Datation of faults for Markovian Stochastic DESs

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***Abstract*—This technical note concerns the fault diagnosis of stochastic discrete event systems. Specifically, the goal is to characterize a detected fault by estimating its occurrence date. For that purpose, partially observed stochastic Petri nets are used to model the system, the failure processes and the sensors. From the proposed modelling and collected dated measurements, the probabilities of consistent trajectories are computed and diagnosis in terms of faults probability is established as a consequence. For each detected fault, the probability density function of its occurrence date is approximated. This estimation improves the diagnosis by providing the most probable time interval of the fault occurrence. The interest of fault datation and the applicability of the proposed approach are showed through a case study that represents a distribution system.**

***Index Terms*—Fault datation, fault detection and diagnosis, stochastic Petri nets.**

I. INTRODUCTION

At a certain level of abstraction, most systems can be considered as Discrete Event Systems (DESs) [2], ranging from manufacturing and traffic systems to biological and chemical processes. Developing efficient Fault Detection and Diagnosis (FDD) methods for such systems is a major stake. A FDD method decides whether a fault has occurred or not. It also provides as much information as possible on the fault such as the fault type, its location, the faulty component(s), its occurrence date…etc. The fault characterization improves the understanding of the fault, reduces the system downtime and helps to take appropriate decisions after the detection.

In the domain of DESs, many studies have addressed the problem of FDD [3,4]. This paper deals with the faulty model-based techniques that start with building a model that includes the faulty behaviors of the system to be modelled. In particular, we are interested in diagnosis techniques based on Petri nets (PNs) models [5] where faults are represented by unobservable events [4]. This is a well-studied problem in untimed context [6-11] compared to the timed context [12-15] despite the fact that considering time is crucial for detection and diagnosis. The existing approaches in timed context are based on the State Class Graphs [12, 13] and Modified State Class Graphs [16]. In [15], stochastic dynamics are considered and Partially Observed Stochastic Petri Nets (POSPNs), with partial measurements on both markings and events, are exploited. In order to evaluate the likelihood of the fault occurrence, fault probabilities are estimated taking inspiration from fault beliefs [11] used when the diagnosis decision is ambiguous. The motivation to use Petri nets as a modeling tool is that such models are physics-based and provide a realistic and comprehensive representation of the considered systems. They are modular and evolutive in the sense that they can easily be updated when the system specifications change. In addition, the POSPNs take into account not only the system operations but also the sensors used to collect the information about operations.

In terms of safety, rapid detection of faults is required to take appropriate actions and reconfigurations, preventing equipment damages and human injuries. However, there usually exist a delay between the time of detection and the exact time when the fault has occurred. To the best of our knowledge, most of the existing works on FDD of DESs ignores this delay in untimed and also in timed context. The fault (or event) date estimation is rarely explored [16, 17] despite the fact that it is important in FDD process. This is the objective of this technical note that is motivated as follows:

- Fault severity assessment: as the delay between a fault occurrence and its detection moment increases, the consequences and impacts of this fault on the system become significant and may lead to serious damages [18]. Estimating the fault occurrence date is important to assess the severity of the fault and to evaluate its consequences.
- Traceability issues: fault datation is also important for traceability issues. Traceability is defined as the ability to describe and follow the life of an equipment, in both a forward and a backward directions, thus ensuring that a requirement can be "traced" from its origins, through its specification and development, to its subsequent deployment and use [19]. To be efficient, traceability should include dated events and time information. For example in pharmaceutical or food industry, it is crucial to identify the date at which a fault has occurred and has changed for example the composition of a product. It is then possible to recall all affected products and protect the consumers. It is also mandatory for law and legal responsibility.

In this work, we are interested in FDD of DESs modelled with POSPNs. Particularly, when a fault is detected, the objective is to estimate its occurrence date. In our previous works, the consistent behaviors with a measurement trajectory have been computed and their probabilities have been assessed [1]. The probability of faults was obtained consequently. In this paper, when a fault is detected, a method is proposed to evaluate the Probability Density Function (PDF) of the fault occurrence date as a function. It is based on the analysis of the consistent behaviors and allows computing the most probable time interval where the fault has occurred. Our results are consolidated by using a Monte Carlo discrete event simulation approach. Not that such approaches and also particle filtering (that can be viewed as a sophisticated Monte Carlo simulation) [27] can be used as non-analytical alternative methods for the fault detection.

This technical note is organized as follows. In Section II, POSPNs, measurement trajectories and consistent trajectories are described. In Section III, the fault diagnosis method of POSPNs developed in [1] is first recalled. The approach for estimating the PDFs of the faults dates is then detailed and an example is provided. In Section IV, a case study representing a distribution system is considered to show the advantages of fault datation. Finally, some conclusions and perspectives are presented in Section V.

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II. Modeling of Markovian Processes with PNs

A. Stochastic Petri Nets

Petri nets are graphical and mathematical modelling tools [2, 5]. In addition, Stochastic Petri Nets (SPNs) are characterized by random firing delays associated with the transitions [20]. Formally, a SPN structure is defined as $G_s = < P, T, W_{PR}, W_{PO}, \mu >$, where $P = \{P_1, \dots, P_n\}$ is a set of n places and $T = \{T_1, \dots, T_q\}$ is a set of q transitions, $W_{PR} \in (\mathbb{N})^{n \times q}$ and $W_{PO} \in (\mathbb{N})^{n \times q}$ are the pre and post incidence matrices ($\mathbb{N}$ is the set of non-negative integer numbers), and $W = W_{PO} - W_{PR}$ is the incidence matrix. $\mu = (\mu_j) \in (\mathbb{R}^+)^q$ ($\mathbb{R}$ is the set of real numbers) is the firing rate vector which characterizes the transition firing periods. $< G_s, M_I >$ is a SPN system with initial marking $M_I \in (\mathbb{N})^n$. A transition T_j is enabled at marking M if and only if (iff) $M \geq W_{PR}(:, j)$, where $W_{PR}(:, j)$ is the column j of the pre incidence matrix; this is denoted as $M[T_j >$. Thus, for each transition T_j, enabled at marking M, the firing periods are given by a Random Variable (RV) with an exponential PDF of parameter $n_j(M).\mu_j$ where $n_j(M)$ stands for the enabled degree of transition T_j at marking M which is given by:

$$n_j(M) = \min\left\{\left\lfloor\left(\frac{m_k}{w_{PR}(k,j)}\right)\right\rfloor, P_k \in {}^\circ T_j\right\} \quad (1)$$

where ${}^\circ T_j$ is the set of input places or preset (the set of places with output arcs connected with T_j) and m_k is the marking of P_k, $w_{PR}(k, j)$ is the element in row k and colomn j of matrix W_{PR}. Finally, $\lfloor . \rfloor$ stands for the lower rounded value of (.). When T_j is enabled, it may fire, and when T_j fires once, the marking varies according to $\Delta M = M' - M = W(:, j)$. This is denoted as $M[T_j > M'$. A SPN system has a time semantic [21, 22] that is defined according to (a) an *infinite server* policy: each transition is considered as a server for firings and in a given marking, each transition may fire simultaneously several times depending on its enabling degree; (b) a *race policy*: the transition whose firing delay elapses first is assumed to be the one that will fire next; (c) a *resampling memory* policy: at the entrance in a marking, the remaining delays associated with all transitions are forgotten. Under some specific assumptions, the SPN behaves as a Markov model [20, 23].

A *timed firing sequence* σ of length $h = |\sigma|$ fired at marking M in time interval $[t_0, t_{end}]$ is defined as: $\sigma = T(t_1)T(t_2) \dots T(t_h)$ where t_k, $k = 1, \dots, h$ represent the firing dates of transitions $T(t_k) \in T$ that satisfy $t_0 \leq t_1 \leq t_2 \leq \dots \leq t_h \leq t_{end}$. This leads to the *timed trajectory* (2):

$$(\sigma, M) = M(t_0)\,[T(t_1) > M(t_1) \dots [T(t_h) > M(t_{end}) \quad (2)$$

with $M(t_0) = M$. Note that *untimed firing sequences* and *untimed trajectories* can be considered in a similar way by making abstraction of time. In this case $M(t_k)$ and $T(t_k)$ are simply denoted as $M(k)$ and $T(k)$.

B. Exponential Random Variable

The firing dates of the transitions in a SPN are given by a sum of exponential RVs. Let us consider the Cumulative Distribution Function (CDF) of the sum of n independent RVs $X_i, i = 1, \dots, n$ having exponential PDFs with parameters $\lambda_i, i = 1, \dots, n$ respectively. Let us denote by S_n the sum of these RVs: $S_n = \sum_{i=1}^{n} X_i$. In the case all parameters λ_i are equal, S_n is modelled by an Erlang distribution [24]. Now, consider the general case where the set of parameters λ_i is composed of a distinct values represented by the two sets $\{\beta_1, \beta_2, \dots \beta_a\}$ and $\{r_1, r_2 \dots r_a\}$ where β_j is the value of the parameter and r_j its multiplicity such that $r_1 + r_2 \dots + r_a = n$. The CDF $G_a(t)$ of S_n for $t \geq 0$ is given by [25]:

$$G_a(t) = 1 - \left(\prod_{j=1}^{a} \beta_j^{r_j}\right) \sum_{k=1}^{a} \sum_{l=1}^{r_k} \frac{\Psi_{k,l}(-\beta_k) t^{r_k - l} \exp(-\beta_k t)}{(r_k - l)!(l-1)!} \quad (3)$$

$$\Psi_{k,l}(x) = -\frac{\partial^{l-1}}{\partial x^{l-1}}\{\prod_{j=0, j\neq k}^{a} (\beta_j + x)^{-r_j}\} \text{ and } \beta_0 = 0, r_0 = 1.$$

C. Partially Observed SPNs and Measured Trajectories

POSPNs are SPNs that include the sensor specifications. For this purpose, a labeling function $\mathcal{L} : T \to E \cup \{\varepsilon\}$ is introduced to assign a label to each transition. $E = \{e_1, \dots, e_{q_o}\}$ is the set of q_o labels that are assigned to observable transitions and ε is the null label. The labeling function is extended to firing sequences and the concatenation of labels obviously satisfies: $\varepsilon.\varepsilon = \varepsilon$ and $\varepsilon.e_k = e_k.\varepsilon = e_k$. The measured label is denoted by $e_O(\tau_i)$ and τ_i is the measurement date. In addition, a marking sensor matrix H is also introduced to collect continuously the weighted sum of the markings over n_o subsets of measured places. $H = (h_{kj}) \in (\mathbb{R})^{n_o \times n}$ such that $h_{kj} \neq 0$ if the marking of the place P_j is considered in the k^{th} subset, and $h_{kj} = 0$ otherwise. Thus, matrix H represents the local marking measurement in single place or the global weighted marking measurement in subnets. The measured part of the marking at date τ_i is denoted as (M_O, τ_i) with $M_O = H.M(\tau_i)$.

Definition 1: A POSPN system is defined as $< G_s, \mathcal{L}, H, M_I >$ where G_s is a SPN structure, $\mathcal{L}$ is the labeling function, H is the marking sensor matrix and M_I is the initial marking. $\mathcal{L}$ and H determine the sensor configuration.

Finally, $F = \{f_1, \dots, f_s\}$ is the set of s fault classes and the function $\mathcal{F}: T \to F \cup \{\emptyset\}$ is introduced to assign a fault class to each faulty transition such that $\mathcal{F}(T_j) = \{f_i\}$ if the fault class f_i is assigned to the transition T_j and $\mathcal{F}(T_j) = \emptyset$ if T_j is a non-faulty transition and corresponds to the expected behavior. The function $\mathcal{F}$ is extended to firing sequences σ such that $\mathcal{F}(\sigma) = \cup \{\mathcal{F}(T_k): T_k \in \sigma\}$.

A measurement function Γ [1, 15] collects the successive dated marking and event measurements of a timed trajectory (σ, M) within time interval $[\tau_0, \tau_{end}]$ and organizes them in the *measurement trajectory* (4) composed of successive dated labels and marking measurements:

$$\Gamma(\sigma, M) = (M_{O0}, \tau_0) e_O(\tau_1)(M_{O1}, \tau_1) \dots e_O(\tau_K)(M_{OK}, \tau_{end}) \quad (4)$$

where K is the length of the measured trajectory and τ_j, $j = 1, \dots, K$ refer to the dates of measurements, $e_O(\tau_j) \in E \cup \{\varepsilon\}$ is the j^{th} label. The label ε is used when a silent transition fires and the firing is indirectly detected (but the transition is not necessarily isolated) by the modification of the measured part of the marking. (M_{Oj}, τ_j) is the j^{th} marking measurement and $M_{O0} = H.M(\tau_0)$. The form given by (4) with an alternation between measured markings and labels is a representation of the collected measurements. Given a measured trajectory TR_O, a trajectory (σ, M) that satisfies $\Gamma(\sigma, M) = TR_O$ is said to be consistent with TR_O in $[\tau_0, \tau_{end}]$.

