Time-varying consumption tax, productive government spending, and aggregate instability

Mauro Bambi, Alain Venditti

Time-varying consumption tax, productive government spending, and aggregate instability

Mauro Bambi* and Alain Venditti†

In this paper we investigate if government balanced-budget rules together with endogenous taxation may lead to aggregate instability in an endogenous growth framework. After highlighting the differences with the exogenous growth framework, we prove that under counter-cyclical consumption taxes, while there exists a unique balanced growth path, sunspot equilibria based on self-fulfilling expectations occur through a form of global indeterminacy. In addition, we argue that this result is empirically plausible for a large set of OECD countries and that it may also emerge with endogenous income taxes.

Key words endogenous growth, time-varying consumption tax, global indeterminacy, self-fulfilling expectation, sunspot equilibria

JEL classification C62, E32, H20, O41

1 Introduction

Balanced-budget rules recommendations to governments have been a recurrent debate since the last financial crisis, concerning mainly their consequences in terms of government debt sustainability. As shown by Schaechter, Kinda, Budina, and Weber (2012), in 2012 approximately 60 countries, mostly advanced, have adopted a type of balanced-budget rule either at the national or supra-national level. Balanced-budget rules can be implemented as overall balance, structural or cyclically adjusted balance, and balance "over the cycle". Since the beginning of the Great Recession in 2008, the rule most widely debated throughout Europe and adopted by some countries like Germany or Switzerland is that known as the "Golden Rule".

But this type of balanced-budget rule has been criticized in regards to its economic stabilization features. Since the study of Schmitt-Grohé and Uribe (1997), it has been a well established fact that balanced-budget rules may lead to belief-driven aggregate instability and endogenous sunspot fluctuations. However, depending on fiscal policy, aggregate instability occurs under different types of

* Durham University Business School, UK.

† Aix-Marseille University., CNRS, EHESS, Centrale Marseille, AMSE & EDHEC Business School, France. Email: alain.venditti@univ-amu.fr

This work was supported by French National Research Agency Grants ANR-08-BLAN-0245- 01 and ANR-17-EURE-0020. We thank the Editor and an anonymous referee together with Yves Balasko, Jess Benhabib, Jean-Pierre Drugeon, Fausto Gozzi, Jean-Michel Grandmont, Herakles Polemarchakis, Neil Rankin, Xavier Raurich, Thomas Seegmuller, Mich Tvede, Yiannis Vailakis and Bertrand Wigniolle for useful comments and suggestions. We are also grateful to the seminar participants at the Newcastle-York Workshop on Economic Theory and at the internal seminar at the University of York. This paper benefited also from a presentation at the International Conference "Financial and Real Interdependencies: volatility, inequalities and economic policies", Católica Lisbon School of Business & Economics, Lisbon, May 28-30, 2015.

preferences. While it requires a large enough income effect when labor income taxes are considered (see Abad, Seegmuller, and Venditti 2017),[1] low enough income effects are necessary under consumption taxes (see Nourry, Seegmuller, and Venditti 2013).[2] Such a conclusion has strong policy implications as for a given specification of preferences, one type of fiscal policy must be preferred to the other if the government is willing to avoid endogenous fluctuations. For instance, under a standard additively-separable CRRA utility function, Giannitsarou (2007) suggests that consumption taxes must be favored with respect to income or capital taxes as they reduce the possible occurrence of aggregate instability.

These results can be criticized from two perspectives. First, as clearly mentioned by Schmitt-Grohé and Uribe (1997), they are partially based on the assumption that tax rates are not predetermined,[3] while taxes are in practice typically set in advance.[4] Second, they are established within stationary models without long-run growth. The aim of this paper is to revisit the issue of aggregate instability coming from balanced-budget rules focusing on consumption taxes compatible with endogenous growth. In practice, this requirement implies that the tax rate depends on de-trended consumption to have a constant tax on a balanced growth path. As a consequence, we consider a time-varying consumption tax which is a predetermined variable and we are thus able to solve the two main weaknesses of Giannitsarou's results, and to prove that even if standard additively-separable CRRA preferences are considered, sunspot fluctuations matter under a balanced-budget rule with consumption taxes.[5]

We consider a standard neoclassical growth model augmented with government policy that provides a constant stream of expenditures financed through consumption taxes and a balanced budget rule. Endogenous growth is obtained from assuming a Barro-type (1990) production function in which government spending acts as an external productive input. In order to have a constant tax on a balanced growth path, the tax rate needs to depend on de-trended consumption and thus becomes a state variable with a given initial condition. Finally, we consider a representative household characterized by a CRRA utility function and inelastic labor. Such a formulation is known to rule out the existence of expectation-driven fluctuations in exogenous growth models (see Giannitsarou 2007). The aim of this paper is to show that this result is not robust to the consideration of endogenous growth.

We first prove that there exists a unique Balanced Growth Path (BGP) along which the common growth rate of consumption, capital, GDP, and government spending is constant. The BGP specifies that the equilibrium tax rate is just equal to its initial value. A consequence of this is that, as in the Barro (1990) model, there are no transitional dynamics with respect to this uniquely unstable BGP and, therefore, there exists a unique initial choice of consumption such that the economy evolves along its BGP. This conclusion is thus similar to the one reached by Giannitsarou (2007): there is *a priori* no room for endogenous fluctuations.

---

1 Actually, local indeterminacy requires that consumption and labor are Edgeworth substitutes or weak Edgeworth complements (see also Linnemann 2008). These properties are associated with a Jaimovich and Rebelo (2008) utility function characterized by a large enough income effect.

2 See also Nishimura, Nourry, Seegmuller, and Venditti (2013) for similar results in two-sector models.

3 The initial value of the tax rate is indeed a function of a forward (non pre-determined) variable (i.e., consumption or labor).

4 It is however claimed in Schmitt-Grohé and Uribe (1997) that their main conclusions are robust to the consideration of a discrete-time reformulation of their model with tax rates set $k \geq 1$ periods in advance so that in each period $t \geq 0$, the tax rates for periods $t, \cdots, t+k-1$ are pre-determined.

5 We will also prove that the same aggregate instability can be generated by a balanced-budget rule with income taxes compatible with endogenous growth.

However, we can prove that the BGP is not the unique long run solution of our model. Indeed, if the tax rule is counter-cyclical with respect to consumption, for any arbitrary initial value of the tax rate, close enough to its initial condition, there exists a corresponding value for the tax rate, consumption, capital, and the constant growth rate that can be an asymptotic equilibrium of our economy, namely an Asymptotic Balanced Growth Path (ABGP). An ABGP is not itself an equilibrium as it does not respect the initial conditions. However we prove that some transitional dynamics exist with a unique equilibrium path converging toward this ABGP. Moreover, we show that there exists a continuum of such ABGP and of equilibria, each of them converging over time to a different ABGP.[6]

The existence of an equilibrium path converging to an ABGP is associated with the existence of consumers' beliefs that are different from those associated with the BGP. Indeed, they may believe that the consumption tax profile will not remain constant but rather change over time and eventually converge to a positive value different from the initial condition. A specific form of global indeterminacy emerges since, from a given initial tax rate, the representative agent can choose an initial consumption to be immediately on the unique BGP or alternatively an initial consumption consistent with any other equilibrium converging to an ABGP. Again different choices reveal different consumers' beliefs on the long run outcome of the economy.

Because this specific form of global indeterminacy is fundamentally related to expectations, one may wonder about the possible existence of sunspot equilibria and endogenous fluctuations based on self-fulfilling beliefs. To this purpose, we adapt existing results (e.g., Shigoka 1994; Cazzavillan 1996; Benhabib, Nishimura, and Shigoka 2008) and we show that sunspot equilibria can be obtained by randomizing over the deterministic equilibria converging to the ABGPs. From an analytical viewpoint, we assume that the sunspot variable is a continuous time homogenous Markov chain and we use the generator of the chain as proposed by Grimmett and Stirzaker (2009) to prove the existence of sunspot equilibria. We then conclude that in an endogenous growth framework, contrary to the conclusions of Giannitsarou (2007), endogenous sunspot fluctuations may arise under a balanced-budget rule and consumption taxes although there exists a unique underlying BGP equilibrium. It is also worth noting that contrary to Drugeon and Wigniolle (1996) or Nishimura and Shigoka (2006), our methodology allows us to prove the existence of sunspots in a non-stationary economic environment while the steady state (BGP) is unstable.

Our results can be compared to some recent conclusions provided by Angeletos and La'O (2013) and Benhabib, Wang, and Wen (2015) within infinite horizon models with sentiments. They show that endogenous fluctuations, based on a certain type of extrinsic shocks called "sentiments", can be accommodated in unique-equilibrium, rational-expectations, macroeconomic models like those in the RBC/DSGE paradigm, provided there is some mechanism that prevents the agents from having identical equilibrium expectations. Of course, our framework is still based on the existence of externalities as we need to generate a form of Ak technology to get endogenous growth. But, contrary to the standard literature, which is based on the existence of local indeterminacy (see Benhabib and Farmer 1994), we find sunspot fluctuations while there exists a unique deterministic BGP without transitional dynamics. The existence of the continuum of ABGPs and of equilibrium paths converging to these, is also fundamentally based on the expectations of agents.

[6] On the other hand, we can prove that if the consumption tax is pro-cyclical then the BGP is the unique equilibrium path.

