Second Order Sliding-Mode Observer for Estimation of Vehicle Dynamic Parameters

N. M'Sirdi, Abdelhamid Rabhi, L. Fridman, J. Davila, Y Delanne

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N.K. M'sirdi[1], A. Rabhi[1], L. Fridman[2], J. Davila[2] and Y. Delanne[3]
[1]LSIS, CNRS UMR 6168. Dom. Univ. St Jérôme, Av. Escadrille Normandie - Niemen
13397. Marseille Cedex 20. France. e-mail: msirdin@lsis.org
[2]UNAM Dept of Control, Division of Electrical Engineering,Faculty of Engineering, Ciudad Universitaria,
Universidad Nacional Autonoma de Mexico, 04510, Mexico, D.F., Mexico
[3] LCPC Nantes: Division ESAR BP 44341 44 Bouguenais Cedex

***Abstract*—This paper uses second-order sliding mode observers to build up an estimation scheme allowing to identify the tire longitudinal equivalent stiffness and the effective wheel radius using the existing ABS angular sensors. This estimation strategy, based on use of the proposed observer could be used with data acquired experimentally to identify the longitudinal stiffness and effective radius of vehicle tires. The actual results show effectiveness and robustness of the proposed method.**

***Index Terms*—Sliding modes, nonlinear observers, robust state and stiffness estimation, wheel slip estimation.**

I. INTRODUCTION

Car accidents occur for several reasons which may involve the driver or components of the vehicle or environment. Such situations appears when the vehicle is driven beyond the adherence or stability limits. However new active safety systems are developed, improved, and installed on vehicles for real-time monitoring and controlling the dynamic stability (Electronic Braking Systems (EBS), Anti-lock Braking Systems (ABS), Electronic Stability Program (ESP)). The active safety becomes more important in recent research on Intelligent Transportation Systems (ITS) technology. Nevertheless, the possibility of rectifying an unstable condition can be compromised by physical limits. Therefore, it is extremely important to detect (on time) a tendency towards instability. This has to be done without adding expensive sensors, so it requires quite robust observers looking forward based on the physics of interacting systems (the vehicle, the driver and the road).

The tire forces properties affect the vehicle dynamic performance. The control of ground - vehicle interactions becomes important due to research efforts on intelligent transportation systems, and specially, on automated highway systems. The design of traction controller is based on the assumption that vehicle and wheel angular velocities are both available on-line by direct measurements and/or estimations. Thus the knowledge of tire parameters and variables (stiffness, forces, velocities, wheel slip and radius) is essential to advanced vehicle control systems such as ABS, Traction Control Systems (TCS) and ESP [1]–[3]. However, tire forces and road friction are difficult to measure directly and to represent precisely by some deterministic model equations. In the literature, their values are often deduced by some experimentally approximated models [4]–[10]. This work is focused to the on-line estimation of the tires sleep, adherence, stiffness and effective radius. The vehicle state is estimated and the tire forces are identified[3]. The main contribution is the robust on-line estimation of the tire effective radius, wheel sleep and velocities, needed for a control, by using only simple low cost sensors (ABS sensors).

Recently, many analytical and experimental studies have been performed on estimation of the frictions and contact forces between tires and road [11], [12], [6]. **Tire forces can be represented by the nonlinear (stochastic) functions of wheel slip. The deterministic tire models encountered are complicated and depend on several factors (as load, tire pressure, environmental characteristics, etc.) [10],[13]–[17].** This makes on-line estimation of forces and parameters difficult for vehicle control applications and detection and diagnosis for driving monitoring and surveillance [18]. In [19], [20], [18], application of sliding mode control is proposed. Observers based on the sliding mode approach have been also used in [22]. In [11] an estimation based on least squares method and Kalman filtering is applied for estimation of contact forces. In [6] presented a tire/road friction estimation method based on Kalman filter to give a relevant estimates of the slope of μ versus slip (λ), that is, the relative difference in wheel velocity. The paper [21] presented an estimator for longitudinal stiffness and wheel effective radius using vehicle sensors and Global Positioning System (GPS) for low values of slip.

Observers robust to unknown inputs are efficient for estimation of road profile and the contact forces [18], [22]. Acceleration and braking maneuvers modify the wheel slip. This phenomenon could be controlled by means of its regulation while using sliding mode approach [2], [22]. This methods enhances the road safety leading better vehicle adherence and maneuvers ability but the vehicle controllability in its environment along the road admissible trajectories still remain an important open problem.

¿From the other hand, it is necessary to remark that observers for mechanical systems with unknown inputs based on standard first order sliding mode approach (as for example [23], [24], [25]) has the following disadvantages:

•for observation of the velocity a filtering is needed corrupting the results;

•the need of filtering in the observation process destroys the finite time convergence property, and the separation principle must be taken into account to design a control;

•for the uncertainties and parameters identification a second filtering is necessary. This leads to a bigger corruption of results.