In order to deal with the problem of fault datation, we exploit the fault diagnosis method of POSPNs that has been proposed in our previous works [1, 15] which is based on two steps:

- The computation of the set of consistent trajectories with the measurements TR_O, denoted by $\Gamma^{-1}(TR_O)$.
- The computation of the probabilities of the consistent trajectories (i.e. the probability that a consistent trajectory $(\sigma, M) \in \Gamma^{-1}(TR_O)$ corresponds to the trajectory that truly provides the measurements). The fault probability is obtained as a consequence.

This method is based on two assumptions:

Assumption 1. The finite set $\Upsilon(\tau_0)$ of possible markings M at time τ_0 and the probabilities $\pi_0(M), M \in \Upsilon(\tau_0)$ are assumed to be known.

Assumption 2. The length of unobservable firing sequences that do not change the observable part of the marking is bounded.

Assumption 1 is required to avoid the enumeration of all reachable markings in order to find those which are consistent with the initial measurement M_{O0}. Since several firing sequences σ with various lengths h (h may eventually be infinite) may be consistent with a given measured trajectory TR_O, Assumption 2 is required to guarantee that the consistent trajectories are of finite length. This assumption can be ensured by the sensor configuration and this assumption is sufficient to ensure that the silent part of the net is acyclic which is an usual assumption for DESs diagnosis. The maximal number of events within $]\tau_i, \tau_{i+1}]$ is denoted by h_{max} such that the maximal number of silent events equals $h_{max} - 1$. Considering Assumptions 1 and 2, $\Gamma^{-1}(TR_O)$ is thus of finite cardinality. This set is composed of the exhaustive list of *partially timed trajectories* $(\sigma, M) \in \Gamma^{-1}(TR_O)$ with σ of form (5). (σ, M) is said to be partially timed since only the firing dates of the transitions $\boldsymbol{T}(\boldsymbol{\tau_k})$ in σ, that correspond to measurements, are known. Variables are associated to the other dates that are unknown.

$$\sigma = T(t_{1,1}) \dots T(t_{1,h_1-1})\boldsymbol{T}(\boldsymbol{\tau_1})T(t_{2,1}) \dots T(t_{2,h_2-1})\boldsymbol{T}(\boldsymbol{\tau_2}) \dots \boldsymbol{T}(\boldsymbol{\tau_{K-1}})T(t_{K,1}) \dots T(t_{K,h_K-1})\boldsymbol{T}(\boldsymbol{\tau_K})T(t_{K+1,1}) \dots T(t_{K+1,h_{K+1}-1}) \quad (5)$$

with $h_k \leq h_{max}$, $k = 1, \dots, (K+1)$. The dates $t_{k,1}, t_{k,2}, \dots, t_{k,h_k}$ satisfy $\tau_{k-1} \leq t_{k,1} \leq t_{k,2} \leq \dots \leq t_{k,h_k-1} \leq \tau_k$ for all $k = 1, \dots, (K+1)$. Only the dates $\tau_k = t_{k,h_k}$, $k = 1, \dots, K$, are measured and all other dates $t_{k,1}, t_{k,2}, \dots, t_{k,h_k-1}$, $k = 1, \dots, (K+1)$ are unknown and correspond to silent events. The transitions $T(t_{K+1,1}) \dots T(t_{K+1,h_{K+1}-1})$ correspond to the silent closure.

In Section III.A, the computation of the set of consistent trajectories and their probabilities are briefly recalled and the fault probability is obtained as a consequence. The problem of the fault datation is then considered. In fact, when a fault is detected, its occurrence date $t_\alpha = t_{k,i}$ is unknown and could have occurred within any time interval $[\tau_k, \tau_{k+1}]$ depending on the considered faulty consistent trajectory. The main issue is thus to analyze the consistent trajectories in order to estimate the PDF of the unknown fault date. The most probable time interval of the fault event date is obtained as a consequence. This will be discussed in Section III.B.

III. Fault Datation

A. Some results on fault diagnosis of POSPNs

From a timed measurement trajectory TR_O, the first step for fault diagnosis is to compute the set of consistent trajectories $\Gamma^{-1}(TR_O)$. It has been obtained in our previous works [1, 15] with a method based on Linear Matrix Inequalities (LMIs). These inequalities reformulate, thanks to linear algebra, the conditions induced by each new measurement. Note that the complexity to compute $\Gamma^{-1}(MT)$ using the set of LMIs is discussed in [15] and given by $\mathcal{O}\left((q+1)^{N.(K+1).h_{max}}\right)$. The complexity is thus exponential wrt the number N of reachable markings and to the length K of the measurement sequences. An incremental method was also proposed in [1] that incrementally constructs the consistent trajectories. Its complexity is given by $\mathcal{O}\left(|\Upsilon(\tau_0)|.q^{(K+1).h_{max}}\right)$. Both methods lead to the computation of the consistent trajectories $(\sigma, M) \in \Gamma^{-1}(TR_O)$ with σ of form (5).

The second step consists in calculating the probability of each partially timed trajectory wrt the measurement dates to deduce the faults probabilities. Considering a consistent trajectory $(\sigma, M) \in \Gamma^{-1}(TR_O)$ with σ of form (5), its probability is given by:

$$P(\sigma, M) = \frac{\pi(\sigma, M)}{\sum_{(\sigma', M') \in \Gamma^{-1}(TR_O)} \pi(\sigma', M')} \quad (6)$$

with:

$$\pi(\sigma, M) = \pi_0(M).P_U(\sigma, M).\left(\prod_{i=1\dots K} P_{\sigma_i}\right).P_{\sigma_{SC}} \quad (7)$$

The different terms of equation $\pi(\sigma, M)$ are detailed in [1] and recalled in the follows.

$\pi_0(M)$ is the probability that the marking at time τ_0 is M. Thanks to Assumption 1, this probability is assumed to be known.

