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In the lattice Boltzmann method (LBM), the modeling of the collision operator is of paramount importance to recover a correct macroscopic fluid behavior, as well as for numerical stability purposes. The most simple and efficient choice is based on the Bhatnagar-Gross-Krook (BGK) approximation [1]. This collision operator is known to suffer from a lack of robustness when increasing the Mach and/or Reynolds numbers of the fluid flow. However, the origins of these instabilities remain an open question.

In this context, the von Neumann analysis is a powerful tool to predict the stability bounds of a numerical scheme. It aims at evaluating the response of a system, described by a given set of equations, to linear perturbations. Originally widely used on Navier-Stokes (NS) based solvers, Sterling and Chen were among the firsts to apply it to the LBM by linearizing the collision operator around a base steady flow [2].

In our work, the standard analysis is extended to the study of the eigenvectors of the linearized system [3]. The computation of their moments provides the macroscopic information carried by each wave, which can be helpful to understand the physical behavior of the LBM. It leads to the identification of three kinds of waves: (1) physical waves, that are physically expected by the NS equations, (2) ghost waves, that have no macroscopic contribution in the linear approximation, and (3) numerical waves, that advect some macroscopic but unphysical information. Moreover, modal interactions occuring between physical and numerical modes can be highlighted by the analysis. Using the BGK operator, when increasing the Mach number of the mean flow, two phenomena progressively appear: curve veering and eigenvalue collision. These two kinds of modal interactions are illustrated on a simple case of harmonic oscillators and may be responsible for the strong instabilities observed with the BGK collision operator.

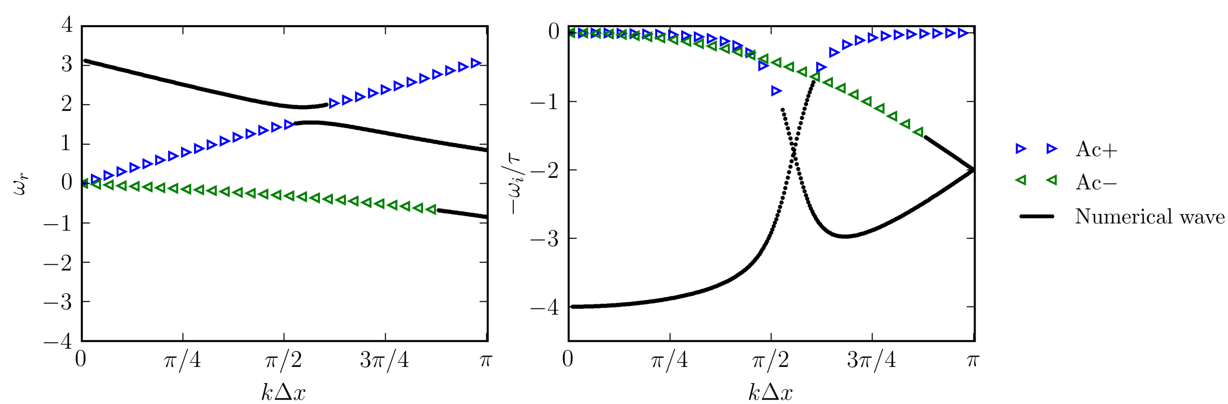

Figure 1: *Curve veering* phenomenon observed in the stability analysis of the D1Q3 lattice at Ma = 0.7.

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[2] J. D. Sterling, S. Chen, Stability Analysis of Lattice Boltzmann Methods, Journal of Computational Physics 123 (1996).

[3] G. Wissocq, P. Sagaut, J.-F. Boussuge, An extended spectral analysis of the lattice Boltzmann method: modal interactions and stability issues, Journal of Computational Physics, submitted (2018).