed sets. An answer set may corresponds to several potential removed sets. □

Corollary 1. *Let $R \subseteq \Sigma$ be such that $(\Sigma\backslash R) \cup \{\mu\}$ is consistent. If it does not exist $R' \subseteq R$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent, there then exists a set of atoms S such that S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ and $F_O(R^+ \cap S) = R$.*

Proposition 9. *If there exists a set of atoms S such that S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$ then $(\Sigma\backslash F_O(R^+ \cap S)) \cup \{\mu\}$ is consistent.*

Proof. Let S be a set of atoms such that S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$. We show that $(\Sigma\backslash F_O(R^+ \cap S)) \cup \{\mu\}$ is consistent.

From S, we build an interpretation $I_S = \{a | a \in S\} \cup \{\neg a | a' \in S\}$. According to the Proposition 5, we know that all the rules r_f which do not appear in S correspond to formulae f satisfied by the interpretation I_S. Thus $(\Sigma\backslash F_O(R^+ \cap S)) \cup \{\mu\}$ is consistent. □

Proof (One-to-one correspondence). Let $S_w(\Pi_{\Sigma\cup\{\mu\}})$ be the set of preferred answer sets of $\Pi_{\Sigma\cup\{\mu\}}$ according to the comparator $\trianglelefteq_C$. Let $R_w(\Sigma \cup \{\mu\})$ be the set of removed sets of $\Sigma \cup \{\mu\}$. We want to show that: $\{F_O(R^+ \cap S) | S \in S_w(\Pi_{\Sigma\cup\{\mu\}})\} = R_w(\Sigma \cup \{\mu\})$.

1. We show that $\{F_O(R^+ \cap S) | S \in S_w(\Pi_{\Sigma\cup\{\mu\}})\} \subseteq R_w(\Sigma \cup \{\mu\})$. Let S be a preferred answer set of $\Pi_{\Sigma\cup\{\mu\}}$, i.e. $S \in S_w(\Pi_{\Sigma\cup\{\mu\}})$ such that $R = F_O(R^+ \cap S)$. We want to show that $R \in R_w(\Sigma \cup \{\mu\})$ i.e.:
   - R is a potential removed set of $\Sigma \cup \{\mu\}$: $(\Sigma\backslash R) \cup \{\mu\}$ is consistent.
   - R is minimal according to the inclusion: $\nexists R' \subseteq \Sigma$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent and $R' \subset R$.
   - R is preferred according to the comparator $\trianglelefteq_C$: $\nexists R' \subseteq \Sigma$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent and $R' \triangleleft_w R$.

   According to the Proposition 9, $(\Sigma\backslash R) \cup \{\mu\}$ is consistent. Suppose that $R \notin R_w(\Sigma \cup \{\mu\})$ then, we have two possibilities:
   (a) R is not minimal according to the inclusion: $\exists R' \subseteq \Sigma$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent and $R' \subset R$.
   (b) R is not preferred according to the comparator $\triangleleft_C$:$\exists R' \subseteq \Sigma$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent and $R' \triangleleft_w R$.
   Let $R' \subseteq \Sigma$ such that $(\Sigma\backslash R') \cup \{\mu\}$ is consistent, then there exists an interpretation $I_{R'}$ which satisfies $(\Sigma\backslash R') \cup \{\mu\}$. Let $S' = \{a | a \in I_{R'}\} \cup \{a' | \neg a \in I_{R'}\} \cup \{\rho | I_{R'} \not\models F_O(\rho)\}$. Clearly, $I_{S'} = \{a | a \in S'\} \cup \{\neg a | a' \in S'\}$ is an interpretation of V^+ which satisfies $(\Sigma\backslash R') \cup \{\mu\}$ and such that $R' = F_O(R^+ \cap S')$. According to the Corollary 1, S' is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$. Moreover,
   (a) if $R' \subset R$, then $F_O(R^+ \cap S') \subset F_O(R^+ \cap S)$ which contradicts the hypothesis $S \in S_w(\Pi_{\Sigma\cup\{\mu\}})$.
   (b) if $R' \triangleleft_w R$ then $F_O(R^+ \cap S') \triangleleft_w F_O(R^+ \cap S)$ which contradicts the hypothesis $S \in S_w(\Pi_{\Sigma\cup\{\mu\}})$.
   Thus $R \in R_w(\Sigma \cup \{\mu\})$.
2. We show that $R_w(\Sigma \cup \{\mu\}) \subseteq \{F_O(R^+ \cap S) | S \in S_w(\Pi_{\Sigma\cup\{\mu\}})\}$. Let R be a removed set of $\Sigma \cup \{\mu\}$ according to the comparator $\trianglelefteq_C$. We have $(\Sigma\backslash R) \cup \{\mu\}$ is consistent, so, there exists an interpretation I_R which satisfies $(\Sigma\backslash R) \cup \{\mu\}$. Let $S = \{a | a \in I_R\} \cup \{a' | \neg a \in I_R\} \cup \{\rho | I_R \not\models F_O(\rho)\}$ be a set of atoms. Clearly, $R = F_O(R^+ \cap S)$. According to the Corollary 1, S is an answer set of $\Pi_{\Sigma\cup\{\mu\}}$. Suppose that S is not a preferred answer set of $\Pi_{\Sigma\cup\{\mu\}}$, i.e. $S \notin S_w(\Pi_{\Sigma\cup\{\mu\}})$, we then have two possibilities:
   (a) S is not minimal according to the inclusion: $\exists S' \in S(\Pi_{\Sigma\cup\{\mu\}})$ such that $F_O(S' \cap R^+) \subset F_O(S \cap R^+)$. Since $R = F_O(R^+ \cap S)$, $F_O(S' \cap R^+) \subset R$. Moreover, $F_O(S' \cap R^+) \subseteq \Sigma$ and $(\Sigma\backslash F_O(S' \cap R^+)) \cup \{\mu\}$ is consistent. So R is not a removed set of $\Sigma \cup \{\mu\}$ which contradicts the hypothesis.
   (b) S is not preferred according to the comparator $\triangleleft_C$:$\exists S' \in S(\Pi_{\Sigma\cup\{\mu\}})$ such that $F_O(S' \cap R^+) \triangleleft_w F_O(S \cap R^+)$. As $R = F_O(R^+ \cap S) = F_O(R^+ \cap S), F_O(S' \cap R^+) \triangleleft_w R$. Moreover, $F_O(S' \cap R^+) \subseteq \Sigma$ and $(\Sigma \backslash F_O(S' \cap R^+)) \cup \{\mu\}$ is consistent. So, R is not a removed set $\Sigma \cup \{\mu\}$ which contradicts the hypothesis.

