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# Estimation of Contact Forces and Tire Road Friction

A.Rabhi<sup>1</sup>, N.K. M'Sirdi<sup>2</sup> and A.Elhajjaji<sup>1</sup>

<sup>1</sup>C.R.E.A, 7 Rue du Moulin Neuf, 80000 Amiens, France

<sup>2</sup>LSIS, CNRS UMR 6168. Dom. Univ. St Jérôme,

Av. Escadrille Normandie - Niemen 13397. Marseille Cedex 20. France.

abdelhamid.rabhi@u-picardie.fr

*Abstract*— In this paper a robust differentiator via sliding mode is used to build up an estimation scheme allowing to identify the tire road friction. The estimations are produced in three steps as cascaded observers and estimator. The first produces estimations of velocities and accelerations. The second estimate the tire forces and the last reconstruct the friction coefficient. The actual results show effectiveness and robustness of the proposed method.

*Keywords*— Vehicle dynamics, Sliding Modes observer, Robust nonlinear observers, tire forces, estimation.

## I. INTRODUCTION

In recent years, the increasing demand for the safety of an automobile has promoted research and development of the technology of active safety. One of the important factors determining vehicle dynamics including safety is road friction. Advanced control of automotive vehicle systems is more and more discussed in the open literature and widely developed in industry. International Workshops and Journal special issues have been dedicated to automotive control.

The tire forces properties affect the vehicle dynamic performance. The control of ground-vehicle interactions becomes important. The design of traction controller is based on the assumption that vehicle and wheel angular velocities are both available on-line by direct measurements and/or estimations. Thus the knowledge of tire parameters and variables (forces, velocities, wheel and slip) is essential to advanced vehicle control systems.

However, tire forces and road friction are difficult to measure directly and to represent precisely by some deterministic model equations. In the literature, their values are often deduced by some experimentally approximated models. Recently, many analytical and experimental studies have been performed on estimation of the frictions and contact forces between tires

and road.

In [2][1] application of sliding mode control is proposed. Observers based on the sliding mode approach have been also used in [3]. In [4] an estimation based on least squares method and Kalman filtering is applied for estimation of contact forces. In [5] presented a tire/road friction estimation method based on Kalman filter to give a relevant estimates of the slope of  $\mu$  versus slip ( $\lambda$ ), that is the relative difference in wheel velocity.

The paper [6] presented an estimator for longitudinal stiffness and wheel effective radius using vehicle sensors and GPS for low values of slip. Robust observers with unknown inputs are efficient for estimation of road profile and for estimation of the contact forces [3].

In this work, we deal with a simple vehicle model in order to estimate of tire-road friction. This estimation can be used to detect a critical driving situation to improve the security. It can be used also in several vehicle control systems such as Anti-lock brake systems (ABS), traction control system (TCS), diagnosis system, etc...

The main characteristics of the vehicle longitudinal dynamics are taken into account in the model used to design robust observer and estimations.

The paper is organised as follows: in section 2 we present the vehicle model and in section 3, the design of the second order sliding mode observer is presented. Some results about the states observation, tire forces and tire road friction are presented in section 4. Finally, some remarks and perspectives are given in a concluding section.

## II. VEHICLE MODELING

### A. Complete model

In literature, many studies deal with vehicle modelling [7][8]. These are complex and nonlinear systems constituted with many subsystems: wheels, mo-

tor and system of braking, suspensions, steering, more and more electronic embarked.

The vehicle is a body topic to strengths and external moments following the three axes:

- Longitudinal,
- Lateral,
- Vertical,

We can define as dynamic equations of the vehicle by applying the principles fundamental of the dynamics. The dynamic equations for the movements of transfer of the case are:

$$\begin{aligned} m\dot{v}_x &= \sum F_x \\ m\dot{v}_y &= \sum F_y \\ m\dot{v}_z &= \sum F_z \end{aligned} \quad (1)$$

where:

$m$  is the total mass of the vehicle.

$v = [v_x, v_y, v_z]^T$  : vehicle velocities along  $x$ ,  $y$ ,  $z$ ,

$\sum F_x$ ,  $\sum F_y$  and  $\sum F_z$ : forces with respect to  $x$ ,  $y$  and  $z$ ,

The balance of the moments, following the three directions, is given by:

$$J \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix} \quad (2)$$

$\sum M_x$ ,  $\sum M_y$ ,  $\sum M_z$ : moments with respect to  $x$ ,  $y$  and  $z$  axes.