The rest of the paper is organized as follows. Section 2 presents the firms and households' behaviors. Section 3 discusses fiscal policies under balanced-budget rules comparing exogenous and endogenous growth models, and presents the consumption tax rule that we consider in the paper. Section 4 defines the intertemporal equilibrium. Section 5 proves the existence and uniqueness of a BGP. In Section 6 its stability is investigated and it is also shown that depending on agents' expectations, there may exist a continuum of other equilibria that converge toward some ABGPs. Based on this conclusion, we prove in Section 7 that sunspot equilibria and endogenous fluctuations based on self-fulfilling beliefs occur, and we show through a numerical exercise that the existence of aggregate instability and sunspot fluctuations driven by consumption tax rates is empirically plausible for a large set of OECD countries. Section 8 proves the robustness of our results showing that the same conclusions can be obtained under income taxes. Section 9 contains a conclusion.[7]

2 Description of the private sector

In this section the endogenous growth model originally developed by Barro (1990) is modified by assuming that the government levies a time-varying consumption tax to finance its spending. In this economy the government spending is productive since it is a public good provided by the government to the firms that use it as an essential input of production. For this reason, our paper differs from Giannitsarou (2007) and Nourry, Seegmuller, and Venditti (2013) in which the government spending is just a pure waste of resources. As in Barro (1990), productive government spending is the source of endogenous growth in our model.

2.1 Firms

A representative firm produces the final good y using a Cobb-Douglas technology with constant returns at the private level but which is also affected by a public good externality, $y = Ak^{\alpha}(L\mathcal{G})^{1-\alpha}$, where $\alpha \in (0, 1)$ is the share of capital income in GDP, $\mathcal{G}$ is the per capita quantity of government purchases of goods and services and A is the constant TFP. We assume that population is normalized to one, $L = 1$, so that we get a standard Barro-type (1990) formulation such that $y = Ak^{\alpha}\mathcal{G}^{1-\alpha}$.[8] Profit maximization then respectively gives the rental rate of capital and the wage rate:

$$r = A\alpha \left(\frac{k}{\mathcal{G}}\right)^{\alpha-1}, \quad w = A(1-\alpha)k\left(\frac{k}{\mathcal{G}}\right)^{\alpha-1} \tag{1}$$

As usual, we assume throughout the paper that $\alpha < 1/2$ in order to match the empirical estimates of the share of capital in total income.

[7] All the proofs of the results are provided in the Working Paper version available on the AMSE website at the address: https://www.amse-aixmarseille.fr/en/research/working-papers.

[8] It is worth noting that all our results do not depend on the consideration of a public spending externality in production and could be obtained under a standard Ak formulation.

2.2 Households

We consider a representative household endowed with a fixed amount of labor and an initial stock of private physical capital that depreciates at rate $\delta > 0$. Instantaneous utility function is given by the following CRRA specification, which is consistent with endogenous growth:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} \tag{2}$$

with $\sigma > 0$ the inverse of the elasticity of intertemporal substitution in consumption.

The representative household derives income from wage and capital. Denoting $\tau > 0$ the tax rate on consumption, budget constraint is given by:

$$(1+\tau)c + \dot{k} = rk + w - \delta k \tag{3}$$

with r and w as given by (1).

The representative household then solves the following problem taking as given the prices r and w, and the time-varying path of τ:

$$\begin{aligned} &\max \int_0^\infty \frac{c(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \\ &s.t.\ \dot{k}(t) = r(t)k(t) + w(t) - \delta k(t) - (1+\tau(t))c(t) \\ &\qquad k(t) \geq 0,\ c(t) \geq 0 \\ &\qquad k(0) = k_0 > 0 \text{ and } (\tau(t))_{t\geq 0}\ \ given \end{aligned}$$

where the set of admissible parameters is so defined

$$\mathbf{\Theta} \equiv \{(\alpha, \rho, \delta, \sigma) :\ \alpha \in (0,1),\ \ \rho > 0,\ \ \delta > 0 \ and\ \sigma > 0\}.$$

The current value Hamiltonian associated with this problem is

$$\mathcal{H} = \frac{c^{1-\sigma}}{1-\sigma} + \lambda[rk + w - \delta k - (1+\tau)c]$$

where λ is the utility price of the final good. Considering (1), the first order conditions with respect to the control, c, and the state, k, write respectively

$$c^{-\sigma} = \lambda(1+\tau) \tag{4}$$

$$-\frac{\dot{\lambda}}{\lambda} = \alpha A \left(\frac{k}{\mathcal{G}}\right)^{\alpha-1} - \delta - \rho \tag{5}$$

Differentiation of Equation (4) gives

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\left[\frac{\dot{\lambda}}{\lambda} + \frac{\dot{\tau}}{1+\tau}\right] \tag{6}$$

Let us then substitute Equation (5) into (6). It follows that, given an initial capital stock k_0, the tax and government spending path $(\tau(t), \mathcal{G}(t))_{t\geq 0}$, the representative household maximizes utility by choosing any path $(c(t), k(t))_{t\geq 0}$ which solves the system of differential equations

$$\dot{k} = Ak^{\alpha}\mathcal{G}^{1-\alpha} - \delta k - (1+\tau)c \tag{7}$$

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left[\alpha A\left(\frac{k}{\mathcal{G}}\right)^{\alpha-1} - \delta - \rho - \frac{\dot{\tau}}{1+\tau}\right] \tag{8}$$

respects the positivity constraints $k \geq 0,\ c \geq 0$, and the transversality condition

$$\lim_{t\to+\infty} \frac{k}{c^{\sigma}(1+\tau)} e^{-\rho t} = 0 \tag{9}$$

3 Balanced-budget rule and fiscal policy

This section is organized as it follows. First, we emphasize the deep difference between exogenous and endogenous growth models. In particular, we show that global indeterminacy emerges naturally in an endogenous growth model if the government balances its budget in each period and the tax rate is endogenous and time-varying as usually assumed in exogenous growth models (e.g., Giannitsarou 2007). This result does not depend on the type of taxation and, for example, it still holds with an income tax.

Secondly, we argue that a natural attempt to rule out this global indeterminacy consists in introducing a fiscal policy rule. Therefore, we design a fiscal policy rule such that the tax rate is still endogenous, time-varying, and also predetermined. Later, and specifically in Section 7, we prove under which conditions the fiscal policy rule is indeed effective in ruling out global indeterminacy and therefore aggregate instability. To do so, we will have to investigate first the possible equilibria which may exist.

3.1 Exogenous vs endogenous growth models

In exogenous growth models where labor supply is endogenous and government spending is unproductive, local indeterminacy may emerge if the government balances its budget in each period and the tax on labor income is time-varying and endogenous (e.g., Schmitt-Grohe and Uribe 1997). Formally, local indeterminacy may emerge under the following balanced-budget rule $\mathcal{G} = \tau w L$ where $\mathcal{G}$ is usually exogenously given and possibly time-varying.[9] Observe that the tax rate is endogenous and time-varying because it must adjust in each period to balance the budget. In the same framework, changing the labor income tax with a consumption tax rules out local indeterminacy when the utility function is CRRA (see Giannitsarou 2007).

Let us now investigate what happens to the Barro's model if the tax rate is allowed to be time-varying and endogenous, and the government balances its budget in each period. Similar to Giannitsarou (2007), the balanced-budget rule is

$$\mathcal{G} = \tau c. \tag{10}$$

[9] An exception is considered in the quantitative analysis performed by Schmitt-Grohe and Uribe (1997) where they assume that $\mathcal{G}$ is endogenous and depends on the level of income.

Then the dynamics of the economy can be described by combining the Euler equation, the capital accumulation equation and the balanced-budget rule. In particular the following equation in the variable τ and the new variable $z \equiv \frac{k}{\mathcal{G}}$ describes the dynamics of the economy:

$$\frac{\dot{z}}{z} + \frac{\dot{\tau}}{\tau} = \left(1 - \frac{\alpha}{\sigma}\right) A z^{\alpha-1} - \delta \left(1 - \frac{1}{\sigma}\right) + \frac{\rho}{\sigma} + \frac{\dot{\tau}}{\sigma\tau(1+\tau)} - \frac{1+\tau}{\tau}\frac{1}{z} \tag{11}$$

Therefore, the model is globally indeterminate because Equation (11) is underdetermined (i.e., there are more variables than equations). Intuitively different households' beliefs about the evolution of the tax rate can be self-fulfilled by choosing opportunely different paths of the capital-government ratio.

A natural attempt to avoid this form of pervasive indeterminacy consists in specifying a fiscal policy rule. In fact, with a fiscal policy rule we may circumvent the underdetermined issue previously explained. The next section is aimed at designing a fiscal policy rule.

3.2 Designing a fiscal policy rule

The objective of this section is to design a fiscal policy rule such that the tax rate is:

(i) endogenous,
(ii) time-varying but constant in the long run, and
(iii) predetermined.

Features (i) and (ii) on endogeneity and time-variation are necessary to be consistent with the existing literature on aggregate instability. Feature (ii) on the constancy of the tax rate in the long run is justified by the fact that along a balanced growth path characterized by a common growth rate for all variables, the tax rate cannot be forever increasing, nor forever decreasing. The last feature is also desirable because tax rates are typically set in advance (see for example Schmitt-Grohe and Uribe 1997, p. 993).

Consistent with the households problem we need to specify a fiscal policy rule for the consumption tax. In Section 8 we will also provide an insight on what happens if there is an income tax instead of a consumption tax. In order to be compatible with a constant long-run growth rate γ for the main macroeconomic variables, we assume that the fiscal policy rule requires that the tax rate is a function of de-trended consumption, that is, $\tau = \tau(\tilde{c})$, and therefore the government balanced-budget rule becomes:

$$\mathcal{G} = \tau(\tilde{c})c \tag{12}$$

where $\tilde{c} \equiv ce^{-\gamma t}$ indicates de-trended consumption with γ the (endogenous) asymptotic and constant growth rate of the economy.[10] The fiscal instrument τ is clearly time-varying and endogenously determined; in fact, it is similar to the fiscal policy suggested by Nourry, Seegmuller, and Venditti (2013) among others. In particular, Nourry, Seegmuller, and Venditti (2013) study, in an exogenous

[10] More precisely, γ is the (constant) growth rate if the economy is on a BGP or is the asymptotic (constant) growth rate if the economy is not on a BGP but converges over time to an Asymptotic BGP (see Definition 3). In fact, at this stage of the analysis, we cannot exclude *a priori* the existence of a subset of initial conditions such that the economy is not on a BGP at $t = 0$ but rather converges to it over time as happens, for example, in an endogenous growth model with a Jones and Manuelli (1997) production function.

growth model, the case $\mathcal{G}(c) = \tau(c)c$ while the tax rate depends on de-trended consumption to have a constant tax on a balanced growth path.