A robust exact differentiator [30] based on super twisting algorithm ([27]) ensures a finite time convergence to the values of the corresponding derivatives and provides the best possible accuracy of the derivatives for the given value even considering deterministic noise, sampling step and in the case of discrete measurements.

In this paper, a nominal model of the vehicle is considered and the the super-twisting based robust exact observer [26] is applied for estimation of rotational velocities. The stiffness and effective radius are identified by application of a dynamical identification algorithm. The robust exact observer [26] used in this paper allows

•to make the velocity observation without filtering;

•to provide finite time convergence to the exact value of the rotational velocity, ensuring separation principle;

•to identify the uncertainties with only just one filtering;

•to apply a continuous time parameter identification algorithm for system parameters identification.

This work deals with a simple vehicle model coupled with wheel - road contact. It is proposed a vehicle model for the online estimation using robust observers. The main characteristics of the vehicle longitudinal dynamics were taken into account in the developed model. The obtained dynamics equations may be written in a state space allowing to define an observer based on the sliding mode approach (as presented in [18], [2],[22]). The observer has been used to reconstruct the global system state components and then to estimate the tires forces [2], [22]. The use of sliding mode approach has been motivated by its robustness with respect to the parameters and modeling errors and has been shown to cope well with this problem.

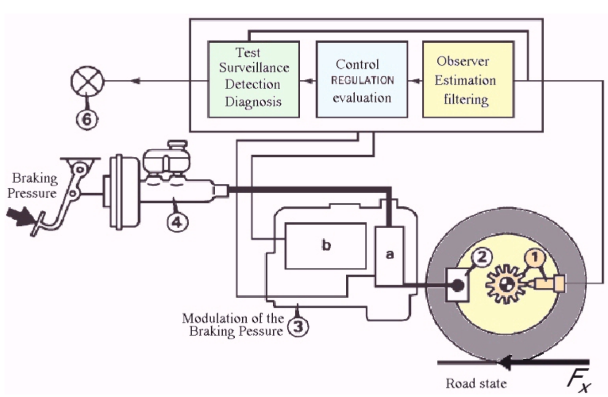

Fig. 1. Wheel dynamics and the ABS system

It is presented, a method to estimate the wheel angular velocities by considering the wheel angular position measurements (produced by an ABS variable reluctance sensor as shown in figure 1). As a second step, we estimate the longitudinal stiffness and wheel effective radius using additional sensors for the accelerating torque and the linear velocity of the vehicle. The proposed method of estimation is verified through one-wheel simulation model with a "Magic formula" tire model and then application results (on a Peugeot 406) show an excellent reconstruction of the velocities, tire forces and radius estimation.

The developed estimations can be used to detect critical driving situations and then improve the security. It can be used also in several vehicle control systems such as ABS, TCS, diagnosis systems, etc...

II. PROBLEM STATEMENT

Consider the simplified motion dynamics of a quarter-vehicle model, capturing only nominal behavior. This model retains the main characteristics of the longitudinal dynamic. For a global application, this method can be easily extended to the complete vehicle and involve the four wheels.

Applying Newton's law to wheel and vehicle dynamics, the equations of nominal motion is given by

$$\dot{\theta} = \omega \tag{1}$$

$$J\dot{\omega} = J\ddot{\theta} = T_f - R_e F_x \tag{2}$$

$$m\dot{v}_x = F_x \tag{3}$$

where m is the vehicle mass and J,R_e are the inertia and effective radius of the tire, respectively. v_x is the linear velocity of the vehicle, θ is the angular position of the considered wheel, ω is the angular velocity of the considered wheel, T_f is the accelerating (or braking) torque, and F_x is the tire/road friction force. The tractive (respectively braking) force, produced at the tire/road interface when a driving (braking) torque is applied to a pneumatic tire, has an opposed direction to relative motion between the tire and road surface. This relative motion determines the tire slip properties. The wheel - slip is due to deflection in the contact patch. The longitudinal wheel-slip λ is generally called the slip ratio and can be described by a kinematic relation like [13]:

$$\begin{cases} \lambda = \frac{R_{ef}\omega}{v_x} - 1 \text{ if } v_x > R_{ef}\omega \text{ (braking)} \\ \lambda = 1 - \frac{R_{ef}\omega}{v_x} \text{ if } v_x < R_{ef}\omega \text{ (traction)} \end{cases} \tag{4}$$