The wheel angular motion is given by:

$$\begin{aligned} \dot{\omega}_{fl} &= \frac{1}{I_\omega} (C_{fl} - R_\omega F_{xf1}) \\ \dot{\omega}_{fr} &= \frac{1}{I_\omega} (C_{fr} - R_\omega F_{xf2}) \\ \dot{\omega}_{rl} &= \frac{1}{I_\omega} (C_{rl} - R_\omega F_{xr1}) \\ \dot{\omega}_{rr} &= \frac{1}{I_\omega} (C_{rr} - R_\omega F_{xr2}) \end{aligned} \quad (3)$$

with:

$\omega_f$  and  $\omega_r$  are the rotation velocities of the front and rear wheel.

$C_{mi}$  is the motor couple applied at wheel  $i$

$T_i$  is the braking couple applied at wheel  $i$

$r_1$  is the distance between the center of gravity and the front axis

$r_2$  is the distance between the center of gravity and the rear axis

The model is nonlinear, we can write it :

$$\dot{x} = f(x, u) \quad (4)$$

## B. Partial model

The complete models are difficult to use in control applications. The majority of applications deals with simplified and partial models. Let consider the simplified motion dynamics of a quart-vehicle model, describing only nominal behavior. This model considers the main characteristics of the longitudinal dynamic. For a global application, this method can be easily extended to the complete vehicle and involve the four coupled wheels.

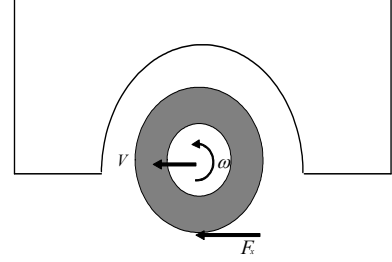


Fig. 1. Vehicle model

Applying Newton's law to wheel and vehicle dynamics gives us the equations of nominal dynamics in the motion.

$$\begin{aligned} m\dot{v}_x &= F_x \\ J_r\dot{\omega} &= T - rF_x \end{aligned}$$

where  $m$  is the vehicle mass and  $J_r$ ,  $r$  are the inertia and effective radius of the tire respectively.

$v_s$  is the linear velocity of the vehicle,  $\omega$  is the angular velocity of the considered wheel.

$T$  is the accelerating (or braking) torque, and  $F_x$  is the tire/road friction force.

The traction (respectively braking) force, produce at the tire/road interface when a driving (braking) torque is applied to pneumatic tire, has opposed the direction of relative motion between the tire and road surface. This relative motion exhibits the tire slip properties.

The wheel-slip is due to deflection in the contact patch. The longitudinal wheel slip is generally called the slip ratio and is given by a kinematic relation as [6].

$$\lambda = \frac{|v_r - v_x|}{\max(v_r, v_x)} \quad (5)$$

where  $v_r$  is the wheel velocity.

Representing the adhesion coefficient as a function of the wheel slip yields the adhesion characteristic  $\mu(\lambda)$ , which primarily depends on the road conditions. Surfaces are shown in the following figure 2.

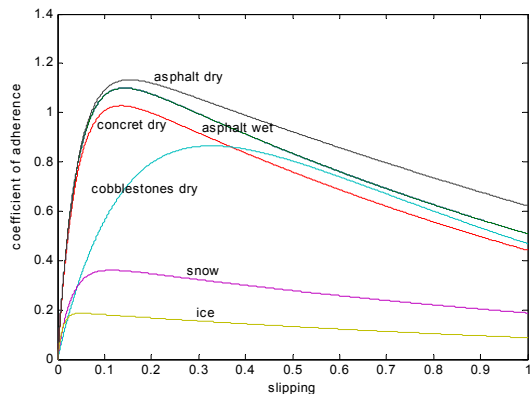


Fig. 2. Wheel slip

The figure 2 shows the variations between coefficient of road adhesion  $\mu$  and longitudinal slip  $\lambda$  for different road surface conditions. It can be observed that all curves  $\mu(\lambda)$  start at  $\mu = 0$  for zero slip, which corresponds to the non-braked, free rolling wheel. With increasing slip ratio of 3% and 20%. Beyond this maximum value the slope of the adhesion characteristic is negative. At a slip ratio of 100% the wheel is completely skidding, which corresponds to the locking of the wheel.

