Targeting the key player: An incentive-based approach

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A B S T R A C T

We consider a network game with local complementarities. A policymaker, aiming at minimizing or maximizing aggregate effort, contracts with a single agent on the network to trade effort change against transfer. The policymaker has to find the best agent and the optimal contract to offer. Our study shows that for all utilities with linear best-responses, it only takes two statistics about the position of each agent on the network to identify the key player: the Bonacich centrality and the self-loop centrality. We also characterize key players under linear quadratic utilities for various contractual arrangements.

1. Introduction

In many economic situations, agents' behaviors depend on their peers. Such interactions are well documented for criminal activities, or for R&D partnerships, or for protective investment against terrorism for instance. This creates opportunities for a policymaker to exploit these interdependencies, either to reduce or to increase the overall activity. For example, effort reduction may be desirable with criminal activities, whereas increased effort may be valuable in R&D investment or protection against terrorism. One possible policy consists in trading effort change against transfers. However in many circumstances, contracting costs substantially increase with the number of contracts. With a limited budget, the policymaker may then resort to make deals with a limited subset of agents.

This article considers agents organized in a network of local complementarities. We study the problem of a policymaker contracting with a single agent in order to minimize (resp. maximize) aggregate effort. The problem can be solved in two steps: first, studying the optimal contract with any agent, and then selecting the best agent, called the key player.

Our analysis shows that, for all utilities with linear best-responses, it only takes two statistics about the position of each agent on the network to identify the key player: the Bonacich centrality, which counts the number of (weighted) walks starting from the agent, and the self-loop centrality, which counts the number of (weighted) closed walks starting from the agent. In more detail, we show that the policy effect is the product of two components: the change in targeted agent effort (what we call the *individual component*) and the change in aggregate effort following a one-unit change in targeted agent effort (the *network component*). The latter is a pure network multiplier effect and is equal to the ratio of the Bonacich centrality over the self-loop centrality. The former is a function of both statistics, the budget level, the shape of utility, and whether the policymaker maximizes or minimizes effort. In total, the key player depends on all these parameters, but only the Bonacich centrality and the self-loop centrality are network-dependent. We also further characterize the key player under the standard case of linear quadratic utilities, and the analysis reveals that the key player is budget-dependent.

We then discuss our modeling assumptions. We first address the issue of contract enforceability. In this model, opportunism is a concern under effort maximization but plays no role under effort minimization. In contrast to enforceable contracts, we find that the key player does not depend on the budget level. We also examine the excess-effort linear contract, which is a natural contract given that agents exert effort in the absence of a principal. This contract puts the inter-centrality index in the spotlight, which is reminiscent of the key player analysis of Ballester et al. (2006). Last, we discuss other policies AMELIORER.

There is already a literature on key-player analysis.[1] In their pioneering work, Ballester et al. (2006) investigate key-player policy in a context of linear quadratic utilities, examining which

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1 See Zenou (2016) for a recent survey.

agents should be dropped from the network so as to minimize aggregate effort. The optimal target is an agent maximizing the inter-centrality index, a specific centrality measure internalizing how much the agent affects others' contribution to the aggregate. Ballester et al. (2010), Liu et al. (2014) and König et al. (2018) elaborate on this seminal paper. Our approach complements Ballester et al. (2006) by considering the situation where a policymaker with a limited budget needs to compensate the agent in order to reduce the agent's effort, whereas in their setup policy intervention is costless. Our approach opens the way to partial effort reduction, a useful device when the policymaker has a limited budget.

The key-player policy fits into a more general literature related to optimal targeting on networks. In a related paper, Belhaj and Deroïan (2018) study optimal contracting on networks without any constraint on the number of contracts established by the principal under linear quadratic utilities. Interestingly, even in this less restrictive environment, it can be optimal to contract with a subset of the population, and in particular with a single agent. Zhou and Chen (2015) examine the benefits of sequentiality in the same game as ours. In their setting, one (forward-looking) agent plays in the first stage and the others in the second stage. A network designer has to find the best agent to play first. Their solution coincides with ours for a zero budget under linear quadratic utilities. Our key-player analysis, in particular Proposition 2 in the present article, generalizes their paper's Proposition 2 to a non-zero budget. Demange (2017) studies the optimal targeting strategies of a planner aiming to increase the aggregate action of agents embedded in a social network, allowing for non-linear interaction. In a recent paper, Galeotti et al. (2017) study optimal targeting in networks, where a principal aims at maximizing utilitarian welfare or minimizing the volatility of aggregate activity. In the drop-out game of Calvò-Armengol and Jackson (2004), the planner subsidizes agents' labor market entry.

The paper is organized as follows. Section 2 presents the model. Section 3 first presents our main result (Theorem 1), applicable to general utilities with linear best-responses, and then characterizes the key player further by focusing on linear quadratic utilities. Section 4 discusses contract enforceability, linear contracts, and alternative policies. Section 5 concludes. All proofs are presented in the end of the article. An Appendix provides a series of tables giving the key player and the aggregate performance according to the nature of the contract and the policymaker's objective.

2. Model

We consider a finite set of agents interacting on a network of local complementarities. A policymaker with a limited budget contracts with a single agent in order to either minimize or maximize the aggregate effort. Effort, contract and network are assumed to be publicly observable. We consider a three-stage game. In the first stage, the policymaker offers a contract to a single agent. In the second stage, the agent decides whether to accept or reject the offer. In the third stage, all agents exert effort, and the transfer is realized. We study the Subgame Perfect Nash Equilibrium (SPNE).

Notation. Numbers are written in lower case, matrices (including vectors) in block letters and in boldface. We denote by $\mathbf{1}$ the n-dimensional vector of ones. We let superscript ^{T} stand for the transpose operator. For instance, we write vector $\mathbf{X} = (x_1, \ldots, x_n)^T$, with x_i as its ith entry, and $x = \mathbf{1}^T\mathbf{X}$ denotes the sum of entries of vector $\mathbf{X}$.

