A theoretical framework for analyzing training situations in mathematics teacher education

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The thoughts on the primary schoolteacher training have led to the production of many resources for primary schoolteachers. Faces of the abundance of such documents, teacher educators need some tools to identify the knowledge potentially at stake in training situations and to allow them to implement such situations according to their own objectives and context. We present a five-level analysis framework that characterizes the training tasks, according to the expected posture of the prospective teacher, to the type of the knowledge at stake and to possible degrees of decontextualization. We illustrate this analysis framework by presenting an example of a training scenario based on the principle of role-play.

Keywords: Teacher education, professional development, primary education, knowledge for teaching, analysis framework.

Introduction

The research about primary schoolteacher education in mathematics and professional development has led to the production of many resources for educators. In France, the COPIRELEM[1] group produced many of them. These resources provide training situations based on various training strategies (Houdement & Kuzniak, 1996), and are generally accompanied with information about their implementation (phases, steps, instructions, elements of institutionalization) with regard to the stakes of the training. But their quality does not guarantee an accurate appropriation by teacher educators. Our questioning is: how is it possible to help teacher educators to exploit training situations in a relevant way, according to their objectives?

The research literature usually provides studies about knowledge for teaching, teacher conceptions, and their evolution (Shulman, 1986; Houdement & Kuzniak, 1996; Ball, Thames, & Phelps, 2008). Other studies present one training situation, and generally focus on its effect on the prospective teachers. For example Horoks and Grugeon (2015) "analyse the contents and methods of an initiation course in research in mathematics education, and […] how it can influence the beginner teachers' practises" (p.2811). To our knowledge, no study focuses on the characteristics of training situations nor provides specific framework in order to analyse any training situation. This led us to develop an analysis framework for training situations. The paper presents this COPIRELEM's work in progress.

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[1] The COPIRELEM is a commission dedicated to the education to the primary school. It is stemming from the network of IREM (French institute of research on mathematical education).

Presentation of the analysis framework

Relying on the Theory of Didactical Situations (TSD) developed by Brousseau (1997) we define a *training situation* as a situation[2] that involves prospective teachers (students, pre-service or in-service teachers) and educators within an institution of teacher education. It is composed of a set of tasks based on a so-called "initiating task" and conducted by a teacher educator. We take into account all the tasks proposed by the teacher educator. Each task corresponds to a type of prospective teachers' activity: we name "study level of a training situation" each type of activity. We distinguish in a training situation five study levels: mathematical activity, (mathematical) analysis of a mathematical activity, didactical and pedagogical analysis of a (mathematical) activity, analysis of a didactical and pedagogical activity, problematization of professional issues.

For each type of activity we take into account three dimensions: the type of knowledge at stake; the degree of decontextualization of this knowledge; the posture of the prospective teachers expected by the teacher educator. These dimensions are specified in next sections.

Three types of knowledge

We rely on the three types of knowledge for teaching mathematics identified by Houdement and Kuzniak (1996): mathematical knowledge, pedagogical knowledge and didactical knowledge.

"Mathematical knowledge corresponds to mathematics that a teacher needs to know in order to prepare, regulate and evaluate his lesson and his students" (Houdement, 2013, p.12). It "includes and specifies the content knowledge" identified by Shulman (1986). Moreover, the specific didactical nature of mathematical knowledge can be identified to the Specialized Content Knowledge (SCK) developed later by Ball and *al.* (2008).

According to (Houdement, 2013), didactical knowledge is linked to the mathematical content and fed by research in the field of mathematics didactics. It corresponds to analysis of teaching and learning phenomenon and to propositions of engineering. Therefore it can be associated with at least two categories (Ball and *al.*, 2008): Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT).

Pedagogical knowledge[3] is characterised as "knowledge of experience" (Portugais, 1995). It is related to teaching and learning conceptions and to the organisation and management of the class. It is less dependent of the mathematical content than other types of knowledge. It is important to take this knowledge into account because schoolteachers deal with various school subjects.

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[2] "A situation is characterised within an institution by a set of relations and mutual roles of one or several subjects (student, teacher, etc.) with a *milieu*, which aims at the transformation of this *milieu* according to a project. The milieu is composed by all what interacts (physical, cultural or social objects, humans) with the subject in a situation." (Brousseau, 2010, p.2) translated.

[3] According to (Houdement, 2013), Ball's, Phelps' and Thames' typology doesn't seem to take into account this type of knowledge.