$P_U(\sigma, M)$ is the probability of the untimed trajectory (i.e. of the trajectory obtained from (σ, M) by making abstraction of time). This probability is obtained by computing the probability to fire each transition of the trajectory before any other enabled transition. It is given by:

$$P_U(\sigma, M) = \prod_{k=1\dots(K+1)} \left(\prod_{\gamma=1\dots h_k} \left(\frac{n_{k,\gamma}(M(k,\gamma-1)) \times \mu_{k,\gamma}}{\sum_{T_j \in T} n_j(M(k,\gamma-1)) \times \mu_j} \right) \right) \quad (8)$$

with $M(1,0) = M$, $M(k,\gamma)$ results from the firing of $T(t_{k,\gamma})$ for $k = 1, \dots, (K+1), \gamma = 1, \dots, (h_k - 1)$ and $M(k,0) = M((k-1), h_k)$ for $k = 2, \dots, (K+1)$.

P_{σ_i} represents, for each group of transitions $\sigma_i = T(t_{i,1}) \dots T(t_{i,h_i-1})\boldsymbol{T}(\boldsymbol{\tau_i})$ $i = 1, \dots, K$ fired within $[\tau_{i-1}, \tau_i]$, the probability to fire all transitions $T(t_{i,1}) \dots T(t_{i,h_i-1})$ before the instant $t_{i,h_i} = \tau_i$ (i.e. $t_{i,h_i-1} \leq \tau_i$) and then to fire $\boldsymbol{T}(\boldsymbol{\tau_i})$ within time interval $[\tau_i \ \tau_i + \Delta t]$ where Δt is an infinitesimally small time increment. This probability is given by:

$$P_{\sigma_i} = \left(\int_{\tau_{i-1}}^{\tau_i} g_{h_i-1}(t).\left(1 - G_{h_i}(\tau_i - t)\right).dt \right).\sum_{T_j \in T} n_j(M(i)).\mu_j.\Delta t \quad (9)$$

where g_{h_i-1} is the PDF that characterizes the duration necessary to fire $T(t_{i,1}) \dots T(t_{i,h_i-1})$. It corresponds to a sum of $h_i - 1$ exponential RVs given by (3). G_{h_i} is the CDF that characterizes the firing of the measured transition $\boldsymbol{T}(\boldsymbol{\tau_i})$. Finally, $M(i)$ is the marking from which $\boldsymbol{T}(\boldsymbol{\tau_i})$ is fired. Note that the increment Δt is simplified thanks to equation (6).

$P_{\sigma_{SC}}$ is the probability to fire the silent transitions (silent closure) $\sigma_{SC} = T(t_{K+1,1}) \dots T(t_{K+1,h_{K+1}-1})$ within $]\tau_K, \tau_{end}]$ and to not fire any enabled transitions until τ_{end}. It is given by:

$$P_{\sigma_{SC}} = \int_{\tau_K}^{\tau_{end}} g_{h_{K+1}-1}(t).\left(1 - G_{h_{K+1}}(\tau_K - t)\right).dt \quad (10)$$

$g_{h_{K+1}-1}$ characterizes the duration necessary to fire $T(t_{K+1,1}) \dots T(t_{K+1,h_{K+1}-1})$ and $G_{h_{K+1}}$ the firing of any enabled transition after the firing of $T(K+1, h_{K+1}-1)$.

Once the probability of each consistent trajectory $(\sigma, M) \in \Gamma^{-1}(TR_O)$ is computed, the probability that a fault of class f_α has occurred wrt TR_O can be deduced. It is given by:

$$P(f_\alpha, TR_O) = \sum_{(\sigma,M)\in\Phi_\alpha(TR_O)} P(\sigma, M) \quad (11)$$

with $\Phi_\alpha(TR_O) = \{(\sigma, M) \in \Gamma^{-1}(TR_O)$ such that $f_\alpha \in \mathcal{F}(\sigma)\}$.
Only trajectories that contain the fault f_α and thus belong to the set $\Phi_\alpha(TR_O)$ are considered and the sum of their probabilities represents the fault probability.

The complexity of the fault probability computation $\mathcal{O}\big(P(f_\alpha, TR_O)\big)$ depends on the number N_{CT} of consistent trajectories and on the complexity $\mathcal{O}\big(P(\sigma, M)\big)$ to compute the probability of a consistent trajectory which mainly depends on the used integration method. The Gauss-Kronrod quadrature was used to compute the integration because it is a nested adaptive method that saves some of the numerical efforts.

B. *Fault Occurrence Dates*

This section will focus on the evaluation of the PDFs of the silent events dates in order to estimate the fault occurrence dates. Let us consider a faulty consistent trajectory (σ, M) where σ is of form (5) and assume that the fault event, denoted by $T_\alpha(t_\alpha)$, occurs before the last measured event. There exists two measurement dates τ_i and τ_{i+1} such that $t_\alpha \in [\tau_i, \tau_{i+1}]$. The sequence σ of the trajectory (σ, M) can be represented as follows:

$$\sigma = \cdots \boldsymbol{T(\tau_i)} T(t_{i,1}) \dots T(t_{i,m}) T_\alpha(t_\alpha) T(t_{i,m+1}) \dots T(t_{i,n}) \boldsymbol{T(\tau_{i+1})} \dots \quad (12)$$

Let us first consider a time interval $[t_s, t_f] \subseteq [\tau_i, \tau_{i+1}]$ and compute the probability that the fault event has occurred within $[t_s, t_f]$ wrt the trajectory (σ, M). This probability will be denoted by $P\left(t_\alpha \in [t_s, t_f]/(\sigma, M)\right)$. As depicted in Fig. 1, the method consists in computing the probability to fire all the transitions $T(t_{i,1}) \dots T(t_{i,m}) T_\alpha(t_\alpha)$ so that $t_\alpha \in [t_s, t_f]$ and to fire the transitions $T(t_{i,m+1}) \dots T(t_{i,n}) T(\tau_{i+1})$ within time interval $[t_\alpha, \tau_{i+1}]$.