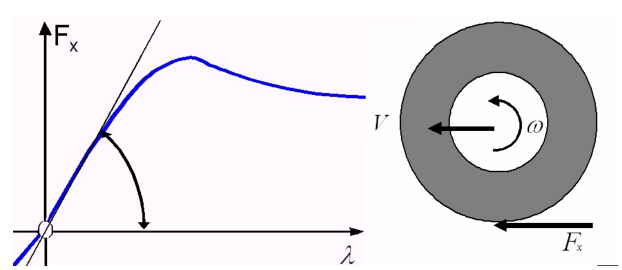

Fig. 2. Wheel slip - forces steady state characteristics

During ordinary driving, however, the tire slip rarely exceeds 5%. By linearizing the model in a small region (around origin), the force slip relation can be characterized as follows

$$F_x = C_x(\frac{v_x - R_e\omega}{v_x}) \tag{5}$$

Where F_x and C_x are, respectively, the force and the longitudinal stiffness of the tire(s). The dynamic equation of the whole system can be written in state space form by defining the following state variables. The angular position $x_1 = \theta$ is measured by the ABS sensor. The angular velocity $x_2 = \dot{\theta} = \omega$ is not measured and can be obtained by observer application. The vehicle velocity $x_3 = v_x$, and the accelerating torque $u = T_f$ are assumed measurable. Note that these expressions assume that velocity is non zero by definition. We can write the system model

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{u}{J} - \frac{R_e C_x}{J} + \frac{R_e^2 C_x}{J}\frac{x_2}{x_3} \\ \dot{x}_3 &= \frac{C_x}{m} - \frac{C_x R_e}{m}\frac{x_2}{x_3} \\ y &= \begin{bmatrix} x_1 & 0 \\ 0 & x_3 \end{bmatrix} \end{aligned} \tag{6}$$

The task is to reconstruct the angular velocity (x_2) of the system by using x_1 and u. The equivalent output injection will be used for parameters identification. An auxiliary system will be introduced for the variable x_3 in order to obtain an equivalent output injection for this variable.

III. State observation

A. States x_1, x_2

Consider the subsystem with state variables $x_1 = \theta$, $x_2 = \dot{\theta} = \omega$, and the control input $u = T_f$ (may be computed in function of the system states or their estimates), this submodel of (6) can be rewritten in the state space form as follows:

$$\begin{aligned} &\dot{x}_1 = x_2, \\ &\dot{x}_2 = f_1(t, x_1, x_2, u) + \xi_1(t, x_1, x_2, u) \\ &y = x_1, \end{aligned} \tag{7}$$

where the nominal part of the system dynamics is represented by $f_1(t, x_1, x_2, u) = \frac{u}{J}$ containing the known nominal functions, while uncertainties are concentrated in the term $\xi_1(t, x_1, x_2, u) = -\frac{R_e C_x}{J} + \frac{R_e^2 C_x}{J}$. The system (7), understood in Filippov's sense [31] is assumed such that the functions $f_1(t, x_1, x_2, u)$ and the perturbation $\xi_1(t, x_1, x_2, u)$ are Lebesgue-measurable and uniformly bounded in any compact region of the state space.

Our task is then to design a finite-time convergent observer of the angular velocity $x_2 = \dot{\theta} = \omega$ assuming that the position $x_1 = \theta$, the torque u, and the nominal model are available. Only the scalar case $x_1, x_2 \in R$ is considered for simplicity. In general case the observers are constructed in exactly the same way for each wheel position variable x_{1j} in parallel.

The proposed super-twisting observer for the system (7) takes the form [26]

$$\begin{aligned} &\dot{\hat{x}}_1 = \hat{x}_2 + z_1 \\ &\dot{\hat{x}}_2 = f_1(t, x_1, \hat{x}_2, u) + z_2 \end{aligned} \tag{8}$$

where $\hat{x}_1$ and $\hat{x}_2$ are the state estimations, and the correction variables z_1 and z_2 are calculated by the super-twisting algorithm

$$\begin{aligned} &z_1 = \lambda|x_1 - \hat{x}_1|^{1/2}\, sign(x_1 - \hat{x}_1) \\ &z_2 = \alpha\, sign(x_1 - \hat{x}_1). \end{aligned} \tag{9}$$

It is taken for ensures observer convergence that at the initial moment $\hat{x}_1 = x_1$ and $\hat{x}_2 = 0$.

Taking $\tilde{x}_1 = x_1 - \hat{x}_1$ and $\tilde{x}_2 = x_2 - \hat{x}_2$ we obtain the error equations

$$\begin{aligned} &\dot{\tilde{x}}_1 = \tilde{x}_2 - \lambda|\tilde{x}_1|^{1/2}\, sign(\tilde{x}_1) \\ &\dot{\tilde{x}}_2 = F(t, x_1, x_2, \hat{x}_2) - \alpha\, sign(\tilde{x}_1) \end{aligned} \tag{10}$$

where $F(t, x_1, x_2, \hat{x}_2) = f_1(t, x_1, x_2, u) - f_1(t, x_1, \hat{x}_2, u) + \xi_1(t, x_1, x_2, u)$. In our case, the system states are bounded, then the existence of a constant f^+ is ensured such that

$$|F(t, x_1, x_2, \hat{x}_2)| < f^+ \tag{11}$$

holds for any possible t, x_1, x_2 and $|\hat{x}_2| \leq 2\sup|x_2|$. The state boundedness is true, because the system (7) is BIBS (Boundary Input - Boundary State) stable, and the control input $u = T_f$ is bounded. The maximal possible acceleration in the system is a priori known and it coincides with the bound f^+. Let α and λ satisfy the following inequalities, where p is some chosen constant, $0 < p < 1$.