Three degrees of decontextualization

In TSD, Brousseau (1997) and Douady (1985) identify three degrees of decontextualization of a mathematical knowledge: implicitly mobilized; explicitly mobilized in context or decontextualized (to become available in other contexts). We extend this notion to didactical and pedagogical knowledge.

A mathematical knowledge is *(implicitly) mobilized in context* (in act) if it is used as tool (Douady, 1985) in a mathematical task. This task can be carried out: what is asked is effectively achieved (manipulation, elaboration and writing a solution for example). But the task can only be evocated: it is mentally achieved. A mathematical knowledge is *explicit in context* if its use (as tool) is identified and formulated. At least, a mathematical knowledge is *decontextualized* if a status of object is given (by the educator) to the concept used previously as tool, usually during an institutionalization phase[4] (Brousseau, 1997).

The didactical/pedagogical knowledge is *mobilized in context* when the didactical/ pedagogical choices are made for the considered mathematical task. It is *explicit in context* during the analysis about the consequences of these choices. At least, it is *decontextualized* when the underlying didactical/pedagogical concepts are highlighted.

Four postures of the prospective teachers

In conjunction with the teacher trainer's relationship to the prospective teachers identified by Sayac (2008), we define four specific postures of prospective teachers, which are expected by the educator during a training situation[5].

Prospective teachers are in a posture of *student* relatively to mathematical knowledge when they have to perform mathematical activity or when they are concerned with the mathematical knowledge of this activity. They are in a *student/teacher* posture when they investigate mathematical tasks for students or students' works, or when they analyse the conditions of implementation of a task in the classroom. They are in a *teacher* posture when entering in a broader questioning on classroom practices and issues of mathematical learning. Finally, they are in a *practitioner/researcher* posture when they problematize a professional issue related to mathematical learning or teaching.

The five study levels

In order to analyse a training situation, we define five study levels. To each level corresponds a type of activity, that induces (implicitly or explicitly) a posture of the prospective teacher (expected by the educator), and that involves different types of knowledge in a certain degree of décontextualisation.

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[4] In institutionalization phase (Brousseau 1997), the teacher gives a cultural (mathematical) status to some knowledge emerging from students' actions during the situation.

[5] We notice that the prospective teachers are not always aware of these postures.

Level 0. A task induces a mathematical activity. This activity can be performed or evocated (mentally performed). The mathematical knowledge is mobilized (implicitly or explicitly) in context. The prospective teachers are in a posture of student (relatively to mathematical knowledge).

Level 1. A task induces a (mathematical) analysis. It highlights decontextualized mathematical knowledge. In this task, didactical and/or pedagogical knowledge can be implicitly mobilized in context. The prospective teachers are in a posture of learning mathematics student.

Level 2. A task induces a didactical and pedagogical analysis of the mathematical activity (analysis of implementation conditions - actual or anticipated only). Didactical and pedagogical knowledge is explicit in context. The prospective teachers are in a student/teacher posture.

Level 3. A task induces an analysis of the didactical and pedagogical activity. It can be a questioning on classroom practice (specific learning tasks, professional actions...) or on issues of mathematical learning for one or several contents (curriculum, progressions...), or even a highlighting of didactical analysis concepts (didactic situation phases, types of tasks...). This analysis leads to the decontextualization of didactical and/or pedagogical knowledge. The prospective teachers are in a posture of teacher.

Level 4. This level corresponds to the problematization of professional issues related to classroom practices, learning issues and/or didactical analysis tools. The prospective teachers are in a posture of practitioner/researcher, especially when it comes to developing an analysis methodology of this issue and to infer results.

The following table summarizes the characteristics of the five study levels.

<table>
<tr><th rowspan="2">Study levels</th><th rowspan="2">Type of activity</th><th rowspan="2">Posture of the PT</th><th>Mathematical</th><th>Didactical</th><th>Pedagogical</th></tr>
<tr><th colspan="3">Knowledge</th></tr>
<tr><td>0</td><td>Mathematical activity (action performed effectively or mentally)</td><td>Student</td><td>Mobilized implicitly or explicitly in context</td><td colspan="2"></td></tr>
<tr><td>1</td><td>Analysis of the mathematical activity</td><td>Student
Student/teacher</td><td rowspan="4">Decontextualized</td><td>Mobilized Implicitly in context</td><td>Mobilized Implicitly in context</td></tr>
<tr><td>2</td><td>Didactical and pedagogical analysis of the mathematical activity</td><td>Student/teacher</td><td>Explicit in context</td><td>Explicit in context</td></tr>
<tr><td>3</td><td>Analysis of the didactical and pedagogical activity</td><td>Teacher</td><td rowspan="2">Decontextua-lized</td><td rowspan="2">Decontextua-lized</td></tr>
<tr><td>4</td><td>Problematization of professional issues related to classroom practices, learning issues and/or didactical analysis tools</td><td>Practitioner/re-searcher</td></tr>
</table>