The occurrence date t_α can be written as $t_\alpha = \tau_i + d_\alpha$ where $d_\alpha \geq 0$ is a RV representing the duration necessary to fire the transitions $T(t_{i,1}) \dots T(t_{i,m}) T_\alpha(t_\alpha)$. We also denote by $d_\beta \geq 0$ the RV that represents the duration necessary to fire transitions $T(t_{i,m+1}) \dots T(t_{i,n}) T(\tau_{i+1})$. Let us denote by $g_{d_\alpha}(x)$ and $g_{d_\beta}(x)$ the PDFs of d_α and d_β. The durations d_α and d_β are the sums of $(m+1)$ and $(n-m+1)$ independent exponential RVs, consequently they are also independent RVs. They are given by Erlang PDFs and can be derived from (3). Proposition 1 characterizes the probability that a fault of class f_α occurs within a time interval $[t_s, t_f] \subseteq [\tau_i, \tau_{i+1}]$ with the condition that (σ, M) is the realized trajectory (i.e. knowing that (σ, M) corresponds to the real behavior of the system).

Proposition 1. Let us consider a DES modelled with a marked POSPN $< G_s, \mathcal{L}, H, M_I >$ that satisfies Assumptions 1 and 2 and a measured trajectory TR_O collected within $[\tau_0, \tau_{end}]$. Let us also consider a partially timed trajectory $(\sigma, M) \in \Gamma^{-1}(TR_o)$. The probability that a fault of class f_α has occurred within a given time interval $[t_s, t_f] \subseteq [\tau_i, \tau_{i+1}]$ knowing that (σ, M) is the realized trajectory and $t_\alpha \in [\tau_i, \tau_{i+1}]$ is given by:

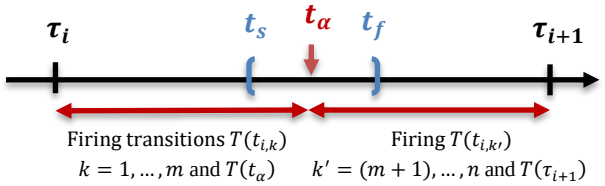

Fig. 1. Probability of fault date $[t_s, t_f] \subseteq [\tau_i, \tau_{i+1}]$.

$$P\left(t_\alpha \in [t_s, t_f]/(\sigma, M)\right) = \frac{\int_{t_s-\tau_i}^{t_f-\tau_i} g_{d_\alpha}(t)\, g_{d_\beta}((\tau_{i+1}-\tau_i)-t)dt}{\int_0^{\tau_{i+1}-\tau_i} g_{d_\alpha}(t)\, g_{d_\beta}((\tau_{i+1}-\tau_i)-t)dt} \text{ if } t_\alpha \in [\tau_i, \tau_{i+1}], \text{ 0 otherwise} \quad (13)$$

$g_{d_\alpha}(t)$ and $g_{d_\beta}(t)$ the PDFs of the RVs d_α and d_β derived from (3).

Proof. Obviously, $P\left(t_\alpha \in [t_s, t_f]/(\sigma, M)\right)$ equals 0 if $t_\alpha \notin [\tau_i, \tau_{i+1}]$ since we consider that $[t_s, t_f] \subseteq [\tau_i, \tau_{i+1}]$. In case $t_\alpha \in [\tau_i, \tau_{i+1}]$, the sequence σ is given by (12) and the durations d_α and d_β satisfy $d_\alpha + d_\beta = \tau_{i+1} - \tau_i$. The objective is then to compute the conditional probability $P\left(d_\alpha \in [t_s - \tau_i,\ t_f - \tau_i]/\big(d_\alpha + d_\beta = \tau_{i+1} - \tau_i\big)\right)$.
Considering two continuous RVs X and Y, the conditional PDF of X knowing that $Y = y$ can be written as :

$$g_{X/Y=y}(x) = \frac{g_{X,Y}(x,y)}{g_Y(y)} \quad (14)$$

where $g_{X,Y}(x,y)$ gives the join density of X and Y and $g_Y(y)$ the PDF of Y. Considering $X = d_\alpha$, $Y = d_\alpha + d_\beta$, $y = \tau_{i+1} - \tau_i$ and knowing that the joint density $g_{d_\alpha, d_\alpha+d_\beta}(x,y)$ is equivalent to $g_{d_\alpha, d_\beta}(x, y-x)$, we obtain :

$$g_{d_\alpha/d_\alpha+d_\beta=\tau_{i+1}-\tau_i}(x) = \frac{g_{d_\alpha,d_\beta}(x,(\tau_{i+1}-\tau_i)-x)}{g_{d_\alpha+d_\beta}(\tau_{i+1}-\tau_i)} \quad (15)$$

Using the fact that d_α and d_β are independent, $g_{d_\alpha,d_\beta}(x,(\tau_{i+1}-\tau_i)-x)$ and $g_{d_\alpha+d_\beta}(\tau_{i+1}-\tau_i)$ are respectively given by the product and the convolution of g_{d_α} and g_{d_β}. Finally, $g_{d_\alpha/d_\alpha+d_\beta=\tau_{i+1}-\tau_i}(x)$ is integrated within $[t_s - \tau_i,\ t_f - \tau_i]$. Thus equation (13) is obtained. ■

Let us now consider the more general case where the time interval $[t_s, t_f] \subseteq [\tau_i, \tau_{i+n}]$ with $n > 1$ and $[\tau_i, \tau_{i+n}]$ is the smallest interval composed of two measured dates and includes $[t_s, t_f]$ (see Fig. 2). Moreover, the result is now extended to the case where a set of trajectories consistent with a given measurement trajectory TR_O is obtained as in II.A with their associated probabilities. The objective is thus to compute the probability that the fault occurs within a time interval $[t_s, t_f]$, that will be denoted by $P(t_\alpha \in [t_s, t_f])$, wrt the measurement trajectory TR_O. One has to consider all the consistent trajectories in $\Gamma^{-1}(TR_o)$ where the fault event may have occurred within any measured time interval $[\tau_k, \tau_{k+1}]$ for each trajectory. The probability $P(t_\alpha \in [t_s, t_f])$ is thus calculated by Proposition 2.

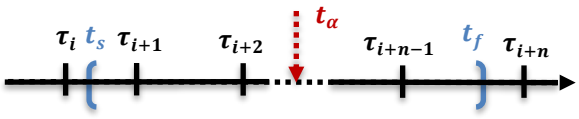

Fig. 2. Probability of fault date with $[t_s, t_f] \subseteq [\tau_i, \tau_{i+n}]$.