Network. We let $N = \{1, 2, \ldots, n\}$ be the set of agents organized in a network of bilateral relationships. The network is formally represented by a symmetric adjacency matrix $\mathbf{G} = [g_{ij}]$, with binary element $g_{ij} \in \{0, 1\}$. The link between agents i and j exists whenever $g_{ij} = 1$, in which case we will say that agents i and j are neighbors. By convention, $g_{ii} = 0$ for all i. By abuse of language we will speak of network $\mathbf{G}$. The network is undirected, i.e. $\mathbf{G}^T = \mathbf{G}$. We let $\mu(\mathbf{G})$ denote the largest eigenvalue of the adjacency matrix $\mathbf{G}$ by symmetry. A walk of length p connecting i and j in network G is a set of agents $i_0 = i, i_1, \ldots, i_p = j$ such that $g_{j,j+1} = 1$ for all $j = \{0, 1, \ldots, p-1\}$. A closed walk of length p originating at i is a walk of length p such that $i_0 = i_p = i$.

Agents' utilities. Agents exert effort and derive utility from own effort as well as from aggregate neighbors' effort. We let vector $\mathbf{X} = (x_i)_{i\in N} \in \mathbb{R}^n_+$ be a given profile of effort, and we define for convenience the vector $\mathbf{Y} = \mathbf{GX}$ as the vector of aggregate neighbors' effort. Individual utilities are homogeneous across agents and generate linear interaction between neighbors in a context of local positive externalities and local complementarities. Formally, agent i's utility is expressed as the function $u(x_i, y_i)$ and is quasi-concave in the first argument, increasing in the second argument. Defining $x_i^{BR}(y_i)$ as agent i's best-response effort when aggregate neighbors effort is equal to y_i, we impose the following assumption:

Assumption 1. The utility function generates a linear best-response of the form

$$x_i^{BR}(y_i) = 1 + \delta y_i \tag{1}$$

where parameter $\delta > 0$ measures the strength of complementarities, or intensity of interaction, between neighbors. This system of linear best-responses generates a unique and interior equilibrium effort profile if and only if $\delta\mu(\mathbf{G}) < 1$, which we assume throughout the article (otherwise there is no equilibrium and effort would escalate to infinity – see Ballester et al. (2006) in the context of linear quadratic utilities).

Example 1. Under linear quadratic utilities (see Ballester et al., 2006), agent i's utility is given by

$$u(x_i, y_i) = x_i - \frac{1}{2}x_i^2 + \delta x_i y_i$$

Parameter δ measures the strength of complementarities or intensity of interaction between neighbors, and agent i's best-response is identical to Eq. (1).

Example 2. Agent i's utility function is

$$u(x_i, y_i) = x_i - f(x_i - \delta y_i)$$

where function f is twice continuously differentiable, increasing, convex and satisfies $f'^{-1}(1) = 1$. Function f represents the cost of effort. Effort cost is lowered by neighbors' effort, higher δ meaning higher impact on own effort cost. Then agent i's best-response is identical to Eq. (1).

Centralities and equilibrium. We let $\mathbf{I}$ denote the n-dimensional identity matrix. We set the n-square matrix $\mathbf{M} = (\mathbf{I} - \delta\mathbf{G})^{-1}$. The condition $\delta\mu(\mathbf{G}) < 1$ guarantees $\mathbf{M} \geq \mathbf{0}$. In that case, the entry m_{ij} of matrix $\mathbf{M}$ represents the weighted number of walks from agent i to agent j in the network, where walks of length k are decayed by the factor δ^k. In particular, the index m_{ii}, which measures the closed walks originating from agent i, will be called *self-loop centrality* thereafter. For any vector $\mathbf{Z} \geq \mathbf{0}$, we let $\mathbf{B_Z} = \mathbf{MZ}$ denote the *Bonacich centrality* of the network weighted by $\mathbf{Z}$ (we avoid references to network $\mathbf{G}$ and parameter δ for convenience). The quantity $b_{Z,i}$ is the number of walks from agent i to others, where walks are weighted as follows: the weight of a walk of length k from agent i to agent j is $\delta^k z_j$. In particular, we let $\mathbf{B} = \mathbf{M1}$ denote the vector of un-weighted Bonacich centralities. By construction the Bonacich centrality of agent i is expressed as $b_i = m_{ii} + \sum_{j\neq i} m_{ij}$.

Let $\mathbf{X}^0 = (x_i^0)_{i\in N}$ be the Nash equilibrium played by agents when there is no policymaker's intervention:

Result 1 (Ballester et al., 2006). *When there is no policymaker's intervention,* $\mathbf{X}^0 = \mathbf{B}$.

Equilibrium effort can be interpreted in terms of Bonacich centrality. By linearity of best-responses, the aggregate play of neighbors depends on Bonacich centralities,[2] so that the utility of agent i at the Nash equilibrium $\mathbf{X}^0$ is given by $u(b_i, \frac{b_i-1}{\delta})$, and is a function solely of the centrality of agent i.

The policymaker's intervention. The policymaker's objective is to either minimize or maximize aggregate effort subject to a limited budget constraint.[3] We define the variable $\o \in \{-1, 1\}$. The policymaker's objective function is given by the quantity $\o\, \mathbf{1}^T\mathbf{X}$, with $\o = -1$ (resp. $\o = 1$) when the policymaker wants to minimize (resp. maximize) aggregate effort. The policymaker offers a contract to a single agent on the network. A contract between the policymaker and agent i specifies an effort $x_i \in \mathbb{R}^+$ and a monetary transfer $t \in \mathbb{R}$ from the policymaker to agent i. We assume that in the case of effort minimization, the budget is so low that the policymaker cannot compensate any agent for exerting a null effort[4] (this will be formalized by Assumption 3 in the next section). We let $\mathbf{X}^*_{-i}(x_i)$ be the Nash equilibrium played by agents in $N \setminus \{i\}$, given x_i, and we let $\mathbf{X}^*(x_i) = (x_1^*, x_2^*, \ldots, x_{i-1}^*, x_i, x_{i+1}^*, \ldots, x_n^*)^T$ be the profile of effort such that all other agents but i play the Nash equilibrium given effort x_i. We also define $\mathbf{Y}^*(x_i) = \mathbf{G}\mathbf{X}^*(x_i)$.