Table 1: Characteristics of the five study levels

Each study level is based on the study of the activity of previous levels and involves mathematical, didactical and/or pedagogical knowledge. The change from study level *n* to study level *n + 1* is linked either to a change of the prospective teachers' posture or to a change of degree of decontextualization for at least one type of knowledge (from implicitly mobilized in context to explicit in context, from explicit in context to decontextualized). But the different activities induced by a training situation don't usually appear in a chronological order (from level 0 to level 4). For examples, see the analysis of various training situations developed in French context by the COPIRELEM group (Guille-Biel Winder, Petitfour, Masselot & Girmens, 2015; Bueno-Ravel and al., accepted). But we think that the analysis could be extended to situations based on different training strategies. That is why we present here the analysis of a training scenario[6] based on the principle of role-play developed in an international context (Lajoie & Pallascio, 2001; Lajoie, accepted).

An example of use of the analysis framework

Definition of role-play

As Lajoie and Pallascio (2001) state "role-play involves staging a problematic situation with characters taking roles". It is used over many years in mathematics education course in UQAM (University of Québec in Montréal) and is organized as follows:

> First, the 'theme' on which students will need to role-play is introduced (introduction time). Second, students then have about 30 minutes to prepare in small groups (preparation time). Third comes the play itself (play time), where students chosen by the educator come in front of the classroom and improvise a teacher-student(s) interaction (sometimes, like in the case reported here, involving the whole class). Finally, we have a whole classroom discussion on the play (discussion time). (Lajoie, accepted)

We designed a role-play on the teaching of proportions based on a problem from a textbook. We use the analysis framework to illustrate an example of analysis aimed at highlight the potential of this situation.

An example of role-play

The role-play presented below is intended for pre-service schoolteacher education. We describe the different phases.

Introduction time. The educator distributes to prospective teachers an excerpt from a fifth grade (10-11 year old pupils) handbook presenting a problem of proportions (Fig. 1), and various productions of pupils. The teaching issue announced by the educator is the following: to manage a class discussion about the pupils' strategies and about their ideas and solutions, in order to share them in the class community and to determine their validity and efficiency.

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[6] We voluntarily distinguish situations from "scenarios" because we intend to underline the dynamic aspect: a scenario is a set of chronologically organized tasks chosen among all the tasks that constitute the training situation.

Preparation time. The prospective teachers have to prepare the discussion class about the pupils' strategies.

Play time. At the end of the preparation time, the educator chooses prospective teachers to play the game: some of them play pupils, one of them plays the teacher, while the others are watching the discussion class and taking notes.

Discussion time. The debate intends to highlight and to analyse the choices of the « teacher » during the play game: what worked well during the implementation of the discussion class? What was difficult? What seemed to be important? What alternative implementations could be realized?

Institutionalization time[7]. The educator institutionalizes the knowledge at stake: he generalizes some elements about how to manage a discussion class or about proportion problems solving.