$$\begin{aligned} &\alpha > f^+, \\ &\lambda > \sqrt{\frac{2}{\alpha - f^+}\frac{(\alpha + f^+)(1+p)}{(1-p)}}, \end{aligned} \tag{12}$$

Theorem 1: ([26]). Suppose that condition (11) holds for system (7), and the parameters of the observer (8) are selected according to (12). Then, the observer (8) guarantees the convergence of the estimated states $(\hat{x}, \dot{\hat{x}})$ to the real value of the states $(x, \dot{x})$ after a finite time transient, and there exists a time constant t_0 such that for all $t \geq t_0$, $(\hat{x}_1, \hat{x}_2) = (x_1, x_2)$. The proof of this theorem is presented in the work [26].

Let f_1, x, z_1, z_2 be measured at discrete times with the time interval δ, and let t_i, t_{i+1} be successive measurement times. Consider a discrete modification of the observer (the Euler scheme)

$$\begin{aligned} &\hat{x}_1(t_{i+1}) = \hat{x}_1(t_i) + (\hat{x}_2(t_i) + \lambda|x_1(t_i) - \hat{x}_1(t_i)|^{1/2}\, sign(x_1(t_i) - \hat{x}_1(t_i \\ &\hat{x}_2(t_{i+1}) = \hat{x}_2(t_i) + (f_1(t_i, x_1(t_i), \hat{x}_2(t_i), u(t_i)) + \alpha\, sign(x_1(t_i) - \hat{x}_1(t \end{aligned} \tag{13}$$

where $\hat{x}_1(t_i)$, $\hat{x}_2(t_i)$ are the estimated variables.

Theorem 2: ([26]). Suppose that the function f_1 is uniformly bounded and condition (11) holds. Then the observation algorithm (13) with parameters (12) ensures the convergence of the estimation errors to the domain $|\tilde{x}_1| \leq \gamma_1\delta^2$, $|\tilde{x}_2| \leq \gamma_2\delta$ where γ_1, γ_2 are some constants, depending on the observer parameters.

This theorem is proved in [26].

B. State x_3

Consider the subsystem with state variable $x_3 = v_f$, in this case an observer will be introduced in order to obtain an equivalent output injection, in the same form that the states x_1 and x_2, the dynamic equation of x_3 could be written as

$$\begin{aligned} \dot{x}_3 &= f_2(t, x_3, u) + \xi_2(t, x_2, x_3, u) \\ y_2 &= x_3, \end{aligned} \tag{14}$$

in this case, the dynamic of the system is considered as unknown $\xi_2(t, x_2, x_3, u) = \frac{C_x}{m} - \frac{C_x R_e}{m}\frac{x_2}{x_3}$ in consequence $f_2(t, x_3, u) = 0$. The system (14), understood in Filippov's sense [31] is assumed such that the perturbation $\xi(t, x_1, x_2, u)$ is Lebesgue-measurable and uniformly bounded in any compact region of the state space.

Our task is then to design a finite-time convergent observer of the linear velocity x_3. The proposed sliding mode observer is given by

$$\dot{\hat{x}}_3 = z_3 \tag{15}$$

where $z_3 = \beta sign(x_3 - \hat{x}_3)$. Defining $\tilde{x}_3 = x_3 - \hat{x}_3$, the dynamic of the error for x_3 becomes

$$\dot{\tilde{x}}_3 = \xi_2(t, x_2, x_3, u) - \beta\, sign(\tilde{x}_3) \tag{16}$$

where β is chosen such that $\beta > \max(\frac{C_x}{m} - \frac{C_x R_e}{m}\frac{x_2}{x_3}) = \eta$.

Theorem 3: Suppose that $|\dot{x}_3| \leq \eta$, and the parameter β is chosen such that $\beta > \eta$. The observer (15) guarantees the convergence of the estimated state ($\hat{x}_3$) to the real value of the states (x_3) after a finite time transient, and there exists a time constant t_1 such that for all $t \geq t_1$, $\hat{x}_3 = x_3$.