We will study the optimal contract, i.e. the contract maximizing the policymaker's objective under individual participation constraint. The contract is assumed to be enforceable (we relax this assumption in Section 4). The optimal contract maximizes the policymaker's objective under the agent's participation constraint. As a basic observation, the agent's participation constraint is binding at optimum, otherwise the policymaker could trade effort change against saved budget. So, recalling that $\o = 1$ (resp. $\o = -1$) when the policymaker wants to maximize (resp. minimize) aggregate effort, the optimal enforceable bilateral contract solves program $(\mathcal{P}_1)$:

$$\max_{i \in N, x_i} \quad \o\, \mathbf{1}^T\mathbf{X}^*(x_i) \tag{2}$$

$$\text{s.t.} \quad u(x_i, y_i^*(x_i)) + t = u(b_i, \tfrac{b_i-1}{\delta})$$

3. Contracting with key players

An optimal policy consists in identifying the best agent and the best deal with this agent. In this section, we examine the optimal contract. We characterize the key-player contract as a function of network structure, intensity of interaction, and available budget.

3.1. General insights

In the absence of policymaker intervention, agents exert their Nash equilibrium effort, equal to their Bonacich centrality. Suppose that a policy targets one agent, and induces an effort change equal to $\o \cdot \alpha$, with $\alpha > 0$ by convention. The next proposition indicates how the network reacts to this change:

Proposition 1. *Consider* $\o \in \{-1, 1\}$. *Suppose that, from the initial equilibrium* b_i, *agent i's effort varies by an amount* $\o \cdot \alpha$. *Then for all utility functions with linear best-responses, the aggregate effort change is equal to*

$$\frac{b_i}{m_{ii}} \cdot \o \cdot \alpha \tag{3}$$

By Proposition 1, an effort change of one unit by agent i induces a final aggregate effort change of magnitude $\frac{b_i}{m_{ii}}$, where b_i represents the Bonacich centrality of agent i, and m_{ii} the weighted number of loops originating from agent i (recall that $b_i = m_{ii} + \sum_{j \neq i} m_{ij}$). This ratio therefore captures *agent i's network effect*, measuring the influence of the agent on aggregate behavior. This result is useful to understand the impact of any key-player policy intervention. The message is that for a given effort change α, the agent with largest impact on aggregate effort maximizes the index $\frac{b_i}{m_{ii}}$. Enhancing this ratio means having a great influence on others. Moreover, measuring the impact of agent i on others only takes a pair of measures (b_i, m_{ii}).

Note that in Ballester et al. (2006), removing an agent from the network is equivalent to decreasing her effort by an amount equal to her effort b_i (so that she exerts no effort in the end); replacing α with the value b_i we get the inter-centrality index $c_i = \frac{b_i^2}{m_{ii}}$. Now, what is crucial in the following analysis is that modifying agent i's effort is costly, and, with a limited budget, it is not always possible to induce zero effort from the key player. In general, the efficiency of a given policy will depend not only on the agent's impact on others, but also on the policy capacity to induce a large change in the agent's effort. We will see now how the nature of the contract, the budget level, and the position of the agent in the network all affect the maximal amount of effort change that the policymaker can obtain from any agent. This will be useful to identify the key player.

To assess how much effort the policymaker can demand from the agent, her participation constraint is key. To evaluate this, we need to quantify how much neighbors' best-responding effort changes when an agent's effort changes by $\o \cdot \alpha$. This is what the next lemma does, exploiting linear best-responses:

Lemma 1. *Consider* $\o \in \{-1, 1\}$. *Suppose that, from the initial equilibrium* b_i, *agent i's effort changes by* $\o \cdot \alpha$. *Then the aggregate Nash effort of agent i's neighbors is given by*

$$y_i^*(b_i + \phi\alpha) = \frac{(b_i - 1)m_{ii} + \phi\alpha(m_{ii} - 1)}{\delta m_{ii}} \tag{4}$$

This simple lemma states that, with linear best-responses, the aggregate response of agent i's neighbors to effort change only depends on the statistics b_i and m_{ii}. This result will simplify the key-player analysis. Knowing neighbors' response to own effort change, we can examine the incidence of the participation constraint on key-player selection. In particular, the next theorem indicates which network statistics need to be used by the policymaker to find the key player for all utilities with linear best-responses. To proceed, we first impose two assumptions. Define

$$h(\alpha) = u\left(b_i + \alpha, \frac{(b_i - 1)m_{ii} + \alpha(m_{ii} - 1)}{\delta m_{ii}}\right)$$

Assumption 2 (*Effort Maximization*).

$$\lim_{\alpha \to +\infty} h'(\alpha) < 0$$

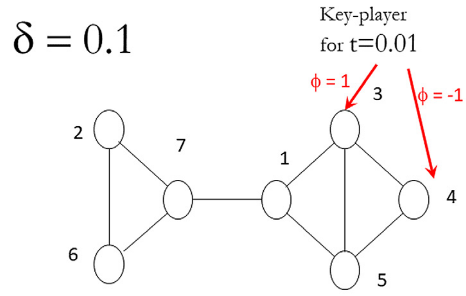
Assumption 2 guarantees, under effort maximization, the existence of an optimal contract. Note that this assumption imposes conditions both on utility and on the intensity of interaction.

2 Indeed, by definition $(\mathbf{I} - \delta\mathbf{G})\mathbf{B} = \mathbf{1}$, so at equilibrium $Y = \mathbf{GB} = \frac{\mathbf{B}-\mathbf{1}}{\delta}$.

3 This program can be part of a more general program with endogenous budget, where policymaker's payoff is a function of the sum of agents' effort net of transfers. We abstract from optimal budget selection considerations and assume that the budget is not larger than the optimal budget.

4 Otherwise the key player would be the agent maximizing the inter-centrality index, as in Ballester et al. (2006).

agent	mii	bi	ci
1	1.034	**1.42**	**1.95**
2	1.022	1.26	1.56
3	**1.036**	1.41	1.92
4	1.023	1.28	1.60
7	1.033	1.39	1.88



ϕ = -1

agent	Xi(0.01)	Xi(1)
1	0.143	**1.94**
2	0.146	1.75
3	0.140	1.92
4	**0.147**	1.77
7	0.146	1.91

ϕ = 1

agent	Xi(0.01)	Xi(1)
1	0.281	**2.08**
2	0.219	1.82
3	**0.284**	2.07
4	0.223	1.85
7	0.273	2.04

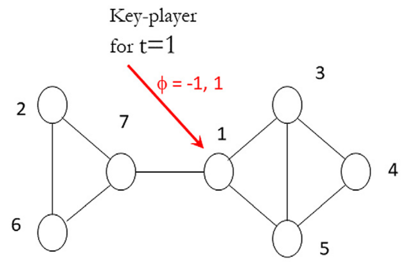


Fig. 1. Key player as a function of the budget t and the principal's objective ϕ. The upper table presents self-loop centrality, Bonacich centrality and inter-centrality; the highest centrality is in bold notation. The lower table presents the index $\mathcal{X}_i(t, ø)$, the key-player is in bold notation.