In addition, we will also assume, from now on, the following:

Assumption 1 *The elasticity of the tax rate with respect to de-trended consumption is constant and given by*

$$\phi \equiv \frac{d\tau}{d\tilde{c}}\frac{\tilde{c}}{\tau} \tag{13}$$

It is worth noting that such a restriction is common in the literature. In their seminal contribution, Schmitt-Grohé and Uribe (1997) consider a tax on labor income with constant government spending such that $\tau(wl) = \mathcal{G}/wl$, which has a constant elasticity with respect to its tax base equal to -1. The same property is assumed by Giannitsarou (2007) with a consumption tax satisfying $\tau(c) = \mathcal{G}/c$. In Nourry, Seegmuller, and Venditti (2013), however, the government spending is assumed to vary with consumption and the elasticity of the tax rate $\tau(c) = \mathcal{G}(c)/c$ is equal to $\eta - 1$ with η the constant elasticity of government spending with respect to consumption.

As a consequence of Assumption 1 we have that

$$\dot{\tau} \equiv \frac{d\tau}{dt} = \frac{d\tau}{d\tilde{c}}\dot{\tilde{c}} = \frac{d\tau}{d\tilde{c}}\tilde{c}\left(\frac{\dot{c}}{c} - \gamma\right) \tag{14}$$

and therefore

$$\frac{\dot{\tau}}{\tau} \equiv \phi\left(\frac{\dot{c}}{c} - \gamma\right) \tag{15}$$

Integrating (15) leads to

$$\tau(t) \equiv B\left(c(t)e^{-\gamma t}\right)^{\phi} \tag{16}$$

with B a generic (and endogenously determined) constant. Therefore, the last expression (16) is rather a menu of fiscal policies. To select just one of them (and avoiding in this way the introduction of a trivial form of indeterminacy in the model) we assume that $\tau(0) = \tau_0 > 0$ is exogenously given. By doing so, we may find the value of B and observe that the last expression, and therefore the fiscal rule (15), is equivalent to

$$\tau(t) \equiv \tau_0\left(\frac{\tilde{c}(t)}{c_0}\right)^{\phi} = \tau_0\left(\frac{c(t)}{c_0 e^{\gamma t}}\right)^{\phi} \tag{17}$$

where $B = \tau_0/c_0^{\phi}$.[11] Clearly the tax rate, τ, is a predetermined variable, in the sense that the initial value τ_0 is given, which is consistent with the fact that tax rates are typically set in advance and it seems even more compelling in our model where the tax base is not predetermined, since it depends on consumption. It is also worth underlining that identities (15) and (17) are a direct consequence

[11] Our specification shares some similarity with that used by Lloyd-Braga, Modesto, and Seegmuller (2008) since in both cases the tax rate is adjusted comparing the level of consumption to a reference level, which is the consumption steady state in the cited contribution while an (asymptotic) BGP in our case.

of Assumption 1. Note that this formulation is consistent with , and indeed includes under the restriction $\phi = 0$, the case of a constant and exogenously given tax rate $\tau = \tau_0$ considered by Barro (1990). Moreover, the fiscal rule (15) is pro(counter)-cyclical if $\phi > 0$ ($\phi < 0$) since it increases (decreases) when consumption grows faster (slower) than γ.[12]

It is also worth noting that while our formulation is very similar tthat of Nourry, Seegmuller, and Venditti (2013), but there is a strong qualitative difference: here we postulate a specific form of the tax function as given by (17) and government spending adjusts accordingly along the balanced-budget rule (12), while in Nourry, Seegmuller, and Venditti (2013) the government spending rule is postulated as in (10) and the tax rate adjusts accordingly since $\tau(c(t)) = \mathcal{G}/c(t)$. In this case, the tax rate is not predetermined as $\tau(0) = \mathcal{G}/c(0)$.

Before concluding this section, we note that one could be tempted to assume an exogenous target value for γ. This would lead to two undesirable consequences: first, the model becomes an exogenous growth model since it emerges immediately from the fiscal rule that the growth rate of consumption (and therefore of capital) will not be determined, as usual in an endogenous growth model, by a combination of parameters, but rather by the target value itself, otherwise the tax rate will grow in the long run.[13] Secondly, it can be easily proved that an equilibrium path will exist only for a zero-measure set of parameters.

4 Intertemporal equilibrium

Given an initial condition of capital $k_0 > 0$ and of the consumption tax $\tau_0 > 0$, an intertemporal equilibrium is any path $(c(t), k(t), \tau(t), \mathcal{G}(t))_{t\geq 0}$ that satisfies the system of Equations (7), (8), (12), and (15), respects the inequality constraints $k \geq 0,\ c \geq 0$, and the transversality condition (9). Put differently, Equations (7)-(9) with (17) then provide a set of necessary and sufficient conditions for an intertemporal equilibrium starting from a given pair (k_0, τ_0).

Therefore, we may define the control-like variable $x \equiv \frac{c}{k}$ and observe that the intertemporal equilibrium can be derived studying the following system of nonlinear differential equations in the variables (x, τ):

$$\frac{\dot{x}}{x} = \frac{[(1+\tau)(1-\sigma) - \phi\tau]\left[\alpha A (x\tau)^{1-\alpha} - \delta - \rho - \sigma\gamma\right]}{\sigma(1+\tau) + \phi\tau} + \gamma(1-\sigma) + (1+\tau)x - (1-\alpha)A(x\tau)^{1-\alpha} - \rho \tag{18}$$

$$\frac{\dot{\tau}}{\tau} = \frac{\phi(1+\tau)}{\sigma(1+\tau)+\phi\tau}\left[\alpha A (x\tau)^{1-\alpha} - \delta - \rho - \sigma\gamma\right] \tag{19}$$

The interested reader may find in the Appendix of the Working Paper version of Bambi and Venditti (2018) the detailed procedure to obtain this system starting from Equations (7), (8), (12), and (15). It is also worth noting that x_0 is not predetermined since it depends on c_0, while τ_0 is exogenously given and therefore predetermined.

[12] The definition of pro(counter)-cyclical is based on a comparison of the growth rates. This is consistent with the real business cycle literature where an economy is said to be in recession if it grows more slowly than at its trend.

[13] To see this point even more explicitly, observe that the fiscal rule could be rewritten as $\tau(t) = \tau_0\,(c(t)/z(t))^{\phi}$ with $z(t) = c_0 e^{\bar{\gamma}t}$. Then all the aggregate variables will grow at the rate of the variable $z(t)$, the growth rate of which, $\bar{\gamma}$, has been given exogenously.

5 Balanced growth paths

A balanced growth path (BGP) is an intertemporal equilibrium where consumption and capital are purely exponential functions of time t, namely:

$$k(t) = k_0 e^{\gamma t} \qquad \text{and} \qquad c(t) = c_0 e^{\gamma t} \qquad \forall t \geq 0. \tag{20}$$

From Equation (15), it follows immediately that along a BGP the consumption tax is constant and equal to

$$\tau(t) = \hat{\tau} = \tau_0 \qquad \forall t \geq 0$$

with the hat symbol indicating, from now on, the value of a variable on a BGP. Along the BGP, the tax rate is therefore constant and equal to its initial value. Also, government spending will be purely exponential with a growth rate equal to γ consistently with the balanced-budget rule (12). Therefore, Equations (18) and (19) rewrite

$$0 = (\alpha - \sigma)A(x\hat{\tau})^{1-\alpha} + \sigma(1+\hat{\tau})x - \rho - \delta(1-\sigma) \equiv g(x) \tag{21}$$

$$\gamma = \frac{1}{\sigma}\left[\alpha A(x\hat{\tau})^{1-\alpha} - \delta - \rho\right]. \tag{22}$$

Studying the zeros of Equation (21) is the necessary step to prove existence and uniqueness of a balanced growth path. Moreover, along a BGP, the transversality condition (9) becomes

$$\lim_{t \to +\infty} \frac{k_0}{c_0^{\sigma}(1+\tau_0)} e^{-[\rho - \gamma(1-\sigma)]t} = 0 \tag{23}$$

It follows that along a BGP, condition (23) holds if and only if $\rho - \gamma(1-\sigma) > 0$. As we restrict our analysis to endogenous positive long-run growth,[14] any value of γ solution of Equation (22) needs to satisfy $\gamma \geq 0$ when $\sigma \geq 1$ and $\gamma \in [0, \rho/(1-\sigma))$ when $\sigma < 1$.

Proposition 1 ***(Existence and Uniqueness of a BGP).*** *Given any initial condition of capital $k_0 > 0$ and the tax rate $\tau_0 > 0$, there exist $\underline{A} > 0$, $\underline{\tau} > 0$ and $\bar{\tau}(\sigma) \in (0, +\infty]$ with $\bar{\tau}(\sigma) > \underline{\tau}$ such that when $A > \underline{A}$ and one of the following conditions holds:*

(i) $\sigma \geq 1$ and $\tau_0 > \underline{\tau}$,

(ii) $\sigma \in (0,1)$ and $\tau_0 \in (\underline{\tau}, \bar{\tau}(\sigma))$,

there is a unique balanced growth path where the ratio of consumption over capital is constant and equal to $\hat{x}$ – the unique positive root of Equation (21) – and the growth rate of the economy is

$$\hat{\gamma} = \frac{1}{\sigma}\left[\alpha A(\hat{x}\hat{\tau})^{1-\alpha} - \delta - \rho\right] > 0, \tag{24}$$

with $\hat{\tau} = \tau_0$.

---

[14] We could indeed also focus on negative growth rates $\gamma < 0$ implying consideration of long-run values for capital and consumption equal to zero. As these solutions are ruled out by the Inada conditions satisfied by the utility and production functions specifications, we do not consider such a possibility.