Proof: Consider the Lyapunov function

$$V(\tilde{x}_3) = \frac{1}{2}\tilde{x}_3^2$$

its time derivative

$$\dot{V}(\tilde{x}_3) = \tilde{x}_3\dot{\tilde{x}}_3 = \tilde{x}_3(\xi_2(t, x_2, x_3, u) - \beta\, sign(\tilde{x}_3)) \tag{17}$$

If β is chosen as was given in the theorem (3), then $\dot{V}(\tilde{x}_3) < 0$. This shows that $\tilde{x}_3$ goes to zero in a finite time, then, there exist a constant t_1 such that for all $t \geq t_1$ holds $\tilde{x}_3 = 0$ ■

IV. Equivalent output injection analysis

For the time t_2 where $t_2 = \max(t_0, t_1)$ and for all $t \geq t_2$ the error dynamics (10) and (16) holds

$$\dot{\tilde{x}}_2 = 0 = F(t, x_1, x_2, \hat{x}_2) - \alpha\, sign(\tilde{x}_1) \tag{18}$$

$$\dot{\tilde{x}}_3 = 0 = \xi_2(t, x_2, x_3, u) - \beta\, sign(\tilde{x}_3) \tag{19}$$

Notice in (18) that at this time $\hat{x}_2 = x_2$ and $f_1(t, x_1, x_2, u) \equiv f_1(t, x_1, \hat{x}_2, u)$ in consequence $F(t, x_1, x_2, \hat{x}_2) = \xi_1(t, x_1, x_2, u)$.

It was assumed that the terms z_2, z_3 change at a high (infinite) frequency. However, in reality, various imperfections make the state oscillate in some vicinity of the intersection and components of z_2, z_3 are switched at finite frequency, this oscillations have high and slow frequency components.

The high frequency terms z_2, z_3 are filtered out and the motion in the sliding mode is determined by the slow components [32]. It is reasonable to assume that the equivalent control is close to the slow component of the real control which may be derived by filtering out the high-frequency component using low pass filter.

The filter time constant should be sufficiently small to preserve the slow components undistorted but large enough to eliminate the high frequency component.

Thus the conditions $\tau \to 0$ where τ is the filter time constant, and $\delta/\tau \to 0$, where δ is the sample interval, fulfilled to extract the slow component equal to the equivalent control and to filter out the high frequency component.

The above reasons allows us to write the equivalent output injection as

$$\bar{z}_2 = \xi_1(t, x_2, x_3, u) \tag{20}$$

$$\bar{z}_3 = \xi_2(t, x_2, x_3, u) \tag{21}$$

where $\bar{z}_2$ and $\bar{z}_3$ are the filtered versions of z_2 and z_3 respectively.

V. System Identification

Assuming that J and m are known, and defining $a_1 = \frac{1}{J}$, it is possible to write the system (6) as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a_1 u + \vartheta_1\varphi_1 \\ \dot{x}_3 &= \vartheta_2\varphi_2 \\ y &= \begin{bmatrix} x_1 & 0 \\ 0 & x_3 \end{bmatrix} \end{aligned} \tag{22}$$

with

$$\begin{aligned} \varphi_1(x) &= \begin{bmatrix} \frac{-1}{J} \\ \frac{x_2^2}{Jx_3} \end{bmatrix} \quad \text{and} \quad \varphi_2(x) = \begin{bmatrix} \frac{1}{m} \\ -\frac{x_2}{mx_3} \end{bmatrix} \\ \vartheta_1 &= \begin{bmatrix} R_e C_x & R_e^2 C_x \end{bmatrix} \quad \text{and} \quad \vartheta_2 = \begin{bmatrix} C_x & R_e C_x \end{bmatrix} \end{aligned} \tag{23}$$

Notice (22) is in a regression form with regressor vectors $\varphi_1(x), \varphi_2(x)$ in (23), and parameters vectors ϑ_1, ϑ_2 in (23).

Using the regression notation of (22) the observer (8) could be written as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\ \dot{\hat{x}}_2 &= a_1 u + \bar{\vartheta}_1\varphi(t, x_1, x_2, u) + z_2 \end{aligned} \tag{24}$$

where $\bar{\vartheta}_1$ is a parameters vector with nominal values of ϑ_1. For all $t \geq t_2$ equations (20), (21) become

$$\bar{z}_2 = \Delta\vartheta_1\varphi_1(t, x_1, x_2, u) \tag{25}$$

$$\bar{z}_3 = \vartheta_2\varphi_2(t, x_2, x_3, u) \tag{26}$$

where $\Delta\vartheta_1 = \vartheta_1 - \bar{\vartheta}_1$.

To proceed we will consider, for clarity of presentation only, the estimation procedures in two steps, one for x_2 and one for x_3 in order to estimate respectively ϑ_1 and then ϑ_2.

A. Identification of ϑ_1

It is possible to apply a dynamic form of the Least Square identification algorithm to estimate the parameter vector with the knowledge of $\bar{z}_2$ the regression vector deduced from the measurements and observations of φ_1.