Assumption 3 (*Effort Minimization*)**.** The condition

$$u(0, \frac{b_i - m_{ii}}{\delta m_{ii}}) + t \leq u\left(b_i, \frac{b_i - 1}{\delta}\right)$$

holds for all $i \in N$.

Assumption 3 guarantees that, under effort minimization, the budget is so small that the policymaker cannot offer a contract with null effort to any agent. We get the following theorem:

Theorem 1. *There exists a positive function $\alpha()$ such that the key player maximizes the index*

$$\frac{b_i}{m_{ii}} \cdot \alpha(b_i, m_{ii}, t, ø) \tag{5}$$

Note that function α in Theorem 1 is defined by the agent's participation constraint. A sharp message from Theorem 1 is that, for a given budget and a given policymaker objective, the Bonacich centrality and the self-loop centrality are sufficient network statistics to identify the key player for all contracts and all utility functions with linear best-responses. In particular, when agreeing to contract, agent i's effort change only depends on these two network statistics for all utilities. Of course, the way of combining these two statistics, together with other statistics such as the budget and principal's objective, is highly dependent on the shape of utilities.

It is important to note that when agent i rejects the offer, she still exerts effort and interacts with her neighbors. This interaction aspect determines the cost of changing agent i's effort. In total, interaction matters twice: it determines the magnitude of effort change that agent i can accept for a given budget, contractual arrangement and policymaker objective (the term $\alpha(b_i, m_{ii}, t, ø)$), what we call the individual component of the key-player policy; and it shapes the impact of that change on the aggregate effort change (the term $\frac{b_i}{m_{ii}}$), what we call the network component of the key-player policy. The performance of the key-player policy depends on individual component and network component combined. This is grasped by Eq. (5), which characterizes policy efficiency as the product of the agent network effect multiplied by the maximal amount of effort change she can accept given the available budget.

It is interesting to note that, for utilities given by Example 2, the individual component depends on the sole self-loop centrality m_{ii}; i.e., Bonacich centrality b_i does not affect the effort change of the key player.[5] It follows that the key player maximizes an index of the type $b_i \cdot \frac{\alpha(m_{ii}, t, ø)}{m_{ii}}$. This result holds for all budget levels and every function f.

3.2. Key players under linear quadratic utilities

So far, we have seen that the effort variation of the targeted agent is a function of two network statistics. This function is shaped by the agent's utility, the budget and the policymaker's objective. We will now characterize the key player under the widely studied case of linear quadratic utilities (see Ballester et al., 2006), as presented in Example 1.

To guarantee the existence of an optimal contract under effort maximization, we need to assume $\max_{i \in N} m_{ii} < 2$ (equivalent to Assumption 2 with linear quadratic utility). We define δ_c as the smallest value of δ such that $\max_{i \in N} m_{ii} = 2$, and we assume that $\delta < \delta_c$ throughout the section. We obtain the following characterization for the optimal bilateral contract:

Proposition 2. *Consider linear quadratic utilities. For the optimal bilateral contract, the key player maximizes the index $\mathcal{X}_i(t, ø)$ defined as*

$$\mathcal{X}_i(t, ø) = \frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii} - 1)^2 b_i^2 + 2m_{ii}(2 - m_{ii})\, t} + ø \cdot (m_{ii} - 1) b_i}{2 - m_{ii}} \tag{6}$$

The aggregate effort change is $ø \cdot \mathcal{X}_i(t, ø)$.

The key player maximizes an index which is a sophisticated function of the position. In particular, the index is by no means reducible to a simple centrality measure. From Proposition 2 we get two main messages. First, the key player can vary with the amount of available budget. To illustrate, Fig. 1 depicts how the identity of

[5] Applying the participation constraint given in program $(\mathcal{P}_1)$ with utility function given in Example 2, and taking care of Eq. (4) in Lemma 1, few computation shows that α solves $f(1 + \phi \cdot \frac{\alpha}{m_{ii}}) - \phi \cdot \alpha = t + f(1)$. The value of α solving this equation is therefore independent from b_i.

the key player varies as a function of the budget under both effort minimization and effort maximization in a seven-agent network with fixed intensity of interaction. For instance, under effort minimization and for a low budget, agent 4 is the key player, whereas for a higher budget, the key player is agent 1. Agent 4 is more peripheral than agent 1, which shows how the budget constraint can affect the position of the optimal target. Second, for the same budget, the key player under effort maximization is in general different from the key player under effort minimization: on Fig. 1, for $t = 0.01$ the key player is agent 3 under effort maximization but agent 4 under effort minimization. This is illustrated further in the limit cases of low budget and low intensity of interaction for any network. When t is close to zero, the key player maximizes the index $\frac{1}{(m_{ii}-1)}$ under effort minimization, whereas it maximizes the index $\frac{b_i^2}{m_{ii}} \cdot \frac{m_{ii}-1}{2-m_{ii}}$ under effort maximization. Moreover, when δ tends to zero, the key player is maximized for high-degree agents under both effort minimization and effort maximization. In more detail, we have $b_i \simeq 1 + \delta d_i + o(\delta)$ while $m_{ii} \simeq 1 + \delta^2 d_i + o(\delta^2)$, so the network component is approximately $\frac{b_i}{m_{ii}} \simeq 1 + \delta d_i + o(\delta)$. The individual component is close to $\alpha \simeq \sqrt{2t} + (ø + \sqrt{2t}) d_i \delta^2 + o(\delta^2)$. Hence, as the intensity of interaction tends to zero, the individual component outweighs ($\simeq \sqrt{2t}$) the network component ($\simeq 1$) as soon as $t > \frac{1}{2}$. We summarize the index under various limits in the next corollary. Recall that $c_i = \frac{b_i^2}{m_{ii}}$:

Corollary 1. *When t, δ tend to their limits, the key player maximizes the following index:*