Discussion of these conditions is in order. The requirements of a level of technology greater than $\underline{A}$ and of a tax rate larger than $\underline{\tau}$ guarantee positive economic growth. The first one is indeed a condition similar to the one in the AK model while the second one allows provision of sufficient government spending to sustain growth through the technology $y = Ak^{\alpha}\mathcal{G}^{1-\alpha}$. Also the condition of a tax rate lower than $\bar{\tau}(\sigma)$ when $\sigma < 1$ guarantees that the transversality condition is respected and therefore the utility is bounded. Therefore, Proposition 1 shows that if $A > \underline{A}$ and one of the conditions (i)–(ii) holds, a unique BGP exists. Of course, uniqueness depends on the existence of a unique value $\hat{x}$ which implies a unique specification of initial consumption for any exogenously given initial condition of the capital stock and consumption tax. For any given k_0 and τ_0, we have indeed $c_0 = k_0\hat{x}$ and $\tau(\tilde{c}) = \tau_0$ so that the stationary value of de-trended consumption $\tilde{c}$ corresponding to the BGP is derived from (20), and such that $\tilde{c} = c_0 = k_0\hat{x}$.

Clearly the value of the positive real zero $\hat{x}$ and of $\hat{\gamma}$ depend on the exogenously given parameters but also on the initial condition of the consumption tax τ_0.[15] For this reason we may explicitly write $\hat{x}$ and $\hat{\gamma}$ as continuous and differentiable functions of these values, that is, $\hat{x} = \hat{x}(\alpha, \tau_0, \rho, \delta, \sigma)$ and $\hat{\gamma}(\alpha, \tau_0, \rho, \delta, \sigma)$. Given a generic capital stock k_0, the balanced growth path is

$$\hat{k} = k_0 e^{\hat{\gamma}(\alpha,\tau_0,\rho,\delta,\sigma)t} \qquad \text{and} \qquad \hat{c} = \hat{x}(\alpha, \tau_0, \rho, \delta, \sigma)k_0 e^{\hat{\gamma}(\alpha,\tau_0,\rho,\delta,\sigma)t}$$

where the growth rate is positive if and only if the conditions of Proposition 1 hold.

We conclude this section with some comparative statics results that provide sufficient conditions for the growth rate $\hat{\gamma}$ and welfare to be increasing functions of the tax rate $\tau_0 = \hat{\tau}$. Indeed, we can easily compute welfare along the BGP characterized by the stationary values of the growth rate $\hat{\gamma}$ and the ratio of consumption over capital $\hat{x}$, namely

$$W(\hat{\gamma}, \hat{x}) = \frac{(\hat{x}k_0)^{1-\sigma}}{(1-\sigma)[\rho - \hat{\gamma}(1-\sigma)]} \tag{25}$$

We then get the following result:

Corollary 1 *Let the conditions of Proposition 1 hold. There exist $A_{min} > \underline{A}$ and $\underline{\sigma} > 0$ such that when $\tau_0 < (1-\alpha)/\alpha$, $\sigma > \underline{\sigma}$ and $A > A_{min}$, then*

$$\frac{d\hat{x}}{d\tau_0} > 0, \quad \frac{d\hat{\gamma}}{d\tau_0} > 0 \ \text{ and } \ \frac{dW(\hat{\gamma}, \hat{x})}{d\tau_0} > 0$$

Obviously, as shown by expression (25), along the BGP welfare is an increasing function of both the growth rate $\hat{\gamma}$ and the ratio of consumption over capital $\hat{x}$. Because of the public good externality in the production function, a large growth factor allows to generate an increasing amount of public good, which improves the aggregate production level and thus consumption. Corollary 1 then provides conditions for a positive impact of the tax rate τ_0 on the growth rate, consumption over capital, and welfare. In particular, such a conclusion requires a low enough elasticity of intertemporal substitution in consumption $1/\sigma$, which prevents a too large consumption smoothing over time in order to ensure a larger consumption in the long run, that is, along the BGP.

[15] The fact that the taxation enters in the equation of the consumption-capital ratio and therefore affects also the growth rate is not surprising and consistent with previous contributions (e.g., Barro 1990).

6 Transitional dynamics

6.1 Local determinacy of the steady state $(\hat{x}, \hat{\tau})$

In this section we start by investigating the local stability properties of the steady state $(\hat{x}, \hat{\tau})$ (with $\hat{\tau} = \tau_0$), which characterizes the unique BGP of our economy. Let us recall that in the formulation considered by Barro (1990) where the tax rate τ is constant, there is no transitional dynamics and the economy directly jumps on the BGP from the initial date $t = 0$. In our framework we get similar conclusions:

Proposition 2 *Consider the steady state $(\hat{x}, \hat{\tau})$ (with $\hat{\tau} = \tau_0$) which characterizes the unique BGP of our economy. For any given initial conditions (k_0, τ_0), there is no transitional dynamics, i.e. there exists a unique $c_0 = k_0\hat{x}$ such that the economy directly jumps on the BGP from the initial date $t = 0$.*

As shown in Proposition 2, the steady state $(\hat{x}, \hat{\tau})$ is locally saddle-path stable if and only if $\phi \in (-\sigma(1 + \hat{\tau})/\hat{\tau}, 0)$ and locally unstable otherwise.[16] However, in both cases, we find the same conclusion as in Barro (1990): there is no transitional dynamics with respect to the BGP as any initial choice of $c(0)$ different from $c_0 = k_0\hat{x}$ leads to trajectories diverging from $(\hat{\tau}, \hat{x})$. This is not really surprising since the tax rate on the BGP is exactly $\hat{\tau} = \tau_0$, which is the initial condition of the state variable of our problem.

Note, however, that the reasons for the absence of transitional dynamics in our framework are different from those in Barro (1990). In his model, τ is constant by definition and so is the consumption growth rate $\dot{c}/c$. In our case, the tax rate can *a priori* exhibit counter or procyclicality and there is no transitional dynamics because of the local stability properties of the BGP.

To make this argument more explicit, consider Figure 1, which illustrates Proposition 2 and shows the phase diagrams when the parameters are set as in the previous section. The initial conditions are $k_0 = 1$ and $\tau_0 = 0.2$, $\sigma = 1$ and ϕ is equal to 0.5 (left diagram) and -0.01 (right diagram). According to the directions of the arrows it is clear that in both phase diagrams, any choice of $x_0 \neq \hat{x}_0$ along the vertical line $\hat{\tau} = \tau_0$ leads to paths that cannot converge to $(\hat{x}, \hat{\tau})$. In this case we have local determinacy of the steady state $(\hat{x}, \hat{\tau})$ since given any k_0 and τ_0 satisfying the conditions of Proposition 1, there exists a unique choice of $x_0 = \hat{x}_0$ and therefore of $c_0 = k_0\hat{x}$, which pins down an equilibrium path corresponding to the BGP described in the previous section.

6.2 Existence of other equilibria

As we have shown in the previous subsection, the unique steady state $(\hat{x}, \hat{\tau})$ may be a saddle-point or totally unstable depending on whether $\phi \in (-\sigma(1 + \hat{\tau})/\hat{\tau}, 0)$ or not. While these two possible configurations do not alter the fact that when $\tau_0 = \hat{\tau}$, the economy directly jumps on the BGP from the initial date $t = 0$, we can prove that contrary to Barro (1990), some particular transitional dynamics may occur in our model. Indeed, the BGP, as defined by $(\hat{x}, \hat{\tau})$ and $\hat{\gamma}$, is not the unique possible equilibrium of our economy. In this section, depending on the value of ϕ, we look for the existence of equilibrium paths $(x_t, \tau_t)_{t \geq 0}$ that may eventually converge to an Asymptotic BGP, denoted from now on as ABGP, defined as follows:

[16] The change in stability at $\phi = -\sigma(1 + \hat{\tau})/\hat{\tau}$ occurs through a discontinuity in a similar way as in the model of Benhabib and Farmer (1994) (see for example figure 2, p. 34) since one of the eigenvalues of the Jacobian matrix changes its sign from $+\infty$ to $-\infty$.

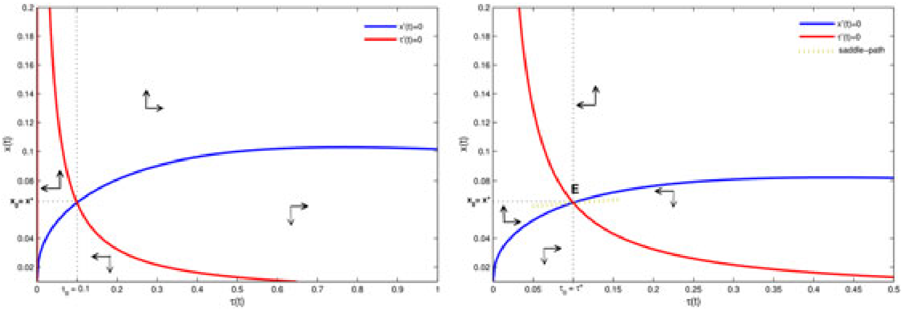

Figure 1 Phase diagrams when $\phi > 0$ (left) and $\phi < 0$ (right) and $\gamma = \hat{\gamma}^0$

Definition 1 ***(ABGP).*** *An ABGP with consumption tax is any path $(x(t), \tau(t))_{t\geq 0} = (x^*, \tau^*)$ such that:*

(a) τ^* is a positive arbitrary constant sufficiently close to (but different from) τ_0;
(b) (x^*, τ^*) is a steady state of (18)-(19) with $x^* > 0$ and $\gamma^* > 0$ solution of

$$0 = (\alpha - \sigma)A(x^*\tau^*)^{1-\alpha} + \sigma(1 + \tau^*)x^* - \rho - \delta(1 - \sigma) \tag{26}$$

$$\gamma^* = \frac{1}{\sigma}\left[\alpha A(x^*\tau^*)^{1-\alpha} - \delta - \rho\right]. \tag{27}$$

(c) (x^*, τ^*) satisfies the transversality condition.