The model structure for the linear regression [35],[36],[26] can be written as in equation (25) where $\bar{z}_2$ is a measurable quantity, $\varphi_1(t)$ is a regression vector made of known quantities and $\Delta\vartheta$ is the unknown parameters vector (difference to the nominal parameters). The application of linear regression algorithms like the Least Squares parameter estimation algorithm can be written as

$$
\begin{aligned}
\bar{z}_2 &= \widehat{\Delta\vartheta}\varphi_1(t, x_1, x_2, u) && (27)\\
\varepsilon(t) &= h(t) - \hat{h}(t) = \widetilde{\Delta\vartheta}\varphi_1(t, x_1, x_2, u) && (28)\\
\dot{\widehat{\Delta\vartheta}} &= \frac{\sigma}{\gamma_t}\Gamma_t\varphi_1(t, x_1, x_2, u)\varepsilon(t) = \Gamma_t\varphi_1\varphi_1^T\widetilde{\Delta\vartheta} && (29)\\
\dot{\Gamma}_t &= -\frac{\sigma}{\gamma_t}\Gamma_t\varphi_1^T(t)\varphi_1(t)\Gamma_t && (30)
\end{aligned}
$$

where $\widehat{\Delta\vartheta}$ is the estimation of $\Delta\vartheta$ the parameters vector and $\hat{h}(t)$ the prediction of the signal $h(t)$. In general γ_t is a normalization term $\gamma_t = 1 + \varphi_1(t)\Gamma_t\varphi_1^T(t)$ and $\sigma \in [0.9, 1]$ a forgetting factor. The initial conditions of the RLS algorithm are $\Gamma_0 = \rho^{-1}I$ initial gain matrix and $\widehat{\Delta\vartheta} = \widehat{\Delta\vartheta_0}$ initial parameters values.

Theorem 1: for the system (27) using the RLS algorithm (27) ensures the following properties:

(i) $\widetilde{\Delta\vartheta}^T\Gamma_t^{-1}\widetilde{\Delta\vartheta}$ is a non increasing function and we have $\left\|\widetilde{\Delta\vartheta}_t\right\|^2 \leq \frac{\lambda\min(\Gamma_O)}{\lambda\max(\Gamma_t)}\left\|\widetilde{\Delta\vartheta}_0\right\|^2$

(ii) $\tilde{\varepsilon}(t) = (\frac{\sigma}{\gamma_t})^{1/2}(h(t) - \hat{h}(t)) \in L_2$

Remark 2: The use of equations (27) ensures the asymptotic convergence of $\widehat{\Delta\vartheta}$ to $\Delta\vartheta$ under the persistent excitation condition [36],[35].

Remark 3: In application, we have considered the delta operator for approximation of the derivation [35].

B. Identification of ϑ_2

The low frequency components of the signal z_3 satisfies (26), using the notation in (22) takes the form

$$\bar{z}_3 = \beta\overline{sign}(\tilde{x}_3) = \vartheta_2\varphi_2 = \frac{\vartheta_{21}}{m} - \vartheta_{22}\frac{1}{m}\frac{x_2}{x_3}$$

Remark 4: In the same way ϑ_{21}, assuming ϑ_{22} known or already estimated, can be identified using the Least Squares algorithm.

Remark 5: Note also that both parameters in ϑ_2 can be estimated by the Least Square Algorithm at this step. This correspond to estimating twice ϑ_{22}, assuming at this step as previous estimation the value produced by the previous step.

Remark 6: Note also that depending on the expression formulated for the forces and wheel slip in (5), (4) several ther variables can be estimated like adherence or longitudinal forces.

VI. EXPERIMENTAL RESULTS

In this section, we present some experimental results to validate our approach. Several trials have been done with a vehicle (P406 of LCPC) equipped with sensors for wheels angular position measurement.

Measures have been acquired with the vehicle rolling at several speeds. The experimental data used here are those of the rear wheel drive. The installed sensors at each wheel are the variable reluctance ones of the Antilock Braking System (ABS see figure). Their resolution is 29 dot per revolution

Fig. 3. Vehicle used for experiments

(ie $\theta(i) = \frac{2\pi n(i)}{29}rad$). An additional encoder (with 1000dot by turn ie $\theta(i) = \frac{2\pi n(i)}{1000}rad$) have been installed for angular position measurements control and validation.

The Fig. 5 shows the installed laser sensor used for measurement of the wheel radius. Data are sampled at 1kHz frequency and several trials have been considered a different running speeds (40 Km/h, 60 Km/h, 80 Km/h, 100 Km/h and varying velocity) with and without using the ABS system.