- *$t \to 0, ø = -1$: $\mathcal{X}_i \to \frac{t}{m_{ii}-1}$ so the key-player minimizes m_{ii}*
- *$t \to 0, ø = 1$: $\mathcal{X}_i \to 2L_i$, where $L_i = \frac{m_{ii}-1}{2-m_{ii}} c_i$ is the key-player index defined in Zhou and Chen (2015).*
- *$t \to \infty, ø \in \{-1, 1\}$: $\mathcal{X}_i \to \sqrt{\frac{2c_i t}{2-m_{ii}}}$ so the key-player maximizes $\frac{c_i}{2-m_{ii}}$*
- *$\delta \to 0, ø \in \{-1, 1\}$: $\mathcal{X}_i \to (1 + \delta d_i)\sqrt{2t}$ (for $t > 0$) so the key-player maximizes the degree d_i.*

Under effort minimization, the policymaker cannot reduce effort with a null budget. Indeed, reducing own effort below best-response effort is costly for the agent, and by complementarities this effort reduction induces a decrease in others' effort too, which by positive externalities further penalizes the agent's utility. Overall, to obtain the agent's consent, the policymaker needs to compensate her with a positive transfer. In contrast, the policymaker with a null budget can enhance effort under effort maximization. Interestingly, the key player under null budget coincides with Zhou and Chen (2015), where a policymaker with zero budget exploits the benefits of sequential play (so $\mathcal{X}_i(0) = L_i$ where L_i is their key player in their notation — see equation (13) in Proposition 2 in their paper). Note however that contracts do (twice) better than sequential move. Indeed, by the commitment aspect of the contract, the agent accepts to increase effort up to a utility level equal to the Nash utility $\frac{b_i^2}{2}$. In contrast, the utility of the first mover in Zhou and Chen is higher than initial utility. The resulting change in aggregate effort is twice that of sequential play.[6]

[6] To be more precise, in the approach by sequential move of Zhou and Chen, the effort selected by the first mover is $\frac{b_i}{2-m_{ii}}$, the first mover's utility is $\frac{b_i^2}{2} \cdot \frac{1}{m_{ii}(2-m_{ii})}$, and the induced change of the aggregate effort is $\frac{b_i^2(m_{ii}-1)}{m_{ii}(2-m_{ii})}$. Applying the approach by contracts with null budget, the effort of the contracting agent is $\frac{2(m_{ii}-1)b_i}{2-m_{ii}}$ and her utility is given by $\frac{b_i^2}{2}$ (note that the utility of the first mover as in Zhou and Chen exceed $\frac{b_i^2}{2}$). The change of the aggregate effort is $2\frac{b_i^2(m_{ii}-1)}{m_{ii}(2-m_{ii})}$.

4. Discussion

In this section, we discuss three aspects of the model. First, we discuss the nature of the environment in which the contract is established, by assuming that the contract is not enforceable. Second, we present the excess-effort linear contract. Finally we discuss other related policies.

4.1. Non-enforceable contract under linear quadratic utilities

In many economic environments, the contract may not be enforceable, i.e. after signing the contract, the agent may behave opportunistically. When the contract is not enforceable, the principal should take care of an incentive-compatibility constraint guaranteeing that the agent finds it not profitable to behave opportunistically once the contract is accepted. In our setup, opportunism can be captured by assuming that the agent considers the opportunity to deviate from the prescribed contracted effort given the current effort choice of other agents. In this case, the incentive-compatibility constraint stipulates that the agent plays a best-response to others' effort, who are currently best-responding to the effort prescribed by the contract. Note that, in this simple formulation, the opportunistic agent does not take care of the possible counter-reactions of other agents, which may reduce incentives to deviate.[7] The optimal non-enforceable contract maximizes program $(\mathcal{P}_2)$:

$$\max_{i \in N, x_i} \quad ø\, \mathbf{1}^T \mathbf{X}^*(x_i)$$

$$\text{s.t.} \begin{cases} u(x_i, y_i^*(x_i)) + t \geq u(b_i, \frac{b_i - 1}{\delta}) \\ u(x_i, y_{-i}^*(x_i)) + t \geq u(x_i^{BR}(y_i^*(x_i)), y_i^*(x_i)) \end{cases}$$

Which constraint is binding at optimum depends on the policymaker's objective. Under effort maximization, the binding constraint is the incentive constraint, because other agents best-respond to increased effort by increasing effort too, which ensures a higher utility level under opportunistic behavior. In contrast, enforceability is not an issue under effort minimization. This is because other agents best-respond to decreased effort by decreasing effort too, which entails a utility under opportunistic behavior lower than the utility without policy intervention. Hence, under effort minimization, whether or not the contract is enforced, program $(\mathcal{P}_2)$ is equivalent to program $(\mathcal{P}_1)$. We thus focus on effort maximization:

Proposition 3 (*Non-enforceable Contract*)**.** *Suppose that the policymaker maximizes aggregate effort (i.e., $ø = 1$) and assume linear quadratic utilities. For the optimal non-enforceable contract, the key player maximizes the index b_i and the aggregate effort change is $\sqrt{2t}\, b_i$.*

Hence, the relevant centrality measure is the Bonacich centrality. This may be good news for the policymaker: observing effort is all it takes to identify the key player. It is worth mentioning that, as opposed to the case where the contract is enforceable, the key player does not depend on the budget. As suggested at the end of the previous section, this means agent i's effort change is the product of a function of network statistics by a function of the budget. This result, tied to the linear quadratic specification of utilities, is explained because the optimal aggregate effort is a

[7] To take these considerations into account, a natural extension would be to introduce a dynamic version of the model where the short-run benefits of behaving opportunistically would be challenged by the long-run negative consequences generated by the counter-moves of other agents. In principle, such dynamics would induce further sequential decrease of agents' effort. In the extreme case of infinitely patient agents, such a dynamics would converge to the initial Nash equilibrium, deterring opportunism.

separable function of budget and network statistics.[8] Of course, the aggregate effort change is increasing in the budget level.

It is possible to evaluate the efficiency loss due to non-enforceability in case of effort maximization. The ratio of the contract performance under non-enforceability over contract performance under enforceability is equal to $\frac{\sqrt{2t}\ \max_j b_j}{\max_i \mathcal{X}_i(t)}$.[9] In the network depicted in Fig. 1, this ratio is roughly equal to 0.70 for $t = 0.01$, and 0.96 for $t = 1$.