The adhesion characteristic plays an essential role for both the design and the validation of ABS. Overall, to improve the functioning of an ABS it is desirable to have some real-time information about the adhesion characteristic.

By assuming that the longitudinal forces are proportional to the transversal ones, we expressed these forces as follows:

$$F_x = \mu F_z \quad (6)$$

where  $F_z$  is the vertical force of the wheel.

The vertical forces that we use in our model are function of the longitudinal acceleration and the height of the center of gravity.

The vertical force can be represented as:

$$F_z = \frac{m}{2(l_f + l_r)}(gl_r - h\dot{v}_x) \quad (7)$$

where:

$h$  is the height of the center of gravity

$l_f$  is the distance between the center of gravity and the front axis center of gravity.

$l_r$  is the distance between the center of gravity and the rear axis center of gravity.

### III. OBSERVER DESIGN

The sliding mode technique is an attractive approach [9]. The primary characteristic of SMC is that the feedback signal is discontinuous, switching on one or several manifolds in the state-space. In what follows, we develop a second order differentiator in order to estimate the tire road friction.

#### A. High Order Sliding Mode Observer (HOSM)

Consider a smooth dynamics function,  $s(x) \in \mathbb{R}$ . The system containing this variable may be closed by some possibly-dynamical discontinuous feedback where the control task may be to keep the output  $s(x(t)) = 0$ . The sliding variable, the  $r^{\text{th}}$  order sliding mode is determined by  $s = \dot{s} = \ddot{s} = \dots = s^{(r-1)} = 0$ , which form an  $r$ -dimensional condition on the state of dynamic system [13][14].

The Levant observer will produce estimates of the successive derivatives. HOSM presents even better robust performance than traditional first order sliding mode.

HOSM dynamics converge toward the origin of surface coordinates in finite time always that the order of the sliding controller is equal or bigger than the sum of a relative degree of the plant and the actuator.

#### B. Robust Differentiation Estimator (RDE)

To estimate the derivatives  $s_1$  and  $s_2$  without its direct calculations of derivatives, we will use the  $2^{\text{nd}}$ -order exact robust differentiator of the form [14]

$$\begin{aligned} \dot{z}_0 &= v_0 = z_1 - \lambda_0 |z_0 - s_\omega|^{\frac{2}{3}} \text{sign}(z_0 - s_\omega) \\ \dot{z}_1 &= v_1 = -\lambda_1 \text{sign}(z_1 - v_0)^{\frac{1}{2}} \text{sign}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\lambda_2 \text{sign}(z_2 - v_1) \end{aligned}$$

where  $z_0$ ,  $z_1$  and  $z_2$  are the estimate of  $s_\omega$ ,  $s_1$  and  $s_2$ , respectively,  $\lambda_i > 0$ ,  $i = 0, 1, 2$ . Under condition  $\lambda_0 > \lambda_1 > \lambda_2$  the third order sliding mode motion will be established in a finite time. The obtained estimates are  $z_1 = s_1 = \dot{s}_\omega$  and  $z_2 = s_2 = \ddot{s}_\omega$  then they can be used in the estimation of the state variables and also in the control.

#### C. Cascaded Observers - Estimators

In this section we develop a robust second order differentiator to build up an estimation scheme allowing

to identify the tire road friction.

The estimations will be produced in three steps as cascaded observers and estimator in order to reconstruct information and system states step by step. This approach allow us to avoid the observability problems dealing with inappropriate use of modeling equations. For vehicle systems it is very hard to build up a complete and appropriate model for observation of all the system states. Thus in our work, we avoid this problem by means of use of simple and cascaded models suitable for robust observers design.

The first step produces estimations of velocities. The second one estimate the tire forces (vertical and longitudinal ones) and the last step reconstruct the friction coefficient.

The robust differentiation observer is used for estimation of the velocities and accelerations of the wheels. The wheels angular positions and the velocity of the vehicles body  $v_x$ , are assumed available for measurements. The previous Robust Differentiation Estimator is useful for retrieval of the velocities and accelerations.

1<sup>st</sup> Step:

$$\begin{aligned}\dot{\hat{\theta}} &= v_0 = \hat{\omega} - \lambda_0 \left| \theta - \hat{\theta} \right|^{\frac{2}{3}} \text{sign}(\theta - \hat{\theta}) \\ \dot{\hat{\omega}} &= v_1 = \dot{\hat{\omega}} - \lambda_1 \text{sign}(\hat{\omega} - v_0)^{\frac{1}{2}} \text{sign}(\hat{\omega} - v_0) \\ \ddot{\hat{\omega}} &= -\lambda_2 \text{sign}(\dot{\hat{\omega}} - v_1)\end{aligned}$$

The convergence of these estimates is garanteed in finite time  $t_0$ .