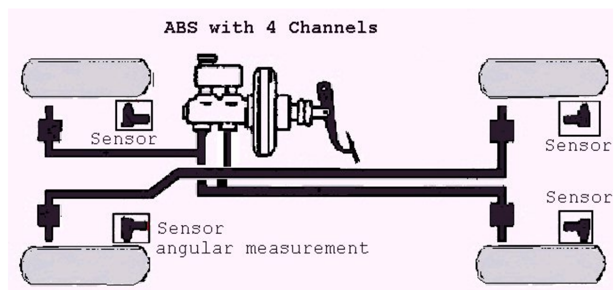

Fig. 4. 4 Sensors used by ABS 29dot/2π

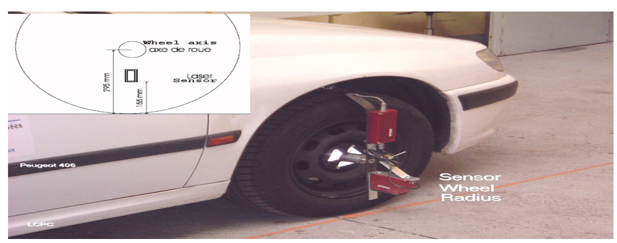

Fig. 5. Sensor for wheel radius measurement

The Fig. 6 shows the measured displacement using the (high and low resolution) sensors installed on the vehicle and the observed one. We can remark that the curve are well superposed despite resolution.

The velocities can be deduced by several ways from the displacement measurements. Here we compare three of them, two standard computation of derivatives and the proposed observer:

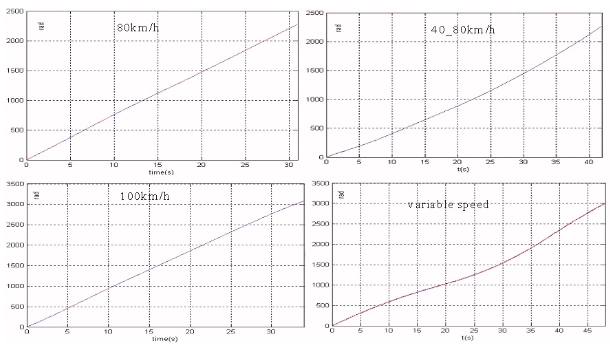

Fig. 6. Measured angular displacements

$$\dot{\theta}(i) = \frac{\theta(i) - \theta(i-1)}{T} \quad (31)$$

$$\dot{\theta}(i) = \frac{\theta(i+1) - \theta(i-1)}{2T} \quad (32)$$

$$\hat{x}_2 = observed(\dot{\theta}) \quad (33)$$

In the upper left and right Fig. (7) we can see that estimation of velocity signal derivation, using (31) and (32) respectively, needs a filter to reduce the noise effect. In Fig. (8) corresponding to low resolution encoders the problem is worse and amplitude of noise has a higher level. Filtering this data will affect the measurement precision.

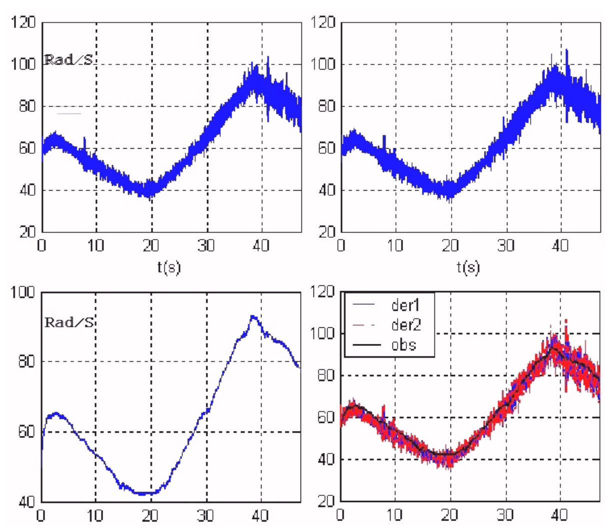

Fig. 7. Estimated velocities using the high sensors (1000dot/t)

We remark that when using the proposed observer (bottom left curve in the two figures) that the estimation remain precise despite the bad resolution of the sensors used by ABS. The observed and reconstructed velocities are compared to the measure provided by a high resolution encoder.