4.2. Effort-change linear contract under linear quadratic utilities

We study the payment scheme that is linear in effort change. This contract is interesting not only because of its simplicity, but also, as we will see, because the effort-change linear contract puts the spotlight on the inter-centrality index, itself familiar to the key player literature. Such a contract with agent i is a transfer function $t_i(x_i) = \gamma_i \cdot ø \cdot (x_i - b_i)$, with $\gamma_i \in \mathbb{R}_+$, so agent i is rewarded whenever effort increases (resp. decreases) under effort maximization (resp. minimization).[10] The participation constraint is an issue under effort minimization but not under effort maximization (see the proof of Proposition 4 for more details). Indeed, when effort is lowered agents' utilities decrease, so the transfer should cover the difference. But given the structure of the payment and the shape of utilities, the transfer covers the difference only when the budget is sufficiently large. Recalling that $c_i = \frac{b_i^2}{m_{ii}}$ represents agent i's *inter-centrality index*, we obtain the following proposition:

Proposition 4 (*Effort-change Linear Contract*)**.** *When* $ø = 1$*, the optimal excess linear contract selects the agent with maximal inter-centrality index* c_i*. When* $ø = -1$*, the optimal excess linear contract selects the agent with maximal inter-centrality index* c_i *among the set of agents satisfying* $\left(\frac{m_{ii}-1}{m_{ii}}\right)^2 c_i \leq \frac{t}{4}$.

At least two messages emerge for the effort-change linear contract. First, this contract highlights the inter-centrality index familiar in standard key-player policies *à la* Ballester et al. (2006), consisting in a costless removal of an agent from the network. However, the key player depends on the policymaker's objective. Under effort minimization, the participation constraint matters and determines the key player. Conversely, participation is not an issue under effort maximization, so the policymaker can always improve on the outcome with any positive budget. Second, the key player is budget-dependent under effort minimization.

4.3. Other policies

So far, we have identified key players under contracting policies. However, our approach can be used under other policies, and we can compare the results in terms of key player identification. For instance, in contexts where the principal wishes to modify effort, suppose that a policymaker has a technology able to modify agent i's private return on effort by $ø \cdot a_i$ within the limits of her budget t, so that $a_i = f(t)$ with f increasing in the budget.[11] Proposition 1 helps identifying the key player. The agent's induced effort change is given by $f(t)m_{ii}$ and, by Proposition 1, the aggregate effort change is given by $f(t)b_i$. That is, the key player maximizes Bonacich centrality, i.e. is the agent with highest effort.

In other contexts, the policymaker is able to modify agents' effort by using a costly technology. For example, the cost may be an increasing function of the difference between modified effort and initial effort, $t = f(|x_i - x_i^0|)$.[12] Applying Proposition 1 again, we deduce that the key player maximizes the index $\frac{b_i}{m_{ii}}$. It is worth mentioning that, in both examples, the key player depends neither on budget level nor on policymaker's objective.

The performance of all these alternative policies can be directly compared to the incentive-based policy examined in this article, and given by Theorem 1, by focusing their impact on the targeted agent.

5. Conclusion

This article explored a key-player policy in which a policymaker contracts with a single agent on the network to induce effort change. Principally, we showed that for all utilities with linear best-responses and all budgets, it only takes two network-related statistics to identify the performance of a given target: the Bonacich centrality and the self-loop centrality.

This analysis leaves interesting issues open. First, it would be challenging to generalize the study to the case of group-player analysis. One difficulty is that not only the group depends on utilities and network structure, but also the sharing of the budget among contracting agents. Second, it would be interesting to study how the agents on the network could protect themselves against the policymaker's intervention, and how this would affect the efficiency of the policy.[13] Third, as pointed out in Ballester et al. (2010), there may be complementarities between key-player policies and other policies affecting incentives to stay on the network, like a policy consisting in increasing wages in the formal market. It would thus be natural to incorporate such complementarities in the comparative analysis of key-player policies.

6. Proofs

Throughout the proofs, we define the vector $\mathbf{1}_i = (0, \ldots, 0, 1, 0, \ldots, 0)^T$ where value 1 is set at the ith entry.

Proof of Proposition 1. Consider $ø \in \{-1, 1\}$. Suppose that agent i's effort varies to the level $x_i = b_i + ø \cdot \alpha$, where $\alpha \geq 0$ by convention. Taking into account that other agents play their best-response, we observe that X solves the following system:

$$(\mathbf{I} - \delta \mathbf{G})\mathbf{X} = \mathbf{1} + ø \cdot \frac{\alpha}{m_{ii}} \mathbf{1}_i \tag{7}$$

We indeed argue that the solution to this system satisfies $x_i = b_i + ø \cdot \alpha$. To see why, note that System (7) can also be written

$$\mathbf{X} = \mathbf{B} + ø \cdot \frac{\alpha}{m_{ii}} \mathbf{M} \mathbf{1}_i$$

or equivalently, for all $j \in N$:

$$x_j = b_j + ø \cdot \alpha \frac{m_{ji}}{m_{ii}} \tag{8}$$

[8] More generally, an immediate implication from Theorem 1 is that the key player is independent from the budget level whenever the function $\alpha()$ is separable in the budget and network statistics, i.e. when there are two functions h_i, h such that $\alpha(b_i, m_{ii}, t, ø) = h_i(b_i, m_{ii}, ø) \cdot h(t)$.

[9] Here we abuse the notation and write $\mathcal{X}_i(t)$ rather than $\mathcal{X}_i(t, 1)$.

[10] Imposing a non-negative return is without loss of generality; as we will see, this constraint is never binding at optimum under both effort maximization and effort minimization.

[11] In Demange (2017), a principal injects cash into a financial system, which corresponds to increasing the agent's best-response by a fixed amount. In the same vein, contextual effects can significantly affect key-player policies (Ballester and Zenou, 2014).

[12] For example, in Galeotti et al. (2017), the adjustment cost of changing initial effort $\mathbf{X}^0$ to $\mathbf{X}$ is equal to $\sum_{i \in N}(x_i - x_i^0)^2$. Focusing on single targets, the corresponding policy cost of changing agent i's effort from x_i^0 to x_i would be equal to $(x_i - x_i^0)^2$.