Proposition 2. Let us consider a DES modelled with a marked POSPN $< G_s, \mathcal{L}, H, M_I >$ that satisfies Assumptions 1 and 2 and a measured trajectory TR_O collected within $[\tau_0, \tau_{end}]$. The probability that a fault of class f_α occurs within the time interval $[t_s, t_f] \subseteq [\tau_i, \tau_{i+n}] \subseteq [\tau_0, \tau_{end}]$ is given by (16) where each conditional probability is given by (13):

$$P(t_\alpha \in [t_s, t_f]) = \sum_{(\sigma,M)\in\Gamma^{-1}(TR_O)} P(\sigma, M) \times P\left(t_\alpha \in [t_s, t_f]/(\sigma, M)\right) \text{ if } n = 1$$

$$P(t_\alpha \in [t_s, t_f]) = \sum_{(\sigma,M)\in\Gamma^{-1}(TR_O)} P(\sigma, M).\Big(P(t_\alpha \in [t_s, \tau_{i+1}]/(\sigma, M)) + P(t_\alpha \in [\tau_{i+1}, \tau_{i+n-1}]/(\sigma, M)) + P\left(t_\alpha \in [\tau_{i+n-1}, t_f]/(\sigma, M)\right)\Big) \text{ if } n > 1 \quad (16)$$

Proof. If $n = 1$, the time interval $[t_s, t_f]$ is included in an interval $[\tau_i, \tau_{i+1}]$ composed of two successive measured dates and the probability $P(t_\alpha \in [t_s, t_f])$ for a fixed trajectory (σ, M) is given by equation (13) multiplied by the probability of the trajectory $P(\sigma, M)$. If $n > 1$, the interval $[\tau_i, \tau_{i+n}]$ can be divided into disjoint time intervals as follows (see Fig. 2):

$$[t_s, t_f] = [t_s, \tau_{i+1}] \cup \cup_{k=1,\dots,n-2}[\tau_{i+k}, \tau_{i+k+1}] \cup [\tau_{i+n-1}, t_f] = [t_s, \tau_{i+1}] \cup [\tau_{i+1}, \tau_{i+n-1}] \cup [\tau_{i+n-1}, t_f]$$

Equation (13) multiplied by the probability of the trajectory is then used to compute the probability that the fault has occurred within each interval. The sum over all the consistent trajectories leads to (16). ■

The complexity of the fault datation method depends on the number N_{fCT} of faulty consistent trajectories to be analyzed and on the complexity $\mathcal{O}\left(P\left(t_\alpha \in [t_s, t_f]/(\sigma, M)\right)\right)$ of equation (13) which is related to the integration method (Gauss-Kronrod quadrature). Note that the estimation of the fault date is performed offline (once the fault is detected) and the complexity issue is less critical for the fault characterization.

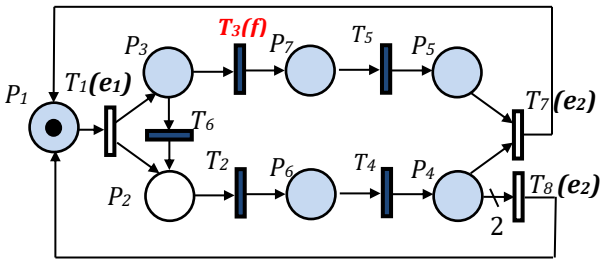

Fig 3. The considered POSPN (unobservable places and transitions are represented in grey).

C. Example

We suppose the POSPN of Fig. 3 has the following parameters μ = $(1\ 1\ 1\ 1\ 1\ 1\ 1)^T$. The initial marking is giving by M_I = $(1\ 0\ 0\ 0\ 0\ 0\ 0)^T$. The set of labels is $E = \{e_1, e_2\}$, the labeling function verifies $\mathcal{L}(T_1) = e_1$, $\mathcal{L}(T_7) = \mathcal{L}(T_8) = e_2$, and the marking sensor matrix is H = $(0\ 1\ 0\ 0\ 0\ 0\ 0)$. A single fault class $F = \{f\}$ is considered that corresponds to the firing of T_3 (i.e. $\mathcal{F}(T_3) = f$). Note that Assumption 2 is satisfied with $h_{max} = 4$. The aim is to validate the fault datation method. For that purpose, we will consider a scenario where the fault has certainly occurred.

Let us consider the two following measured trajectories within $[0, 3]$:

$$TR_{O1} = (0,0)e_1(0.006)(1,0.006)\varepsilon(0.31)(0,0.31)e_2(1.94)(0,3)$$
$$TR_{O2} = (0,0)e_1(0.36)(1,0.36)\varepsilon(1.12)(0,1.12)e_2(2.78)\ (0,3)$$

The successive marking measurements of place P_2 until the observation of event e_2 indicate that the fault f certainly occurred. This is confirmed by computing the sets of consistent partially timed trajectories for TR_{O1} and TR_{O2} given by:

$\Gamma^{-1}(TR_{Ok}) = \{(M_I, \sigma_i)\}$ with

$$\sigma_1 = \boldsymbol{T_1(\tau_1)}T_3(t_{2,1})T_5(t_{2,2})\boldsymbol{T_2(\tau_2)}T_4(t_{3,1})\boldsymbol{T_7(\tau_3)}$$
$$\sigma_2 = \boldsymbol{T_1(\tau_1)}T_3(t_{2,1})\boldsymbol{T_2(\tau_2)}T_5(t_{3,1})T_4(t_{3,2})\boldsymbol{T_7(\tau_3)}$$
$$\sigma_3 = \boldsymbol{T_1(\tau_1)}T_3(t_{2,1})\boldsymbol{T_2(\tau_2)}T_4(t_{3,1})T_5(t_{3,2})\boldsymbol{T_7(\tau_3)}$$
$$\sigma_4 = \boldsymbol{T_1(\tau_1)T_2(\tau_2)}T_3(t_{3,1})T_5(t_{2,2})T_4(t_{3,3})\boldsymbol{T_7(\tau_3)}$$
$$\sigma_5 = \boldsymbol{T_1(\tau_1)T_2(\tau_2)}T_4(t_{3,1})T_3(t_{3,2})T_5(t_{3,3})\boldsymbol{T_7(\tau_3)}$$
$$\sigma_6 = \boldsymbol{T_1(\tau_1)T_2(\tau_2)}T_3(t_{3,1})T_4(t_{3,2})T_5(t_{3,3})\boldsymbol{T_7(\tau_3)}$$

where $(\tau_1, \tau_2, \tau_3) = (0.006, 0.31, 1.94)$ for TR_{O1} and $(\tau_1, \tau_2, \tau_3) = (0.36, 1.12, 2.78)$ for TR_{O2}.