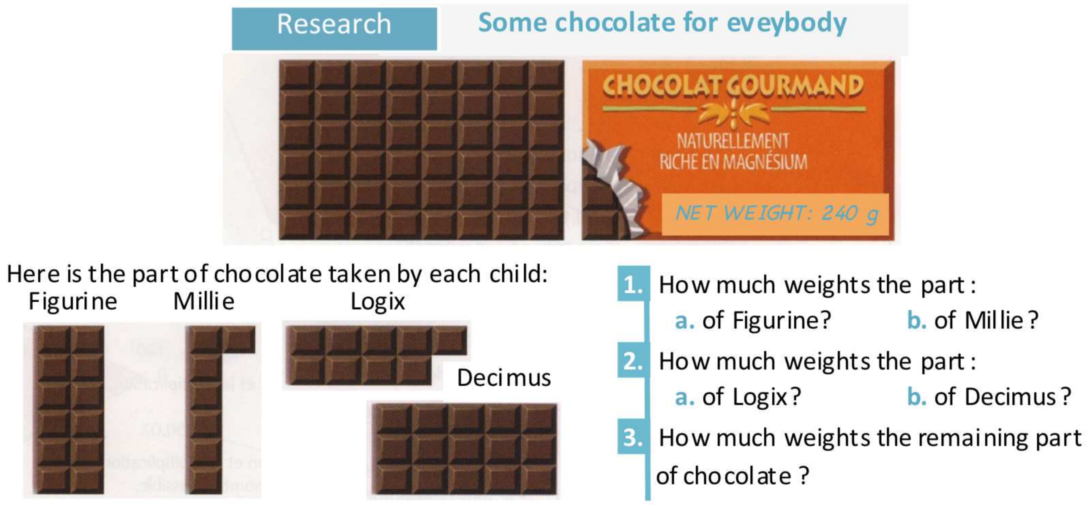

Figure 1: A proportion problem

Analysis of this role-play

The initiating task is a professional situation and corresponds to level 2 of the analysis framework: the prospective teachers are initially in a student/teacher posture. But they will need « to go down » to as student posture and « to go up » to a teacher posture during the phases of the scenario. We describe this more precisely below.

The preparation time of the discussion class leads the prospective teachers "to go back and forth" to the study levels 0, 1 and 2. The problem solving corresponds to level 0 and the mathematical analysis of the problem solving to level 1. Moreover there are various strategies to solve this proportion problem. Preparing the discussion class of the pupils' strategies (level 2) hence needs to analyse and rank them (from the least elaborate to the most elaborate). This analysis corresponds to level 2.

The prospective teachers don't have the same activity during the playtime. The study level is different according to the role to play: mostly levels 0 and 1 for the students' roles and level 2 for the teacher's role.

[7] We add this new time to the four ones proposed by Lajoie and Pallascio (2001).

The discussion time corresponds to level 2 when the prospective teachers analyse how the discussion class has been managed. But it can also correspond to lower levels, when they discuss about pupils' strategies, difficulties, mistakes and their exploitation during the discussion class.

Various institutionalizations can be considered, according to the knowledge that was developed at different study levels. The institutionalized elements will be more or less developed according to the teacher educator's objectives and progression, the prospective teachers' knowledge, etc. Here are some propositions organized in ascending order of study levels. The teacher educator can institutionalize some mathematical knowledge at stake (level 1) and related to the proportionality field: various methods to solve a proportion problem, the mathematical justifications and the mathematical theories they are relied on. He can situate the proportion problems in the more general category of multiplicative problems, or he can explicit some didactical variables usually at stake in proportion problems (level 3). He also can identify some difficulties or mistakes revealed by the pupils' productions as « usual » and highlight mistaken conceptions: identification of quantities, choice of an adapted strategy, persistence of an « additive model », etc. At least, in regard of the announced objective of the role-play, the teacher educator also can institutionalize some didactical knowledge, relatively to the organization of a discussion class (level 3): formulation and validation in mathematics; teacher's tasks before, during and after the discussion class…

Conclusion

The example of role-play situation shows how the analysis framework can be a tool for an *a priori* analysis. Moreover this example shows that the organization of the study levels is not chronological but a hierarchical one: the initiating task can be on level 0, 1, 2, 3 or 4. But the transition to lower levels is often necessary. The conceptual maps of the knowledge for teaching developed by Houdement and Kuzniak (1996) or by Ball and *al.* (2008) have a descriptive, predictive and prescriptive dimension (Ball & *al.,* 2008, p.405). But beyond their interest, (Houdement, 2013, p. 21) stressed the importance of the knowledge reconfiguration in connection with the mathematical content. The analysis framework reports how, during a training situation, the types of knowledge for mathematics' teaching are hinged to one another in connection with the mathematical content.

The analysis framework allows teacher educators to identify the potentialities of a full range of training situations. We intend to extend its use to study other types of training situations (for example e-learning situations). By clarifying the stakes of the various phases of the implementation, the analysis framework reveals various possible strategies for the teacher educator. Thereafter it could be a useful tool for elaborating different training scenarios. Hence, the teacher educator should be able to implement situations in a specific context according to his objectives and constraints (time and period of training, place in a progression which take into account the mathematical, didactical and pedagogical knowledge ever studied…). Besides it is possible to consider a sequence of successive scenarios. The analysis framework can also highlight various possible "training paths", which should reveal the educator's training strategy at a more global scale. A perspective is now to study how teacher educators appropriate this framework and how it supports their teaching practises.

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