Algorithm for Controlling the Transient Behavior of Controlled Generalized Batches Petri Nets

Ruotian Liu, Rabah Ammour, Leonardo Brenner, Isabel Demongodin

Algorithm for Controlling the Transient Behavior of Controlled Generalized Batches Petri Nets

Ruotian Liu, Rabah Ammour, Leonardo Brenner, Isabel Demongodin

LABORATOIRE D'INFORMATIQUE & SYSTÈMES
UMR CNRS 7020

Objective

Compute a control trajectory to reach a target steady state from a given initial state in hybrid / discrete event systems.

Controlled GBPN

: discrete place d_j : discrete transition

: continuous place Φ_j : continuous transition

: batch place $\{V_i, d^{max}_i, s_i\}$ Φ_j : batch transition

- A batch β_k at time τ is: $\beta_k(\tau) = \big(l_k(\tau), d_k(\tau), x_k(\tau)\big)$, where $l_k(\tau) \in \mathbb{R}_{\geq 0}$ is the length, $d_k(\tau) \in \mathbb{R}_{\geq 0}$ is the density and $x_k(\tau) \in \mathbb{R}_{\geq 0}$ is the head position.
- The marking of a batch place p_i is a series of batches: $m_i(\tau) = \{\beta_1(\tau), \dots, \beta_r(\tau)\}$.
- The marking quantity vector is: $\boldsymbol{q} = \mu(\boldsymbol{m})$. For place p_i:

$$q_i(\tau) = \begin{cases} m_i(\tau) & \text{if } p_i \in P^C \\ \sum_{k=1}^{r} l_k(\tau).d_k(\tau) & \text{if } p_i \in P^B \end{cases}$$

- The instantaneous firing flow: $\varphi_j \leq \Phi_j$ represents the firing quantity of transition t_j by time unit.
- The controlled firing flow vector at time τ: $\boldsymbol{u}(\tau)$ is the control input that drives the evolution. $0 \leq u_j(\tau) \leq \Phi_j$

Steady state

The cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$ is in a steady state at time τ_s if for $\tau \geq \tau_s$ the marking $\boldsymbol{m}^s$ and the instantaneous firing flow vector $\boldsymbol{\varphi}^s$ remain constant. Thus a steady state is defined as a pair $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$.

Control of the transient behavior

Given a cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$, a control trajectory is given as $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^n, \tau_n)$ such that the controlled firing flow vector $\boldsymbol{u}^i$ is applied at date τ_i until τ_{i+1}.

Control strategy for reaching the steady state

- OFF: $u_j(\tau) = 0$ if t_j is not enabled or the marking quantity of one of its input places p_i is lower than its steady marking quantity $q_i(\tau) < q_i^s$.
- ON: maximize $\boldsymbol{u}(\tau)$ for approaching the steady marking quantity vector $\boldsymbol{q}^s$.

$$\boldsymbol{q}^s = \boldsymbol{q}^0 + C.\left(\int_{\tau_0}^{\tau_1} \boldsymbol{u}^0(\rho).d\rho + \cdots + \int_{\tau_{n-1}}^{\tau_n} \boldsymbol{u}^{n-1}(\rho).d\rho + \int_{\tau_n}^{+\infty} \boldsymbol{u}^n(\rho)d\rho\right)$$

with $C.\int_{\tau_n}^{+\infty} \boldsymbol{u}^n(\rho)d\rho = 0;\ \boldsymbol{u}^n = \boldsymbol{\varphi}^s$

Assumptions

- No discrete nodes ($P^D = T^D = \emptyset$) and no continuous transitions ($T^C = \emptyset$).
- The steady firing flow vector is positive ($\boldsymbol{\varphi}^s > 0$).

Algorithm

Algorithm: Computation of control trajectory

Input: A cGBPN system $\langle N, \boldsymbol{m}_0 \rangle$ and the steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$.
Output: Control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$