The probability of each partially timed trajectory differs for TR_{O1} and TR_{O2} but in both cases the fault probability equals 1.

We are now interested in the estimation of the occurrence date of the fault. The curves in Fig. 4 and 5 represent respectively the estimation of the PDFs of f occurrence dates for TR_{O1} and TR_{O2} calculated with Propositions 1 and 2. In order to validate the proposed method and consolidate our results, a Monte Carlo simulation method is used. Such a method estimates the PDF by a swarm of points rather than by a function. For this purpose, the POSPN of Fig. 3 is simulated a large number of times and two sets of 500 sequences that fit respectively TR_{O1} and TR_{O2} are collected. From each set, the fault occurrence dates of f are extracted (histograms in Fig 4 and 5). For small to medium size systems, such methods could be considered as a possible alternative approach. Note however that the efficiency of such an approach is directly related to the number of particles: a large number of particles is required to provide a good approximation of the PDF and a further study of particle filters and other Baysian approaches would be necessary to apply them to large systems [27]. One can conclude that $[0TU, 0.3TU]$ is an interval of high probability for the occurrence of f if TR_{O1} is measured (see Fig. 4) while for

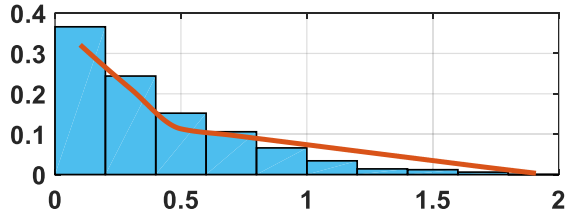

Fig. 4. PDF of the fault occurrence date for TR_{O1}: simulation (histogram), computed with Proposition 2 (curve).

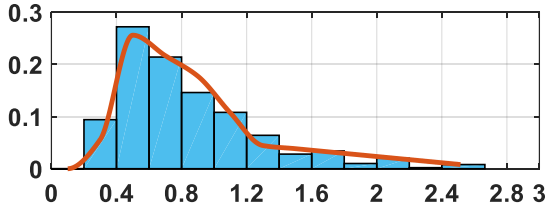

Fig. 5. PDF of the fault occurrence date for and TR_{O2}: simulation (histogram), computed with Proposition 2 (curve).

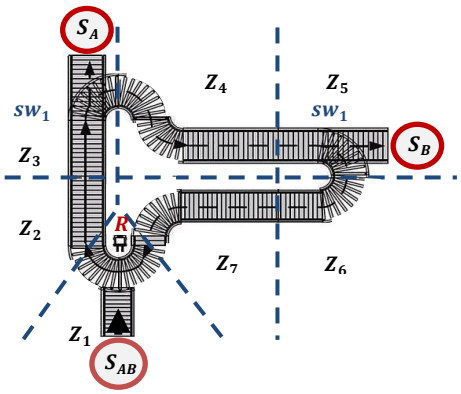

Fig. 6. Scheme of a distribution system [26]

TR_{O2} the interval is $[0.4TU, 0.7TU]$ (see Fig. 5). The method provides accurate indications on the time interval of high probability to characterize when the faults have occurred.

IV. Case Study

In order to show the importance of fault datation in the FDD process, let us consider the distribution system depicted on Fig. 6 similar to the one studied in [26, 15]. This system is devoted to the sorting of goods in supply chain systems. The conveyor is divided into 7 zones where different operations could be considered. It transfers two types of parts A and B from a loading station S_{AB} (area Z_1) to buffers S_A (area Z_3) and S_B (area Z_5). One part is delivered at a time. A bar code reader R distinguishes between parts and transmits the information to a supervisor that directs the parts toward the corresponding buffer by activating two switches: sw_1 and sw_2. In this application, two classes of faults are considered. A fault of class f_a (respectively f_b) occurs when the switch sw_1 (respectively sw_2) is deactivated before the delivery of a part A (respectively B). The POSPN of Fig. 7 models the distribution system. The nominal behavior is represented by transitions T_1 to T_{10} (full line), whereas the faulty behaviors are represented by transitions T_{11} and T_{12} (dotted line). The parameters of the transitions are given by μ = (1 2 3 1 2 2 3 4 3 6 0.3 0.5)^T. Let us consider that the initial marking is known and given by M_I = (10 0 0 0 0 0 0 1 0 0 1 1 0 0)^T. The significance of the transitions and places is provided in Table 1. The sensor configuration is given by the labeling function $\mathcal{L}(T_1) = e_1$, $\mathcal{L}(T_2) = e_2$, $\mathcal{L}(T_j) = \varepsilon$ for $j = 3, \ldots, 12$ and the marking sensor matrix H = (0 1 1 1 1 1 1 0 0 0 0 0). This means that the information given by the reader R on the part type is known (the loading of a part A (resp. B) returns the label e_1 (resp. e_2)). In addition, thanks to the matrix H, an information is obtained when a part is being processed (sum of the markings of $P_2 - P_7$. However, no indication is given to know if the part was successfully delivered or not (due to a fault occurrence).