[13] The literature on defense networks may be a natural starting point in this respect. For instance, in the context of criminal activities, Baccara and Bar-Isaac (2008) examine alternative organizations as optimal reaction to investigation policies. See also Acemoglu et al. (2016) for a model in which nodes invest in defense against a (possibly strategic) single-node attack in presence of contagion, as well as the works of Dziubiński and Goyal (2013, 2017).

and in particular, applying Eq. (8) to agent i, we get $x_i = b_i + ø \cdot \alpha$. Hence, to obtain a change in agent i's effort of $\phi \cdot \alpha$, it is sufficient to consider a linear system of best-responses with an adequately chosen modified constant to agent i's best-response.

Crucially, the interaction pattern between agents in System (7) is identical to the interaction pattern of the model when agent i accepts a contract that entails a change of her effort of $\phi \cdot \alpha$. This means that, to evaluate the consequence of a change in agent i's effort of $\phi \cdot \alpha$ on aggregate effort when agent i accepts a contract, it is sufficient to examine System (7): summing over agents' effort as given in Eq. (8), and remembering that b represents the initial aggregate effort, we get

$$x = b + ø \cdot \alpha \frac{b_i}{m_{ii}} \quad \square$$

Proof of Lemma 1. We have $\mathbf{Y} = \mathbf{G}\,\mathbf{X} = \mathbf{G}\,\mathbf{M}(\mathbf{1} + \frac{\phi\alpha}{m_{ii}}\mathbf{1}_i)$. Taking care that $\mathbf{G}\,\mathbf{M} = \frac{1}{\delta}(\mathbf{M} - \mathbf{I})$, we obtain $\mathbf{Y} = \frac{1}{\delta}(\mathbf{M} - \mathbf{I})(\mathbf{1} + \frac{\phi\alpha}{m_{ii}}\mathbf{1}_i)$. That is, $y_i = \frac{1}{\delta}\left(b_i - 1 + \frac{\phi\alpha}{m_{ii}}(m_{ii} - 1)\right)$. $\square$

Proof of Theorem 1. Consider $ø \in \{-1, 1\}$. Suppose that the policymaker contracts with agent i. Eq. (3) provides the aggregate effort change stemming from agent i's effort change. The change of agent effort, $ø \cdot \alpha$, is given by the binding participation constraint of agent i. Basically, α solves

$$u\big(b_i + ø \cdot \alpha, y_i^*(b_i + ø \cdot \alpha)\big) + t = u_i^R$$

with u_i^R being agent i's reservation utility, which can take two values over all contracts examined in this article: it is equal to either the initial utility $u(b_i, \frac{b_i - 1}{\delta})$ or, in the case of the non-enforceable contract with effort maximization (i.e. $ø = 1$), the utility level given by $u(1 + \delta y_i^*(b_i + \alpha), y_i^*(b_i + \alpha))$. Exploiting Eq. (4), we deduce agent i's participation constraint in all programs. In the case of the optimal non-enforceable bilateral contract under effort maximization, agent i's participation constraint is given by

$$u\left(b_i + \alpha, \frac{(b_i - 1)m_{ii} + \alpha(m_{ii} - 1)}{\delta m_{ii}}\right) + t = u\left(b_i + \alpha\frac{m_{ii} - 1}{m_{ii}}, \frac{(b_i - 1)m_{ii} + \alpha(m_{ii} - 1)}{\delta m_{ii}}\right) \quad (9)$$

For all other cases presented in this article, agent i's participation constraint is written

$$u\left(b_i + ø \cdot \alpha, \frac{(b_i - 1)m_{ii} + ø \cdot \alpha(m_{ii} - 1)}{\delta m_{ii}}\right) + t = u\left(b_i, \frac{b_i - 1}{\delta}\right) \quad (10)$$

The optimal value of α is found by inverting the relevant participation constraint: when $ø = 1$, Assumption 2 guarantees the existence of a solution for Eq. (10), and the existence of a solution for Eq. (9) follows because the best-response utility – the RHS – is positive. When $ø = -1$, we need to impose Assumption 3. It is transparent that all participation constraints only depend on b_i, m_{ii}, t, and of course on the policymaker's objective. $\square$

Proof of Proposition 2. Assume that the policymaker contracts with agent i. Agent i's participation constraint is written

$$x_i - \frac{x_i^2}{2} + \delta x_i y_i + t = \frac{1}{2}b_i^2 \quad (11)$$

with $x_i = b_i + ø \cdot \alpha$ and $y_i = \frac{(b_i - 1)m_{ii} + ø \cdot \alpha(m_{ii} - 1)}{\delta m_{ii}}$. Plugging x_i and y_i into Eq. (11), α solves the second-order equation

$$(2 - m_{ii})\,\alpha^2 - 2ø(m_{ii} - 1)b_i\,\alpha - 2m_{ii}\,t = 0$$

Since $m_{ii} < 2$, there is a unique positive root. It is given by

$$\alpha = \frac{1}{2 - m_{ii}}\left[ø \cdot (m_{ii} - 1)b_i + \sqrt{(m_{ii} - 1)^2 b_i^2 + 2m_{ii}(2 - m_{ii})t}\right]$$

The contract performance being equal to $\alpha\frac{b_i}{m_{ii}}$, the key-player maximizes the index

$$\frac{b_i}{m_{ii}}\left(\frac{ø \cdot (m_{ii} - 1)b_i + \sqrt{(m_{ii} - 1)^2 b_i^2 + 2m_{ii}(2 - m_{ii})t}}{2 - m_{ii}}\right) \quad \square$$

Proof of Proposition 3. Consider $ø = 1$. Suppose that agent i is proposed the contract by the principal (x_i, t). Let $\mathbf{1}_i = (0, 0, \ldots, 0, 1, 0, \ldots, 0)$ with 1 at entry i. When the contract is not enforceable, agent i's participation constraint is written

$$x_i - \frac{x_i^2}{2} + \delta x_i y_i + t = \frac{1}{2}(1 + \delta y_i)^2$$

with $x_i = b_i + \alpha$ and $y_i = \frac{(b_i - 1)m_{ii} + \alpha(m_{ii} - 1)}{\delta m_{ii}}$. Substituting x_i and y_i by their respective values, we get after some development $\alpha^2 = 2tm_{ii}^2$, that is:

$$\alpha = \sqrt{2t}\,m_{ii} \quad (12)$$

Substituting Eq. (12) into Eq. (3), we find

$$x = b + \sqrt{2t}\,b_i \quad \square$$

Proof of Proposition 4. We first present the policymaker's program, and then we proceed to the proof of the proposition.