Therefore, $S \in S_w(\Pi_{\Sigma\cup\{\mu\}})$. □

References

[1] S. Benferhat, D. Dubois, C. Cayrol, J. Lang, H. Prade, Inconsistency management and prioritized syntax-based entailment, in: IJCAI'03, 1993, pp. 640–647.
[2] Salem Benferhat, Sébastien Konieczny, Odile Papini, Ramón Pino Pérez, Iterated revision by epistemic states: axioms, semantics and syntax, in: ECAI'00, pp. 13–17.
[3] S. Benferhat, D. Dubois, S. Lagrue, O. Papini, Making revision reversible: an approach based on polynomials, Fundamenta Informaticae 53 (3,4) (2002) 251–280.
[4] S. Benferhat, S. Lagrue, O.Papini, Revising partially ordered belief, in: Proceedings of the NMR'02, Toulouse, France, 2002.
[5] S. Benferhat, S. Lagrue, O. Papini, Revision with partially ordered information in a possibilistic framework, Fuzzy Sets and Systems (2004) 25–41.
[6] S. Benferhat, S. Lagrue, O. Papini, Revision of partially ordered information, in: Proceedings of the IJCAI'05, Edinburgh, 2005, pp. 376–381.
[7] S. Benferhat, S. Yahi, Complexity and cautiousness results for reasoning from partially preordered belief bases, in: Proceedings of ECSQARU '09, LNCS, vol. 5590, Springer-Verlag, 2009, pp. 817–828.
[8] S. Benferhat, J. Ben-Naim, O. Papini, E. Würbel, Answer set programming encoding of prioritized removed sets revision: application to GIS, Applied Intelligence 32 (1) (2010) 60–87.
[9] P. Besnard, Remedying inconsistent sets of premises, International Journal of Approximate Reasoning 45 (2) (2007) 308–320.
[10] R. Booth, T. Meyer, Admissible and restrained revision, Journal of Artificial Intelligence Research (2006) 127–151.
[11] R. Booth, A. Nittka, Reconstructing an agent's epistemic state from observations about its beliefs and non-beliefs, Journal of Logic and Computation (2008) 755–782.
[12] C. Boutilier, Revision sequences and nested conditionals, in: IJCAI'93, 1993, pp. 519–531.
[13] C. Boutilier, Iterated revision and minimal change of conditional beliefs, Journal of Philosophical Logic 25 (3) (1996) 262–305.
[14] R. Bruni, On exact selection of minimally unsatisfiable subformulae, Annals of Mathematics and Artificial Intelligence 43 (1) (2005) 35–40.
[15] C. Cayrol, M.-C. Lagasquie-Schiex, Non-monotonic syntax-based entailment: a classification of consequence relations, in: Proceedings of ECSQARU'95, LNAI, vol. 946, Springer-Verlag, 1995, pp. 107–114.
[16] A. Darwiche, J. Pearl, On the logic of iterated belief revision, Artificial Intelligence 89 (1–2) (1997) 1–29.
[17] J.P. Delgrande, A. Hunter, T. Schaub, COBA: a consistency-based belief revision system, in: JELIA'02, 2002, pp. 509–512.
[18] J.P. Delgrande, D. Dubois, J. Lang, Iterated revision as prioritized merging, in: KR'06, 2006, pp. 210–220.
[19] J.P. Delgrande, Y. Jin, Parallel belief revision, in: AAAI'08, 2008, pp. 430–435.
[20] D. Dubois, H. Prade, Belief change and possibility theory, in: P. Gärdenfors (Ed.), Belief Revision, Cambridge University Press, UK, 1992, pp. 142–182.
[21] D. Dubois, H. Prade, A synthetic view of belief revision with uncertain inputs in the framework of possibility theory, International Journal of Approximated Reasoning (1997) 295–324.
[22] P. Gärdenfors, Knowledge in Flux: Modeling the Dynamics of Epistemic States, Bradford Books, MIT Press, Cambridge, 1988.
[23] M. Gebser, B. Kaufmann, A. Neumann, T. Schaub, Clasp: a conflict-driven answer set solver, in: LPNMR'07, Springer, 2007, pp. 260–265.