2<sup>nd</sup> Step:

In the second step we can estimate the forces  $F_x$  and  $F_z$ . Then to estimate  $F_x$  we use the following equation,

$$J\dot{\hat{\omega}} = T - R_{ef}\hat{F}_x \quad (8)$$

In the simplest way, assuming the input torques known, we can reconstruct  $F_x$  as follows:

$$\hat{F}_x = \frac{(T - J\dot{\hat{\omega}})}{R_{ef}} \quad (9)$$

$\dot{\hat{\omega}}$  is produced by the *RDE*. Note that any estimator with output error can also be used to enhance robustness versus noise.

After those estimations, their use in the same time with the system equations allow us to retrieve de vertical forces  $F_z$  as follows. To estimate  $F_z$  we use the following equation

$$\hat{F}_z = \frac{m}{2(l_f + l_r)}(gl_r - h.\dot{v}_x) \quad (10)$$

$\hat{v}_x$  is produced by the *RDE*.

3<sup>rd</sup> Step:

At this step it only remains to estimate the adherence or friction coefficient. To this end we assume the vehicle rolling in a normal or steady state situation in order to be able to approximate this coefficient by the following formula

$$\hat{\mu} = \frac{\hat{F}_x}{\hat{F}_z} \quad (11)$$

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

In this section, we give somme results in order to test and validate our approach and the proposed observer. In simulation, the state and forces are generated by use a cae simulateur VEDYNA [15].

The braque torque is shown in figure 3.

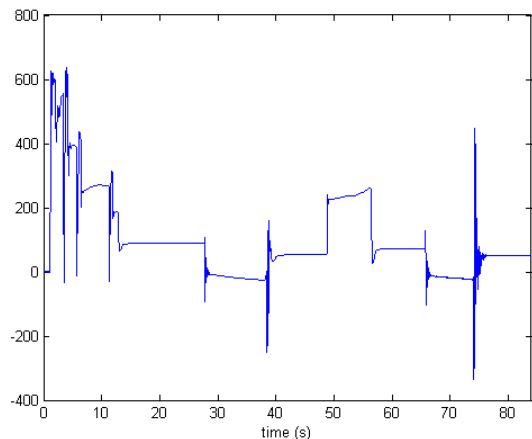


Fig. 3. Braking torque

Figure 4 shows the measured and estimated wheel angular position. This signal is used to estimat velocities and accelerations.

Figure 5 shows the estimated wheel velocity. In the figure 6, we represente the estimation of vehicle velocity. The figure shows the good convergence to the actual vehicle velocity.

Figure 7 shows the obtained vehicle acceleration. The observer allows a good estimation of angular velocity and acceleration.

The last step gives us the estimated longitudinal forces  $F_x$  and normal forces  $F_z$  wich are presented in figure 8 and 9. Finally road friction coefficient is deduced and presented in 10.