Crucially an ABGP is not an equilibrium since it does not satisfy the initial condition $\tau(0) = \tau_0$. An ABGP, in terms of the original variables, is a path

$$k^* = k_0 e^{\gamma^*(\alpha,\tau^*,\rho,\delta,\sigma)t} \qquad \text{and} \qquad c^* = x^*(\alpha, \tau^*, \rho, \delta, \sigma)k_0 e^{\gamma^*(\alpha,\tau^*,\rho,\delta,\sigma)t} \tag{28}$$

which is defined as a steady state (x^*, τ^*) of the system (18)-(19) but is *not an equilibrium* because τ^* is generically different from the exogenously given initial condition of the consumption tax, τ_0. If such an ABGP exists, the asymptotic value (x^*, τ^*) as well as the asymptotic growth rate of the economy γ^* will not be pinned down by τ_0 through Equations (21) and (22) as before, but rather from the asymptotic value of the consumption tax τ^* and then by Equations (43)-(44).

The existence of an equilibrium path converging to an ABGP is associated with the existence of consumers' beliefs that are different from those associated with the BGP. Indeed, they may believe that the consumption tax profile will not remain constant but rather change over time and eventually converge to a positive value $\tau^* \neq \tau_0$. Therefore, the consumption over capital ratio and the growth rate will converge to x^* and γ^* respectively. Based on that we will prove in the next Proposition that under some conditions on ϕ the consumers may indeed decide a consumption path that makes this belief self-fulfilling.

Building on Propositions 1 and 2 we can prove the following result:

Proposition 3 *Given any initial condition $k_0 > 0$ and $\tau_0 > 0$, consider $\underline{\tau}$ and $\bar{\tau}(\sigma)$ as defined by Proposition 1. There exist $\underline{A} > 0$ such that when $A > \underline{A}$, there is a unique equilibrium path $(x_t, \tau_t)_{t\geq 0}$ converging over time to the ABGP (x^*, τ^*) if and only if $\phi \in (-\sigma(1 + \tau^*)/\tau^*, 0)$ and one of the following conditions holds:*

(i) $\sigma \geq 1$ *and* $\tau^* > \underline{\tau}$,
(ii) $\sigma \in (0, 1)$ *and* $\tau^* \in (\underline{\tau}, \bar{\tau}(\sigma))$.

Discussion of these conditions is again in order. First, if the consumers believe that the asymptotic tax rate will be τ^* then a unique ABGP exists if one of the conditions (i)-(ii) holds. As discussed previously, the requirement of a level of technology greater than $\underline{A}$ is a standard condition for AK models and a tax rate larger than $\underline{\tau}$ provides sufficient government spending to sustain growth through the technology $y = Ak^{\alpha}\mathcal{G}^{1-\alpha}$. Also the condition $\tau < \bar{\tau}(\sigma)$ when $\sigma < 1$ guarantees that the transversality condition is respected and thus the utility is bounded.

Furthermore, given (k_0, τ_0) there exists a unique equilibrium path converging to the ABGP (x^*, τ^*) if and only if $\phi \in (-\sigma(1 + \tau^*)/\tau^*, 0)$. Let us assume for simplicity that $\sigma = 1$. Considering the expression of the tax rate as given by (17), let us denote $g(c) \equiv (1 + \tau(\tilde{c}))c$ with $\tilde{c} = ce^{-\gamma t}$. It follows that the elasticity of $g(c)$ is given by

$$\varepsilon_{gc} \equiv \frac{g'(c)c}{g(c)} = 1 + \frac{\phi\tau}{1+\tau}$$

Since $\phi > -(1 + \tau^*)/\tau^*$, we get $\varepsilon_{gc} > 0$. Consider then the system of differential Equations (7) and (8) with $\sigma = 1$, which can be written as

$$\dot{k} = Ak^{\alpha}\mathcal{G}^{1-\alpha} - \delta k - g(c) \quad (29)$$

$$\frac{\dot{c}}{c} = r - \delta - \rho - \frac{\dot{\tau}}{1+\tau} \quad (30)$$

If households expect that in the future the consumption tax rate will be above average, then they expect to consume less in the future and thus, considering that $\varepsilon_{gc} > 0$, we derive from (29) that $g(c)$ is decreasing and thus, for a predetermined k, investment, as given by $\dot{k}$, is increasing. This implies that the rental rate of capital r is decreasing and thus through Equation (30) that consumption is also decreasing. We conclude from the balanced-budget rule with $\phi < 0$ that the tax rate is decreasing and that the initial expectation is self-fulfilling. Of course this mechanism requires a low enough elasticity of intertemporal substitution in consumption, that is, a large enough value of σ, to avoid intertemporal consumption's compensations associated with the initial expected decrease of c.

Figure 2 shows the phase diagrams that implicitly account for the consumers' beliefs of a steady state where the same parameters' values are in Figure 1 except that here $(x^*, \tau^*) = (0.123, 0.25)$ and therefore a growth rate $\gamma = 0.037$.

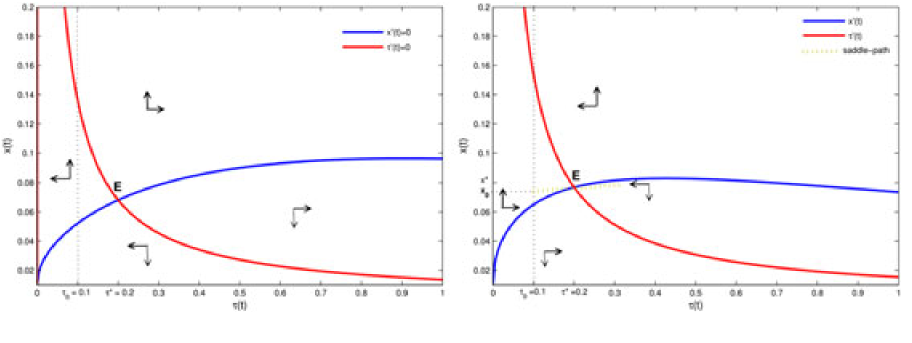

Figure 2 Phase diagrams when $\phi > 0$ (left) and $\phi < 0$ (right) and $\gamma = \hat{\gamma}^1$

Observe that both the locus $\dot{x} = 0$ and $\dot{\tau} = 0$ are shifted with respect to the previous case to reflect the different beliefs.[17] According to the directions of the arrows it is clear that in the case of a pro-cyclical consumption tax (i.e., $\phi = 0.5$), no equilibrium path exists that makes this belief self-fulfilling. On the other hand, in the case of a counter-cyclical consumption tax (i.e., $\phi = -0.01$) an equilibrium path converging to the steady state may exist as shown by the golden path in the Figure. In this case the consumers' belief is indeed self-fulfilling.

6.3 Overall (deterministic) dynamics

Proposition 3 actually proves the existence of a continuum of equilibria, each of them converging to a different ABGP. In fact, any value of τ^* in a neighborhood of the given initial value τ_0 can be a self-fulfilling belief for the consumers if the conditions of the Proposition are met. Of course this implies a form of global indeterminacy since from a given τ_0, one can select either the unique BGP by jumping on it from the initial date or select any other equilibrium converging to an ABGP. Again different choices reveal different consumers' beliefs of the long run outcome of the economy.

Combining the results found in Section 3 and Subsections 4.1 and 4.2 allows for stating the following Theorem, which fully characterizes the dynamics of the economy.

Theorem 1 *Given the initial conditions k_0 and τ_0, let $\tau_{inf} = \tau_0 - \epsilon > 0$ and $\tau_{sup} = \tau_0 + \varepsilon$ with $\epsilon, \varepsilon > 0$ small enough. Consider $\underline{\tau}$ and $\bar{\tau}(\sigma)$ as defined by Proposition 1. There exists $\underline{A} > 0$ such that if $A > \underline{A}$, $\phi \in \left(-\frac{\sigma(1+\tau_{sup})}{\tau_{sup}}, 0\right)$ and one of the following conditions holds:*

(i) $\sigma \geq 1$ and $\tau_{inf} > \underline{\tau}$,

(ii) $\sigma \in (0, 1)$, $\tau_{inf} > \underline{\tau}$ and $\tau_{sup} < \bar{\tau}(\sigma)$,

then there is a continuum of equilibrium paths, indexed by the letter j, departing from (τ_0, x_0^j), each of them converging to a different ABGP (τ^{*j}, x^{*j}) with $\tau^{*j} \in (\tau_{inf}, \tau_{sup})$, that is, the dynamics of the economy are globally, but not locally, indeterminate.