• **The policymaker's program.** Under contract γ_i, all agents play Nash including agent i. We let $\mathbf{X}^*(\gamma_i)$ denote the corresponding equilibrium effort vector,[14] and we set $\mathbf{Y}^*(\gamma_i) = \mathbf{G}\mathbf{X}^*(\gamma_i)$. The principal has to identify the best target and the best reward under limited budget t. The optimal linear contract maximizes the program

$$\max_{i \in N, \gamma_i} \quad ø\,\mathbf{1}^T\mathbf{X}^*(\gamma_i)$$
$$\text{s.t.} \begin{cases} u(x_i^*(\gamma_i), y_i^*(\gamma_i)) + t \geq u(b_i, \frac{b_i - 1}{\delta}) \\ t = \gamma_i \cdot ø(x_i - b_i) \end{cases}$$

When the policymaker wants to maximize effort, the participation constraint is not an issue. Indeed, the agent is rewarded for increasing effort, and by complementarities at optimum all efforts are larger than the effort at the initial equilibrium. And by positive externalities, utilities are mechanically increased. So, under effort maximization, the optimal excess-effort linear contract solves

$$\max_{i \in N, \gamma_i} \quad \mathbf{1}^T\mathbf{X}^*(\gamma_i)$$
$$\text{s.t.} \quad t = \gamma_i\,(x_i - b_i)$$

In contrast, when the policymaker wants to minimize effort, the participation constraint is an issue. Indeed, when effort is lowered, agents' utilities decrease, so the transfer should cover the difference. But by linearity of the reward and because of linear quadratic utilities, the transfer covers the cost only with a sufficiently large budget. This means that with a low enough budget, the agent will always be better off rejecting the offer. In total, under effort minimization, the optimal linear contract is given by

$$\min_{i \in N, \gamma_i} \quad \mathbf{1}^T\mathbf{X}^*(\gamma_i)$$
$$\text{s.t.} \begin{cases} u(x_i^*(\gamma_i), y_i^*(\gamma_i)) + t = u(b_i, \frac{b_i - 1}{\delta}) \\ t = \gamma_i\,(b_i - x_i) \end{cases}$$

[14] Contract execution does not affect the condition of existence and uniqueness of Nash equilibrium.

Table 1
Key player with enforceable contract under linear quadratic utilities as a function of policymaker's objective.

Enforceable contract: Attribute \ objective	$ø = -1$	$ø = 1$
Key player	$\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} - (m_{ii}-1)b_i}{2-m_{ii}}$	$\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} + (m_{ii}-1)b_i}{2-m_{ii}}$
Aggr. effort change	$-\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} - (m_{ii}-1)b_i}{2-m_{ii}}$	$\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} + (m_{ii}-1)b_i}{2-m_{ii}}$

• **The proof of** Proposition 4**.** We consider $ø \in \{-1, 1\}$. Suppose that agent i is proposed a contract γ_i and suppose for now that her participation constraint is satisfied. We let $\mathbf{1}_i = (0, \dots, 0, 1, 0, \dots, 0)$ with 1 at the ith entry. The Nash effort profile is written[15]

$$\mathbf{X}^*(\gamma_i) = \mathbf{M}(\mathbf{1} + ø \cdot \gamma_i \mathbf{1}_i) \tag{13}$$

Noting that $\mathbf{M1}_i = b_i$, the variation of aggregate effort is written:

$$x^* - b = ø \cdot \gamma_i b_i \tag{14}$$

Also, plugging Nash effort into the budget constraint, we get

$$x_i^* - b_i = ø \cdot m_{ii}\breve{\gamma}_i \tag{15}$$

Plugging Eq. (15) into the budget constraint, we find

$$\breve{\gamma}_i = \sqrt{\frac{t}{m_{ii}}} \tag{16}$$

Plugging Eq. (16) into Eq. (15), we obtain

$$x_i^* - b_i = ø \cdot \sqrt{t\ m_{ii}}$$

while plugging Eq. (16) into Eq. (14), we obtain

$$x^* - b = ø \cdot \sqrt{t} \cdot \frac{b_i}{\sqrt{m_{ii}}} = ø \cdot \sqrt{t\ c_i}$$

Hence, conditional on contract participation, the agent with the highest inter-centrality index should be selected.

We turn to the participation constraint, which is an issue only under effort minimization. Suppose then $ø = -1$. Basically, agent i's participation constraint is given by

$$\frac{1}{2}\left(b_i - \sqrt{m_{ii} t}\right)^2 + b_i \sqrt{\frac{t}{m_{ii}}} - \frac{b_i^2}{2} \geq 0$$

i.e.,

$$t \geq 4\left(\frac{m_{ii} - 1}{m_{ii}}\right)^2 c_i$$

Hence, when $t < 4 \cdot \min_{i \in N} \left\{ \left(\frac{m_{ii}-1}{m_{ii}}\right)^2 c_i \right\}$, there is no contract under effort minimization. □

Appendix. Tables

Tables 1–3 present the key player under linear quadratic utilities, according to the nature of the contract, the available budget and the policymaker's objective.

Table 2
Key player with non-enforceable contract under linear quadratic utilities as a function of policymaker's objective.

Non-enforceable contract: Attribute \ objective	$ø = -1$	$ø = 1$
Key player	$\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} - (m_{ii}-1)b_i}{2-m_{ii}}$	b_i
Aggr. effort change	$-\frac{b_i}{m_{ii}} \cdot \frac{\sqrt{(m_{ii}-1)^2 b_i^2 + 2m_{ii}(2-m_{ii})\, t} - (m_{ii}-1)b_i}{2-m_{ii}}$	$\sqrt{2t}\ b_i$

Table 3
Key player with effort-change linear contract under linear quadratic utilities as a function of policymaker's objective.

Effort-change linear contract: Attribute \ objective	$ø = -1$	$ø = 1$
Key player	c_i such that $\left(\frac{m_{ii}-1}{m_{ii}}\right)^2 c_i \leq \frac{t}{4}$	c_i
Aggr. effort change	$-\sqrt{t\ c_i}$	$\sqrt{t\ c_i}$

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[15] The existence of the contract does not affect the conditions of equilibrium existence and uniqueness, the condition still being $\delta\mu(G) < 1$.