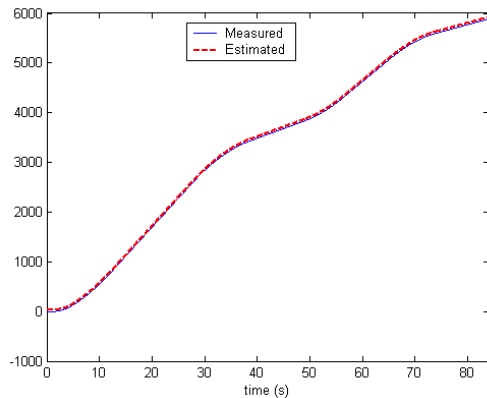


Fig. 4. Angular displacements

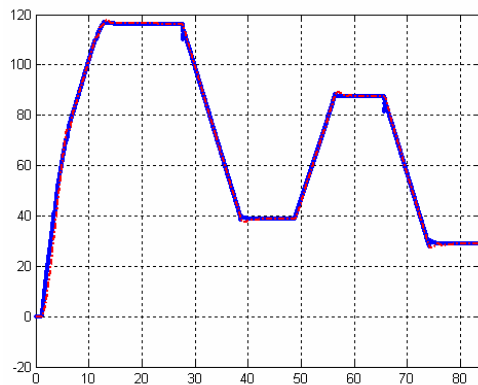


Fig. 5. angular velocity

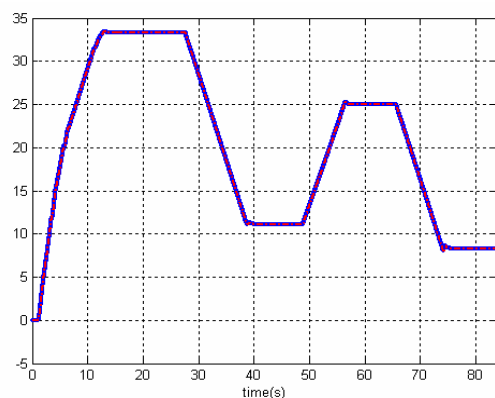


Fig. 6. Vehicle velocity

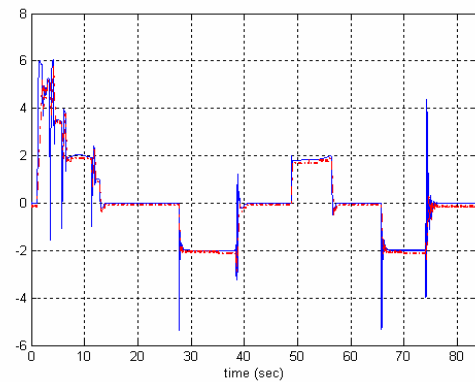


Fig. 7. Estimated and measured acceleration

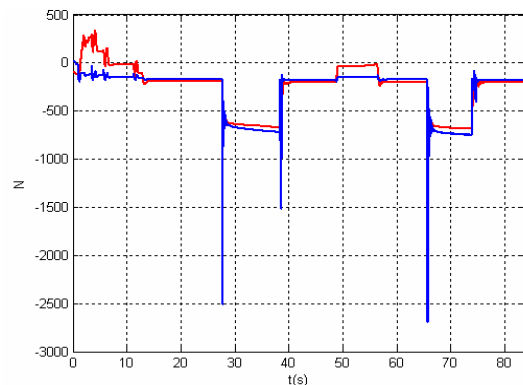


Fig. 8. Longitudinal force

## V. CONCLUSIONS

In this paper, we have proposed an efficient and robust second differentiator to build up an estimation scheme allowing to identify the tire road friction. The estimations produced in three steps by cascaded observers and estimator show good performances. Tire forces (vertical and longitudinal ones) are also. Simulation results are presented to illustrate the ability of this approach to give estimation of both vehicle states and tire forces. The robustness of the sliding mode observer versus uncertainties on model parameters has also been emphasized in simulation. Implementation of this approach to the vehicle is under investigation.

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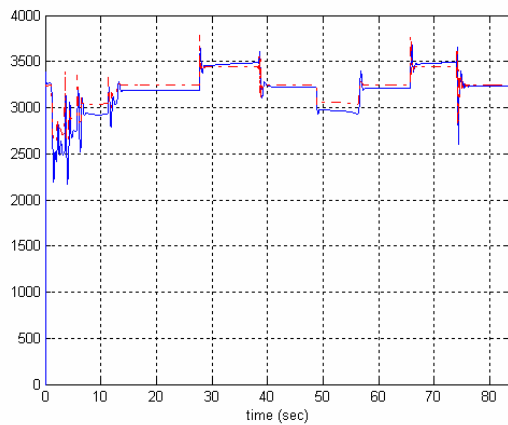


Fig. 9. Normal force  $F_z$

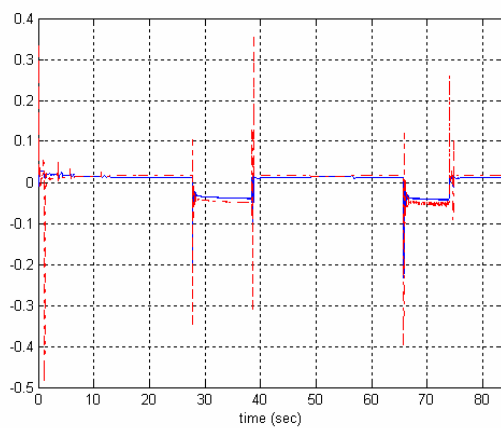


Fig. 10. Road friction

and Road Profile AVCS'04 International Conference on Advances in Vehicle Control and Safety Genova -Italy, October 28-31 2004

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