To fully understand the dynamic behavior of the economy we can write explicitly the solution of the linearized system:

$$\tilde{\tau} = b_1 v_{11} e^{\lambda_1 t} + b_2 v_{21} e^{\lambda_2 t} \tag{31}$$

$$\tilde{x} = b_1 v_{12} e^{\lambda_1 t} + b_2 v_{22} e^{\lambda_2 t} \tag{32}$$

where $\mathbf{v_i} \equiv (v_{i1}, v_{i2})^T$ is the eigenvector associated with the eigenvalue λ_i, with $i = 1, 2$ while b_i are arbitrary constants. If $\phi \in (-\sigma(1 + \tau^*)/\tau^*, 0)$ and assuming without loss of generality that $\lambda_2 > 0$, the saddle-path solution can be easily found imposing $b_2 = 0$. Combining (31) and (32) and imposing $b_2 = 0$ we get under $v_{12} \neq 0$ that

$$\tilde{\tau}_0 = \frac{v_{11}}{v_{12}} \tilde{x}_0 \tag{33}$$

[17] This is indeed obvious from Equations (18) and (19) since the growth rate, γ, enters explicitly in both of them.

with $\tilde{\tau}_0 = \tau_0 - \tau^*$ and $\tilde{x}_0 = x_0 - x^*$. Therefore, given any initial condition k_0 and τ_0 we have the following solution of x converging over time to (x^*, τ^*):

$$x = x^* + \frac{v_{12}}{v_{11}} \tilde{\tau}_0 e^{\lambda_1 t} \tag{34}$$

Of course as $t \to \infty$ we have that $\tilde{\tau} \to \tau^*$, meaning that c converges to the corresponding ABGP since $x \to x^*$ also converges to the corresponding ABGP. Observe also that the initial level of consumption for this equilibrium path can be obtained from (34) evaluated at $t = 0$, taking into account (28), and it is equal to

$$c_0 = c^* + \frac{v_{12}}{v_{11}} (\tau_0 - \tau^*) k_0 \tag{35}$$

Remark 1 *Note that we have a constraint on the initial choice of c_0 (and therefore on x_0) because initial consumption cannot be higher than the initial wealth, $c_0 \leq y_0 - \delta k_0 - \tau_0 c_0$ which at the equilibrium implies that*

$$x_0 \leq A \frac{A \tau_0^{\frac{1-\alpha}{\alpha}}}{(1+\tau_0)^{\frac{1}{\alpha}}} - \delta$$

Figure 3 shows the presence of global indeterminacy in the phase diagram $(x(t), \tau(t))$

The initial tax rate is assumed to be equal to $\tau_0 = 0.2$, and the parameters are chosen as in the balanced growth path section with ϕ set to -0.5. Different initial choices of c_0 pin down different equilibrium paths of the tax rate and of the consumption-capital ratio, each of them converging to a different steady state, characterized by a different growth rate of consumption and capital. Similarly Figure 4 illustrates the emergence of global indeterminacy in the spaces $(t, x(t))$ and $(t, \tau(t))$.

Global indeterminacy arises when the government uses counter-cyclical consumption tax. Under these circumstances, the long run growth rate, as well as the consumption over capital ratio, cannot be univocally determined within the model. Therefore an economy characterized by these features and an initial value of the tax rate $\tau(0)$ can remain on a balanced growth path but can also follow

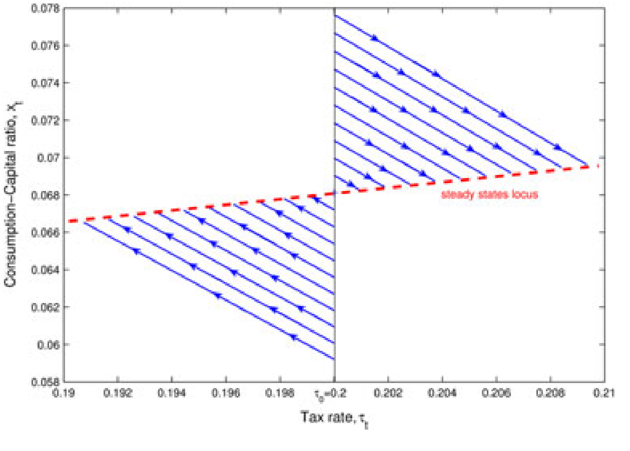

Figure 3 Converging equilibria for different initial values of c_0

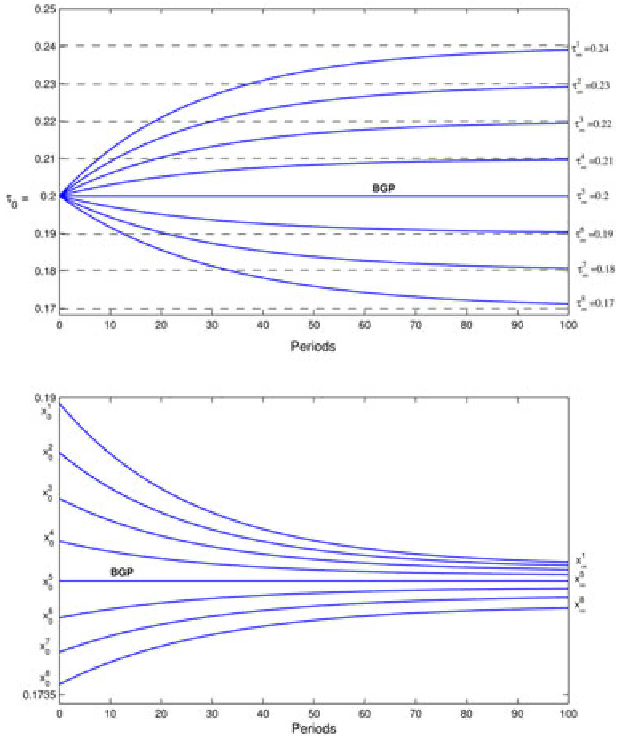

Figure 4 Dynamics of the tax rate and of the consumption-capital ratio for fiscal policy (15)

alternative paths towards different ABGPs, each of them characterized by a different (asymptotic) growth rate.

Remark 2 *As long as the consumption tax is no more a constant, it becomes distortionary as shown by the Euler Equation (4). This means that an optimal taxation analysis becomes much more complex than in the standard Barro model. In his model, Barro addressed the optimal taxation issue by answering the following question: assuming a constant tax rate, what is its welfare-maximizing value? In our framework, a similar exercise could be done by figuring out which among the paths of taxation suggested in Figure 4 leads to the highest welfare. Although this question is clearly very interesting, it requires the solving of a Ramsey problem where the policy maker can choose one of the previously mentioned tax paths. Given the complexity of the issue, we leave this analysis for further research.*[18]

7 Aggregate instability and stabilization policy

7.1 Sunspot equilibria

Suppose that the households choose an initial value of consumption such that they are at $t = 0$ in (τ_0, x_0^j). From Theorem 1, we know that if some conditions on parameters are respected then the

[18] The issue is indeed quite complex since the tax paths were derived from a local analysis and, therefore, a welfare evaluation requires a not-trivial linear-quadratic approximation of the Ramsey problem.

deterministic dynamical system (18)-(19) has a unique solution around the steady state (τ^{*j}, x^{*j}) converging to it over time. Let us call this path $(x, \tau) = \boldsymbol{\phi}_j(t) \equiv (\phi_{j,x}(t), \phi_{j,\tau}(t))$ as shown on the following Figure 5:

As observed before, this is indeed the unique equilibrium consistent with an asymptotic growth rate γ^j. Clearly, aggregate instability cannot emerge unless

(a) countercyclical taxation is implemented;
(b) extrinsic uncertainty is introduced through a sunspot variable.

Condition (a) is obtained by setting appropriately the elasticity of the tax rate with respect to de-trended consumption. This is indeed described in detail in Theorem 1.

On the other hand, extrinsic uncertainty can be introduced in the model through a sunspot variable. Formally, a sunspot variable can be represented by a continuous-time homogeneous Markov chain $\{\varepsilon_t\}_{t\geq 0}$ with $p_{ij}(t-s)$ indicating the transition probability to move from state i at time s to state j at time t with $s \leq t$ while the initial probability distribution of ε_0 is denoted by $\pi = (\pi_1, ..., \pi_N)$ with $\pi_j = \mathbb{P}(\varepsilon_0 = z_j)$. More precisely, we need to consider a probability space $(\Omega, B_\Omega, \mathbb{P})$ where Ω is the sample space, B_Ω is a σ-field associated with Ω, and $\mathbb{P}$ is a probability measure. We assume also that the state space is a countable subset of $\mathbb{R}$: $Z \equiv \{z_1, ..., z_{\bar{\imath}}, ..., z_N\} \subset \mathbb{R}$, with $-\underline{\varepsilon} \leq z_1 < ... < z_{\bar{\imath}} = 0 < ... < z_N \leq \bar{\varepsilon}$. Then each random variable ε_t is a function from $\Omega \to Z$ which we assume to be B_Ω-measurable.

As explained in Shigoka (1994) and Benhabib and Wen (1994), the extrinsic uncertainty modifies the deterministic dynamics described by system (18)-(19), from now on $f(x, \tau)$, as it follows:

$$\begin{pmatrix} dx \\ \dot{\tau} dt \end{pmatrix} = f(x, \tau)dt + m \begin{pmatrix} d\varepsilon_t \\ 0 \end{pmatrix}, \tag{36}$$

where m is a constant measuring the weight of the sunspot variable. Two considerations are appropriate: (i) the deterministic dynamics can be obtained by setting $m = 0$; (ii) the sunspot variable does

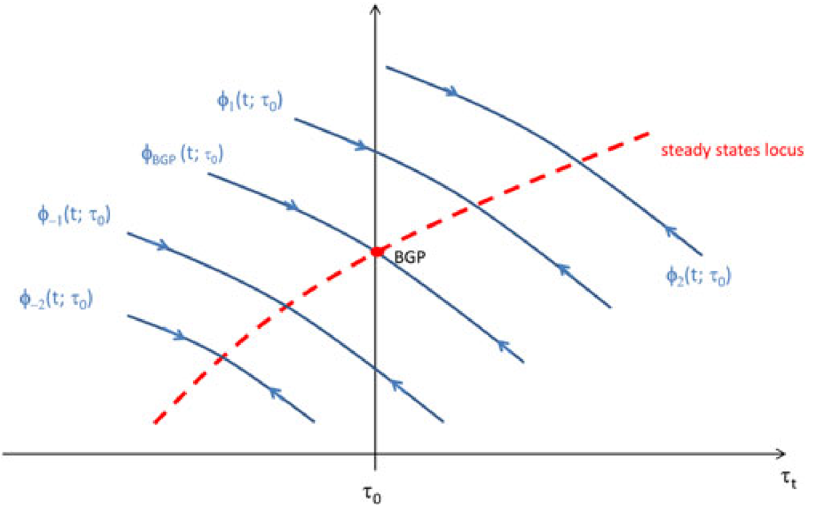

Figure 5 Examples of saddle-paths

not affect the fundamental of the economy. In fact, the sunspot variable is a device to randomize among the deterministic equilibria. See the example in the Supplementary Material.