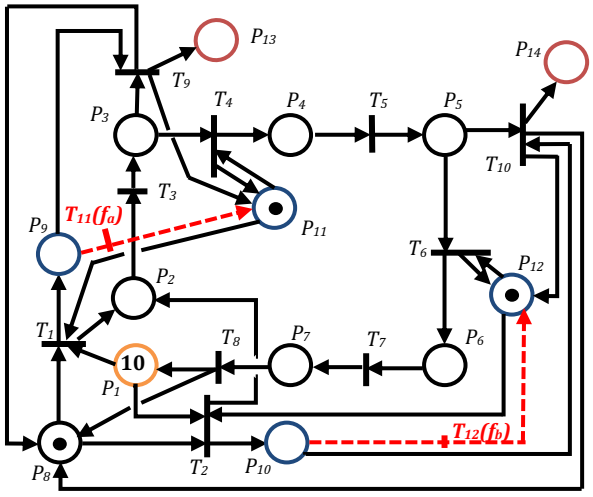

Fig. 7. POSPN of the distribution system

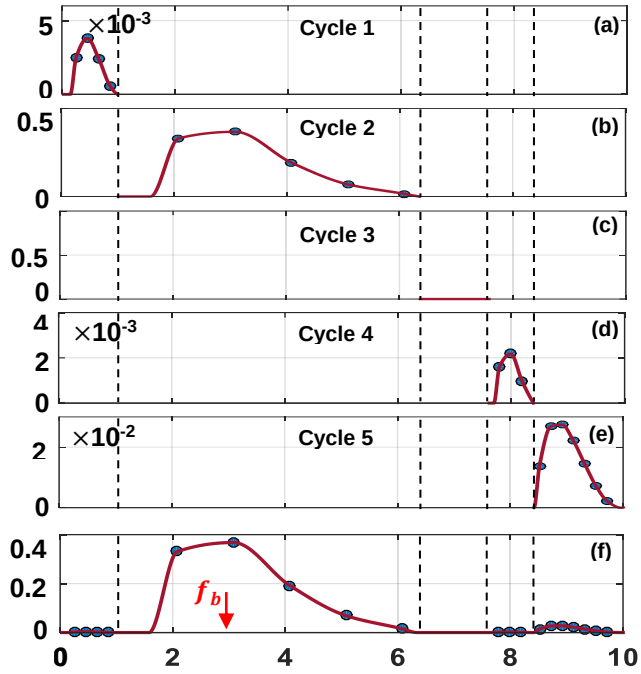

Fig. 8. Estimation of the occurrence date of fault f_b in each cycle (a to e) and for all the cycles (f).

Let us consider the timed trajectory (σ, M_I) obtained within $[0TU, 10TU]$ with:

$$\sigma = \boldsymbol{T_2(0.15)}T_3(0.28)T_4(0.53)T_5(0.97)T_{10}(0.98)\boldsymbol{T_2(1.58)}$$
$$T_3(1.80)\boldsymbol{T_{12}(3.03)}T_4(4.13)T_5(4.29)T_6(5.31)T_7(5.98)T_8(6.34)$$
$$\boldsymbol{T_1(6.37)}T_3(6.78)T_9(7.58)\boldsymbol{T_2(7.69)}T_3(7.72)T_4(7.82)T_5(8.28)$$
$$T_{10}(8.39)\boldsymbol{T_2(8.42)}T_3(8.88)T_4(9.33)T_5(9.51)T_{10}(9.96)$$

The system has performed 5 cycles (a cycle begins with a part loading and ends when this part is either distributed or returned to the loading zone due to an occurrence of a fault). The application of the measurement function Γ allows one to obtain the measurement trajectory given by:

$$\Gamma(\sigma, M) = TR_O =$$
$$(0,0)\boldsymbol{e_2(0.15)}(1,0.15)\varepsilon(0.98)(0,0.98)\boldsymbol{e_2(1.58)}$$
$$(1,1.58)\varepsilon(6.34)(0,6.34)\boldsymbol{e_1(6.37)}(1,6.37)\varepsilon(7.58)(0,7.58)\boldsymbol{e_2(7.69)}$$
$$(1,7.69)\varepsilon(8.39)(0,8.39)\boldsymbol{e_2(8.42)}(1,8.42)\varepsilon(9.96)(0,10)$$

The computation of the partially timed consistent trajectories and their probabilities allows one to obtain the probability of the faults which are given by $P(f_a, TR_O) = 0.017$ and $P(f_b, TR_O) = 0.980$. The fault f_b is thus diagnosed. However, the fault probability does not provide any information on:

- When the fault(s) has occurred
- In which cycle the fault(s) has occurred
- The number of occurrences of fault f_b

In fact, some consistent trajectories have the following form with multiple occurrences of fault f_b within several measurement intervals:

$$\sigma = \boldsymbol{T_2(0.15)} \ldots \boldsymbol{T_{12}}(t_1) \ldots T_8(0.98)\boldsymbol{T_2(1.58)} \ldots \boldsymbol{T_{12}}(t_2) \ldots$$

$T_8(6.34)\boldsymbol{T_1(6.37)}\ldots\boldsymbol{T_2(7.69)}\ldots\boldsymbol{T_{12}}(t_3)\ldots T_8(8.39)\boldsymbol{T_2(8.42)}\ldots$
$\boldsymbol{T_{12}}(t_4)\ldots T_8(9.96)$

In order to estimate the occurrence date of the fault f_b, Propositions 1 and 2 are used and the results are depicted on Fig. 8. from which one can deduce the following information that improves the faults characterization and the FDD process:

- $[2TU, 3.5TU]$ is the interval of highest probability for the occurrence of the fault(s) f_b. One can notice that the true fault date $(3.03TU)$ is within $[2TU, 3.5TU]$
- According to Fig. 8, only cycle 2 is affected by fault(s).
- Due to the high probability within only one cycle and to the fact that, in this system, only one fault can occur within a given cycle because only one product is delivered at each cycle, the fault f_b has occurred only once.

The fault datation helps also to decide which parts were impacted by the occurred fault (parts processed from cycle 2 in this example). In addition, in this example, the part concerned by the fault simply returns to the loading area. If one considers that, due to the fault occurrence, this part quits the conveyer in a bad buffer and impacts other operations that depend on the delivered parts then the fault datation eases the checking of the operations that have been done after the fault occurrence date. Moreover, it will facilitate the identification of this faulty part that could have been used in the continuation of the process.

TABLE 1. Places and transitions of the SPN model

PN element	Significance
P_1	Number of parts in the loading area (Z_1)
$P_2 - P_7$	Conveyor in area $Z_2 - Z_7$
P_8	A new part can be loaded
$P_9 - P_{10}$	Switch sw_1 (resp. sw_2) is activated
$P_{11} - P_{12}$	Switch sw_1 (resp. sw_2) is deactivated
$P_{13} - P_{14}$	Output buffers B_A - B_B
$T_1 - T_2$	Part A (resp. B) exits Z_1
$T_3 - T_8$	Operations in area $Z_2 - Z_7$ is over
$T_9 - T_{10}$	A part is stored in buffer B_A (resp. B_B)
T_{11}	Fault f_a : sw_1 erroneously deactivated
T_{12}	Fault f_b : sw_2 erroneously deactivated

V. CONCLUSION AND PERSPECTIVES

In this technical note, we evaluate the fault occurrence date in DESs modeled with partially observed stochastic Petri nets. The main contribution is the use of a timed probabilistic model to assess the faults occurrence date probabilities. Due to a computation effort that remains high for large systems, numerical complexity will be considered in our future studies. Also, it would be worthwhile to compare our fault detection method with approaches based on Markov processes (*swarm particle and recursive Bayesian filters*). The challenge is also to extend this work to non-Markovian dynamics and to consider large systems.

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