[24] M. Gelfond, V. Lifschitz, The stable model semantics for logic programming, in: Robert A. Kowalski, Kenneth Bowen (Eds.), Proceedings of the ICLP'88, The MIT Press, 1988, pp. 1070–1080.
[25] E. Gregoire, B. Mazure, C. Piette, Extacting MUSes, in: Proceedings of ECAI'06, 2006, pp. 387–391.
[26] J.Y. Halpern, Defining relative likelihood in partially-ordered structures, in: UAI'96, 1996, pp. 299–306.
[27] J. Hué, E. Würbel, O. Papini, Removed sets fusion: performing off the shelf, in: Malik Ghallab, Constantine D. Spyropoulos, Nikos Fakotakis, Nikos Avouris (Eds.), Proceedings of the ECAI'08, 2008.
[28] Y. Jin, M. Thielscher, Iterated belief revision, revised, Artificial Intelligence, 2007, pp. 1–18.
[29] U. Junker, G. Brewka, Handling partially ordered defaults in TMS, in: Proceedings of IJCAI'89, 1989, pp. 1043–1048.
[30] H. Katsuno, A. Mendelzon, Propositional knowledge base revision and minimal change, Artificial Intelligence (1991) 263–294.
[32] S. Lagrue, Gestion d'informations partiellement ordonnées: raisonnement, révision et information géographique, Université du Sud Toulon – Var, 2003.
[33] J. Lang, P. Marquis, Resolving inconsistencies by variable forgetting, in: Proceedings of KR'02, Morgan Kaufman Publishers, 2002, pp. 239–250.
[34] D. Lehmann, Belief revision revisited, in: IJCAI'95, 1995, pp. 1534–1539.
[35] D.K. Lewis, Counterfactuals, Harvard University Press, Cambridge, Mass, 1973.
[36] F. Lin, R. Reiter, Forget it!, in: Proceedings of AAAI'94, 1994, pp. 154–159.
[37] A.C. Nayak, Iterated belief change based on epistemic entrenchment, Erkenntnis (1994) 353–390.
[38] B. Nebel, Base revision operations and schemes: semantics, representation, and complexity, in: Proceedings of the 11th European Conference on Artificial Intelligence, John Wiley & Sons, 1994, pp. 341–345.
[39] I. Niemelä, Logic programs with stable model semantics as a constraint programming paradigm, Annals of Mathematics and Artificial Intelligence 25 (3–4) (1999) 241–273.
[40] O. Papini, Iterated revision operations stemming from the history of an agent's observations, in: M.A. Williams, H. Rott (Eds.), Frontiers of Belief Revision, Kluwer Acad. Publisher, 2001, pp. 279–301.
[41] R. Reiter, A theory of diagnosis from first principles, Artificial Intelligence 32 (1) (1987) 57–95.
[42] N. Rescher, R. Manor, On inference from inconsistent premises, Theory and Decision 1 (1970) 179–219.
[43] M. Sérayet, O. Curé, P. Drap, O. Papini, J. Seinturier, WP3R2 application ontology for archaeological information – specification, Final product, VENUS project, July, 2009.
[44] P. Simons, Extending and implementing the stable model semantics, CoRR, cs.LO/0005010, 2000.
[45] M.-A. Williams, Transmutations of knowledge systems, in: KR'94, 1994, pp. 619–629.
[46] M.A. Williams, Iterated theory base change: a computational model, in: IJCAI'95, Montreal, Quebec, 1995, pp. 1541–1547.
[47] M.A. Williams, D. Williams, A belief revision system for the world wide web, in: Proceedings of the IJCAI Workshop of the Future of Artificial Intelligence and the Internet, 1997, pp. 39–51.
[48] E. Würbel, R. Jeansoulin, O. Papini, Revision : an application in the framework of GIS, in: Anthony G. Cohn, Fausto Giunchiglia, Bart Selman (Eds.), Proceedings of the KR'00, 2000, pp. 505–516.
[49] S. Yahi, S. Benferhat, S. Lagrue, M. Sérayet, O. Papini, A lexicographic inference for partially preordered belief bases, in: Gerhard Brewka, Jérôme Lang (Eds.), Proceedings of the KR'08, 2008, pp. 507–516.