Before proving the existence of sunspot equilibria, several intermediary steps need to be done. In the following we explain carefully these steps and at the end we provide a Theorem that proves our main result.

Different from the discrete-time case, the evolution of a continuous-time Markov chain cannot be described by the initial distribution π and the $n - step$ transition probability matrix, $\mathbf{P}^n$, since there is no implicit unit length of time. However, it is possible to define a matrix $\mathbf{G}$ (generator of the chain) that takes over the role of $\mathbf{P}$. This procedure can be found in Grimmett and Stirzaker (2009) among others and, as far as we know, our paper represents the first economic application of this procedure.

Let $\mathbf{P}_t$ be the $N \times N$ matrix with entries $p_{ij}(t)$. The family $\{\mathbf{P}_t\}_{t\geq 0}$ is the *transition stochastic semigroup* of the Markov chain (see Supplementary Material) and the evolution of $\{\varepsilon_t\}_{t\geq 0}$ depends on $\{\mathbf{P}_t\}_{t\geq 0}$ and the initial distribution π of ε_0. Let us also assume from now on that the transition stochastic semigroup $\{\mathbf{P}_t\}_{t\geq 0}$ is standard, that is, $\lim_{t\to 0} \mathbf{P}_t - \mathbf{I} = \mathbf{0}$ or

$$\lim_{t\to 0} p_{ii}(t) = 1 \qquad \textit{and} \qquad \lim_{t\to 0} p_{ij}(t) = 0 \ \textit{for } i \neq j$$

Under these assumptions on the semigroup the following result is derived from Grimmett and Stirzaker (2009) (see chapter VI, pp. 256-258):

Proposition 4 *Consider the interval $(t, t+h)$ with h small. Then*

$$\lim_{h\to 0} \frac{1}{h}(\mathbf{P}_h - \mathbf{I}) = \mathbf{G}$$

that is, there exists constants $\{g_{ij}\}$ such that

$$p_{ii}(h) \simeq 1 + g_{ii}h \qquad \textit{and} \qquad p_{ij}(h) \simeq g_{ij}h \ \ \textit{if } i \neq j \tag{37}$$

with $g_{ii} \leq 0$ and $g_{ij} > 0$ for $i \neq j$. The matrix $\mathbf{G} = (g_{ij})$ is called the generator of the Markov chain $\{\varepsilon_t\}_{t\geq 0}$.

Therefore, the continuous-time Markov chain $\{\varepsilon_t\}_{t\geq 0}$ has a generator $\mathbf{G}$, which can be used together with the initial probability distribution π to describe the evolution of the chain. For this purpose, the following definition will turn out to be useful:

Definition 2 *Let $\varepsilon_s = z_i$, we define the "holding time" as*

$$\mathcal{T}_i \equiv \inf\{t \geq 0 : \varepsilon_{s+t} \neq z_i\}$$

Therefore the "holding time" is a random variable describing the further time until the Markov chain changes its state. The following Proposition, derived from Grimmett and Stirzaker (2009) (see chapter VI, pp. 259-260) is crucial to understand the evolution of the chain from a generic initial state $\varepsilon_s = z_i$.

Proposition 5 *Under the assumptions on the Markov chain introduced so far, the following results hold:*

(1) The random variable $\mathcal{T}_i$ is exponentially distributed with parameter g_{ii}. Therefore,

$$p_{ii}(t) = \mathbb{P}(\varepsilon_{s+t} = z_i | \varepsilon_s = z_i) = e^{g_{ii}t}.$$

(2) If there is a jumps, the probability that the Markov chain jumps from z_i to $z_j \neq z_i$ is $-\frac{g_{ij}}{g_{ii}}$.

Through the last Proposition we can fully describe the evolution of the Markov chain and therefore we have all the ingredients to build sunspot equilibria. Before doing that we define a sunspot equilibrium as it follows:

Definition 3 ***(Sunspot Equilibrium).*** *A sunspot equilibrium is a stochastic process $\{(\tau_t, x_t, \varepsilon_t)\}_{t\geq 0}$ that solves the system of stochastic differential Equation (36), respects the inequality constraints $\tau_t, x_t > 0$, and the transversality condition.*

We are now ready to prove the following theorem on the existence of sunspot equilibria while Figure 6 provides an example of one of these equilibria.

Theorem 2 (Existence of Sunspot Equilibria). *Assume that all the conditions for indeterminacy in Theorem 1 hold and a sunspot variable is introduced in the model through the continuous-time Markov chain $\{\varepsilon_t\}_{t\geq 0}$. Then sunspot equilibria exist.*

Theorem 1 shows how to build sunspot equilibria starting from our deterministic model characterized by a unique deterministic and locally determinate BGP and a continuum of other equilibria each of them converging over time to a different ABGP.

Contrary to the standard literature where sunspot equilibria are based on the existence of local indeterminacy (see Benhabib and Farmer 1994), we find sunspot fluctuations while there exists a unique unstable BGP as well as a continuum of other equilibria converging to the ABGPs. From this point of view, our conclusions share some similarities with Farmer (2013) where expectations-driven fluctuations, in an economy with a continuum of steady states, are generated from the existence of

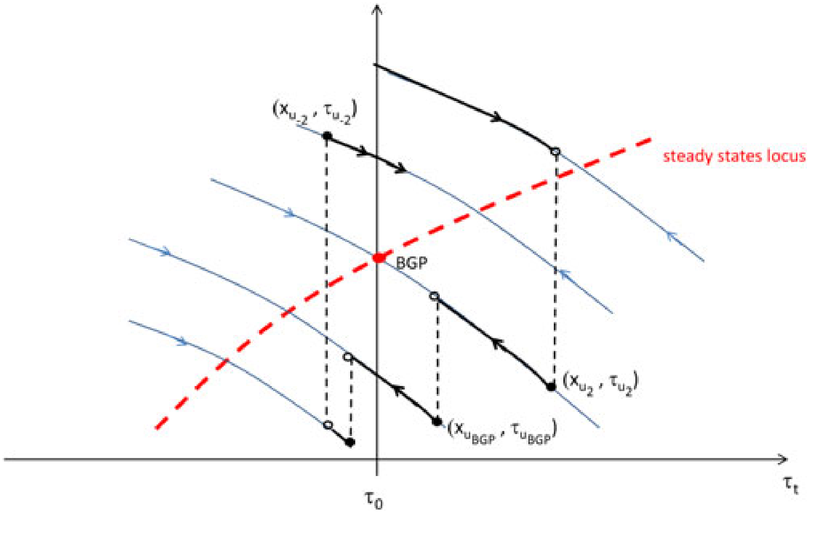

Figure 6 Example of a sunspot equilibrium

a continuum of equilibrium unemployment rates. However the presence of a continuum of steady states differs significantly in the two frameworks because in our case it crucially depends on the presence of endogenous growth and of endogenous countercyclical taxation when the government balances its budget. In Farmer (2013), the existence of a continuum of steady states comes from the fact that there is one less equation than unknown. This under-determinacy arises from the absence of markets to allocate search intensity between the time of searching workers and the recruiting activities of firms. To close the model, Farmer then needs to introduce beliefs about the future value of asset prices, measured relative to the wage, and this justifies the existence of sunspots.

The existence of sunspot fluctuations with a unique deterministic equilibrium is also obtained by Dos Santos Ferreira and Lloyd-Braga (2008) but again under a quite different mechanism. Here, the authors consider free entry oligopolistic equilibria where firms, producing under increasing returns to scale, compete in prices in contestable markets. Multiple free entry equilibria may exist, each one characterized by a number of producing firms that varies according to the (correct) conjectures of all the competitors. This multiplicity generates a static indeterminacy on which sunspot equilibria may be constructed while the intertemporal equilibrium is unique.

A direct consequence of Theorem 1 and of the description of sunspot equilibria so far, lead to the following result.

Corollary 2 ***(Stabilizing fiscal policy).*** *Assume that the fiscal policy is procyclical, i.e.* $\phi \in (0, +\infty)$, *then aggregate instability cannot emerge.*

Of course the reason behind this result is that the dynamics of the economy with pro-cyclical taxation are globally and locally determinate and therefore it is not possible to use a sunspot variable to randomize among the deterministic equilibria because there is only one of them. In fact, in this case we have two strictly positive eigenvalues and therefore two explosive paths to be ruled out by setting $b_1 = b_2 = 0$ in systems (31) and (32). In this case the economy has no transitional dynamics and the only solution is the balanced growth path solution described in the previous section.

7.2 Policy implications

In order to check the empirical plausibility of our results, we provide now a simple numerical exercise. On the basis of quarterly data, we consider the benchmark parameterization $(\alpha, \delta, \rho) = (1/3, 0.025, 0.01)$. Concerning the elasticity of intertemporal substitution in consumption, there is no agreement in the empirical literature about its precise value. While early studies such that Campbell (1999), Kocherlakota (1996) and Vissing-Jorgensen (2002) suggest quite low values, more recent contributions, for example, Mulligan (2002), Vissing-Jorgensen and Attanasio (2003), and Gruber (2013), provide robust estimates of this elasticity between 1 and 2. In light of all these studies, we may assume that a plausible range for ϵ_{cc} is (0.5, 2). Consumption tax rates have been estimated by Mendoza *et al.* (1994) and (1997),[19] and more recently by Volkerink and De Haan (2001). They provide ranges of tax rates for each OECD countries as given in Table 1:[20]

Choosing the value $A = 2.3$ compatible with Propositions 1 and 3 implies a minimal level of tax rate for indeterminacy such that $\underline{\tau} = 0.139$, with $\hat{\gamma} = 1.84\%$ when $\sigma = 2$, $\hat{\gamma} = 2.39\%$ when $\sigma = 1$ and $\hat{\gamma} = 2.5\%$ with $\bar{\tau}(\sigma) = 0.2$ when $\sigma = 0.6$. Except for Japan, the US, and Switzerland, we then

[19] Updated estimates up to 1996 are available online from the authors.