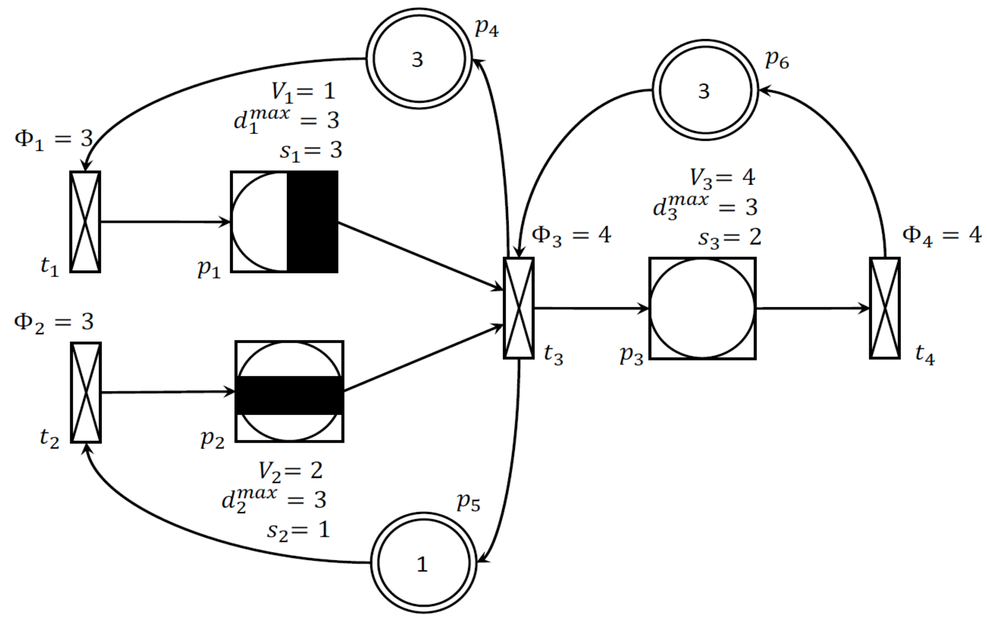
Initialize : $\boldsymbol{q}^0 = \mu(\boldsymbol{m}_0)$, $\boldsymbol{u}^0 = \boldsymbol{0}$, $\tau_0 = 0$, $i = 0$;
while $\boldsymbol{m}^i \neq \boldsymbol{m}^s$ *and* $\boldsymbol{u}^i \neq \boldsymbol{\varphi}^s$ **do**
  Determine the sets $T_{\mathcal{N}}(\boldsymbol{m}^i)$, $T_L(\boldsymbol{m}^i)$, $S_G(\boldsymbol{m}^i)$, $S_E(\boldsymbol{m}^i)$;
  Solve the following LPP: $\max \boldsymbol{1}^T \cdot \boldsymbol{u}^i$ *s.t.*

$$\begin{cases} \text{(a)} & 0 \leq u_j^i \leq \varphi_j^s & \forall t_j \in T \\ \text{(b)} & u_j^i = 0 & \forall t_j \in T_{\mathcal{N}}(\boldsymbol{m}) \cup T_L(\boldsymbol{m}) \\ \text{(c)} & C(p_k, \cdot) \cdot \boldsymbol{u}^i \leq 0 & \forall p_k \in S_G(\boldsymbol{m}) \\ \text{(d)} & C(p_k, \cdot) \cdot \boldsymbol{u}^i = 0 & \forall p_k \in S_E(\boldsymbol{m}) \\ \text{(e)} & Pre(p_k, \cdot) \cdot \boldsymbol{u}^i \leq V_k \cdot d_k^{out} & \forall p_k \in P^B \end{cases}$$

  Determine all the next timed events from considered ones, select the nearest at time τ_{i+1} ;
  Determine the new marking $\boldsymbol{m}^{i+1}$ and the marking quantity vector $\boldsymbol{q}^{i+1}$;
  i = i + 1;
Return $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$

Example

$\boldsymbol{m}_0 = [\{(2,3,3)\}\ \{(1,2,1)\}\ \emptyset\ 3\ 1\ 3]^T$
$\boldsymbol{m}^s = [\{(1,2,1),(2,3,3)\}\ \{(1,1,1)\}\ \{(2,0.5,2)\}\ 1\ 2\ 2]^T$
$\boldsymbol{\varphi}^s = [2\ 2\ 2\ 2]^T$



Control trajectory: $(\boldsymbol{u}^0, 0), (\boldsymbol{u}^1, 1), (\boldsymbol{u}^2, 1.5), (\boldsymbol{u}^3, 2)$ with:

$$\underset{\tau_0 = 0}{\underset{\boldsymbol{u}^0}{\begin{pmatrix}2\\0\\0\\0\end{pmatrix}}} \xrightarrow{\tau_1 = 1} \underset{\boldsymbol{u}^1}{\begin{pmatrix}2\\0\\2\\0\end{pmatrix}} \xrightarrow{\tau_2 = 1.5} \underset{\boldsymbol{u}^2}{\begin{pmatrix}2\\2\\2\\2\end{pmatrix}} \xrightarrow{\tau_3 = 2} \underset{\boldsymbol{u}^3 = \boldsymbol{\varphi}^s}{\begin{pmatrix}2\\2\\2\\2\end{pmatrix}}$$

[1] I. Demongodin and A. Giua. Dynamics and steady state analysis of controlled generalized batches Petri nets. NAHS, 2014.
[2] L. Wang, C. Mahulea, J. Julvez and M. Silva. On/off strategy based minimum-time control of continuous Petri nets. NAHS, 2014.

Aix*Marseille université Initiative d'excellence — cnrs dépasser les frontières — UNIVERSITÉ DE TOULON