These curves show the robustness of our observer based on second order sliding modes and super twisting algorithm versus measurement noise and additional perturbations. Recall that the term $\xi(t, x_1, x_2, u) = \varphi_1(z)\vartheta_1$ is not known and correspond to a perturbation to be rejected in a first step;

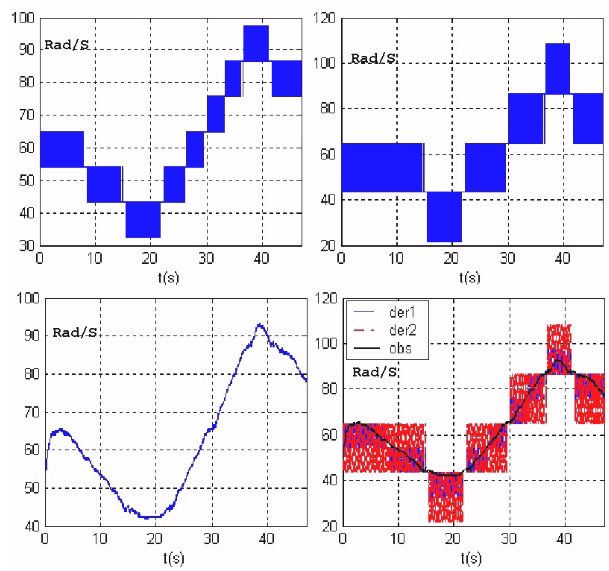

Fig. 8. Estimated velocities using the ABS sensors (29dot/t)

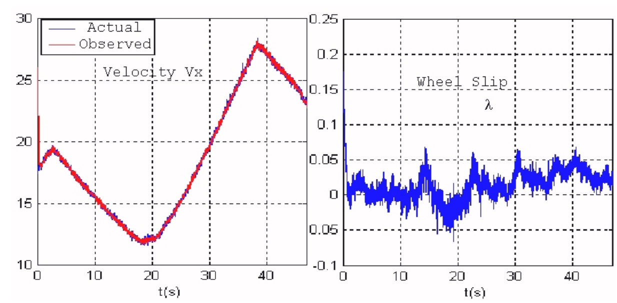

Fig. 9. Estimated velocity using the ABS sensors

thank to the finite time convergence. In a second step (after the convergence time) this perturbation is retrieved by use of a low pass filtering and them the parameters ϑ_1 can be estimated. The second step estimations are the wheel radius and its longitudinal equivalent stiffness. The estimations are shown in Fig.(10), Fig.(9)

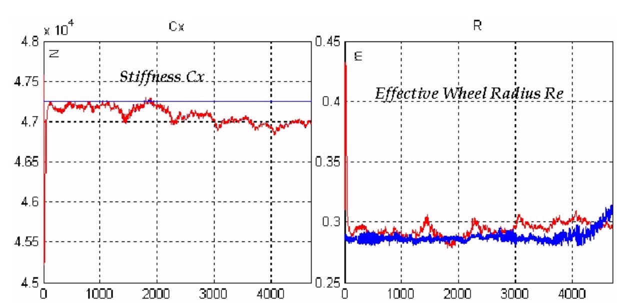

Fig. 10. Estimated wheel Stiffness and Radius using the ABS sensors and observations

The estimated parameters are quite good and the algorithm is very easy to apply and is not difficult to tune its parameters.

VII. CONCLUSION

The super-twisting second-order sliding-mode algorithm is modified in order to design a velocity observer for vehicle using only the ABS sensors already placed in standard vehicles nowadays. The finite time convergence of the observer is proved and consequently the separation principle can be considered as avoided. The gains of the proposed observer are chosen very easily ignoring the system parameters. This observer is compared, using experimental data, to classical derivation methods and is proven robust despite the bad resolution of the encoders. Its robustness combined with a sliding mode estimation of the vehicle velocity allow us to reconstruct the wheel sleep. In this way, the observability problems are avoided by means of cascaded finite time converging observer instead of additional sensors. It can be shown that contact forces can also be estimated by this way.

The finite time convergence of state observations in the same time as robustness and perturbation rejection allows to solve the problem of parameter identification using the equivalent control method (by retrieval of the rejected signal). The use of the equivalent control, which provides a linear regression model, allows to apply the classical parameter identification methods (RLS) to estimate the systems dynamic parameters like the tire longitudinal equivalent stiffness and the effective wheel radius.

The estimation scheme build up using a Second Order Sliding Mode observers and a Sliding Mode velocity estimator has been tested on experimental data (acquired with a P406 vehicle) and shown to be very efficient using only standard sensors. The actual results prove effectiveness and robustness of the proposed method. In our further investigations we consider also the case of complete vehicle in a road with changing adherence. The estimations produced on-line will be used to define a predictive control to enhance the safety.

ACKNOWLEDGMENTS

J. Davila and L. Fridman gratefully acknowledge the financial support of the Mexican CONACyT (Consejo Nacional de Ciencia y Tecnologia), grant no. 43807-Y, the Programa de Apoyo a Proyectos de Investigacion e Innovacion Tecnolgica (PAPIIT) UNAM, grant no. 117103, as well as, DGSCA - DTD. This work has been done in a collaboration managed by members of the LSIS inside the GTAA (Groupe Thématique Automatique et Automobile). The GTAA is a research group supported by the CNRS. Thanks are addressed to the LCPC of Nantes for experimental data and the trials with their vehicle Peugeot 406.

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