[20] Tables 1 and 2 are taken from Nourry, Seegmuller, and Venditti (2013).

Table 1 Consumption tax rates of OECD countries

$\tau \in (0.05, 0.1)$	Japan, US, Switzerland
$\tau \in (0.1, 0.15)$	Australia, Canada, Italy, Spain
$\tau \in (0.15, 0.2)$	Belgium, Germany, Greece, Netherlands, New Zealand, Portugal, UK
$\tau \in (0.2, 0.25)$	Austria, France, Iceland, Luxembourg, Sweden
$\tau \in (0.25, 0.3)$	Finland, Ireland
$\tau \in (0.3, 0.35)$	Denmark, Norway

conclude that, depending on the value of σ, our conditions on the tax rate for the existence of aggregate instability are empirically plausible for most OECD countries.

Let us finally consider empirically plausible values of the elasticity ϕ of the consumption tax rate. To our knowledge there is no direct estimates of this parameter available in the literature. However, Lane (2003) provides some empirical estimates of the elasticity of government expenditures with respect to output growth. Since consumption is almost perfectly correlated with output, we use Lane's results as a proxy for the elasticity of government expenditures with respect to consumption, as given by

$$\eta \equiv \mathcal{G}'(c)c/\mathcal{G}(c) \tag{38}$$

In the following we will say that public expenditures are counter(pro)-cyclical when $\eta < 0$ ($\eta > 0$). Consider then the balanced-budget rule (12) together with the fiscal rule (17). We get

$$\mathcal{G}(c) = \tau_0 \left(c_0 e^{\gamma t}\right)^{-\phi} c^{1+\phi}$$

and thus

$$\eta = 1 + \phi \text{ or equivalently } \phi = \eta - 1$$

From Lane's approximation of η we can then evaluate empirically plausible values of ϕ. The tax rule (17) is therefore counter-cyclical for $\eta < 1$, pro-cyclical for $\eta > 1$ and constant for $\eta = 1$.

Using annual data over 1960–1998 for 22 OECD countries, Lane (2003) shows that in most OECD countries government spending is countercyclical, that is, $\eta \leq 0$.

We conclude that the elasticity of the consumption tax rate ϕ is negative implying countercyclicality for most OECD countries. As a whole we have shown that the existence of aggregate instability and sunspot fluctuations driven by consumption tax rates is empirically plausible for a large set of OECD countries. According to the predictions of our model (see Corollary 2), the output volatility of these countries would reduce in response to a switch from a countercyclical to a procyclical taxation.

Table 2 Elasticity of government expenditure of OECD countries

$\eta \in (-0.1, 0]$	Australia, Greece, Japan, New Zealand, Spain
$\eta \in (-0.2, -0.1)$	Austria, US
$\eta \in (-0.4, -0.2)$	Belgium, Finland, Germany, Netherlands, Sweden
$\eta \in (-0.7, -0.4)$	Canada, France, Italy, UK

8 Income tax vs consumption tax

One could think that our main results strongly rely on the consideration of a consumption tax. This is actually not the case. If we assume that government expenditures are financed through a tax on income, that is, $\mathcal{G} = \tau(\tilde{y})y$, with $\tilde{y} = ye^{\gamma t}$, and that the elasticity of the tax rate with respect to de-trended output is constant and equal to ϕ, then we find similar results. Indeed, the model is basically the same except that the capital accumulation equation becomes now

$$\dot{k} = (1-\tau)Ak^{\alpha}\mathcal{G}^{1-\alpha} - \delta k - c$$

We consider here

$$\frac{\dot{\tau}}{\tau} \equiv \phi\left(\frac{\dot{y}}{y} - \gamma\right) \tag{39}$$

and thus the fiscal rule

$$\tau(t) \equiv \tau_0\left(\frac{\tilde{y}(t)}{y_0}\right)^{\phi} = \tau_0\left(\frac{y(t)}{y_0 e^{\gamma t}}\right)^{\phi} \tag{40}$$

where $B = \tau_0/y_0^{\phi}$. Moreover, solving $\mathcal{G} = \tau y = \tau A k^{\alpha}\mathcal{G}^{1-\alpha}$ with respect to $\mathcal{G}$ gives $\mathcal{G} = (\tau A)^{1/\alpha}k$. From the corresponding first order conditions, straightforward computations then lead to the following dynamical system

$$\frac{\dot{x}}{x} = \frac{1}{\sigma}\left[(1-\tau)A(\tau A)^{\frac{1-\alpha}{\alpha}}(\alpha-\sigma) - \delta(1-\sigma) - \rho + \sigma x\right] \tag{41}$$

$$\frac{\dot{\tau}}{\tau} = \frac{\phi\alpha}{\alpha - \phi(1-\alpha)}\left[(1-\tau)A(\tau A)^{\frac{1-\alpha}{\alpha}} - \delta - \gamma - x\right] \tag{42}$$

with $x \equiv \frac{c}{k}$. Along a BGP as defined by (20), we find again that τ is constant and equal to its initial value τ_0. As in the case with a consumption tax rate, we get the following results:

Proposition 6 *Given any initial condition of capital $k_0 > 0$ and the tax rate $\tau_0 > 0$, there exist $\bar{A} > \underline{A} > 0$, $\bar{\tau} > \underline{\tau} > 0$ and $\bar{\tau}(\sigma) > \underline{\tau}(\sigma) > 0$ such that if one of the following conditions holds:*

(i) $\sigma \geq 1$, $A > \underline{A}$ and $\tau_0 \in (\underline{\tau}, \bar{\tau})$,

(ii) $\sigma \in (0,1)$, $A \in (\underline{A}, \bar{A})$ andn $\tau_0 \in (\underline{\tau}(\sigma), \bar{\tau}(\sigma))$,

there is a unique balanced growth path where the ratio of consumption over capital is constant and equal to

$$\hat{x} = \frac{1}{\alpha}\left[\delta(1-\alpha-\sigma) + \rho(1-\sigma) + (\sigma-\alpha)\alpha(1-\hat{\tau})A(\hat{\tau}A)^{\frac{1-\alpha}{\alpha}}\right]$$

and the growth rate of the economy is

$$\hat{\gamma} = \frac{1}{\sigma}\left[\alpha(1-\hat{\tau})A(\hat{\tau}A)^{\frac{1-\alpha}{\alpha}} - \rho - \delta\right]$$

with $\hat{\tau} = \tau_0$. Moreover, for any given initial conditions (k_0, τ_0) satisfying the previous conditions, there is no transitional dynamics, that is, there exists a unique $c_0 = k_0\hat{x}$ such that the economy directly jumps on the BGP from the initial date $t = 0$.

Under an income tax, we also observe that the unique BGP is not the only possible long run outcome of the model and we can similarly define ABGPs:

Definition 4 *An ABGP with income tax is any path* $(x(t), \tau(t))_{t\geq 0} = (x^*, \tau^*)$ *such that:*

(a) τ^* *is a positive arbitrary constant sufficiently close to (but different from)* τ_0*;*
(b) (x^*, τ^*) *is a steady state of (41)-(42) with*

$$x^* = \frac{1}{\alpha}\left[\delta(1-\alpha-\sigma) + \rho(1-\sigma) + (\sigma-\alpha)\alpha(1-\tau^*)A(\tau^* A)^{\frac{1-\alpha}{\alpha}}\right] \quad (43)$$

$$\gamma^* = \frac{1}{\sigma}\left[\alpha(1-\tau^*)A(\tau^* A)^{\frac{1-\alpha}{\alpha}} - \rho - \delta\right]. \quad (44)$$

(c) (x^*, τ^*) *satisfies the transversality condition.*

As previously, the existence of an equilibrium path converging to an ABGP is associated with the existence of consumers' beliefs that are different from those associated with the BGP. Under some conditions on ϕ, we can again show that the consumers may decide a consumption path which makes this belief self-fulfilling.

Proposition 7 *Given any initial conditions* $k_0 > 0$ *and* $\tau_0 > 0$*, let* $\tau_{inf} = \tau_0 - \epsilon > 0$ *and* $\tau_{sup} = \tau_0 + \varepsilon$ *with* $\epsilon, \varepsilon > 0$ *small enough. Consider* $\bar{A} > \underline{A} > 0$*,* $\bar{\tau} > \underline{\tau} > 0$ *and* $\bar{\tau}(\sigma) > \underline{\tau}(\sigma) > 0$ *as defined by Proposition 6. Then, if* $\phi \in (-\infty, 0) \cup \left(\frac{\alpha}{1-\alpha}, +\infty\right)$ *and one of the following conditions holds:*

(i) $\sigma \geq 1$*,* $A > \underline{A}$*,* $\tau_{inf} > \underline{\tau}$ *and* $\tau_{sup} < \bar{\tau}$*,*
(ii) $\sigma \in (0, 1)$*,* $A \in (\underline{A}, \bar{A})$*,* $> \underline{\tau}(\sigma)$ *and* $\tau_{sup} < \bar{\tau}(\sigma)$*,*

then there is a continuum of equilibrium paths, indexed by the letter j, departing from (τ_0, x_0^j)*, each of them converging to a different ABGP* (τ^{*j}, x^{*j}) *with* $\tau^{*j} \in (\tau_{inf}, \tau_{sup})$*, that is, the dynamics of the economy is globally, but not locally, indeterminate.*

The main difference with the results derived under a consumption tax relies on the fact that global indeterminacy is not only possible under a countercyclical tax rate but is also compatible with a sufficiently pro-cyclical tax rate. However, as shown by Lane (2003), countercyclicality is the most empirically relevant configuration. A policy implication is here that a slightly pro-cyclical income tax rate would be sufficient to stabilize the economy. We have then proved that all our results do not depend on the specific consumption tax but also hold under an income tax.

9 Conclusion

We have considered a Barro-type (1990) endogenous growth model in which a government provides as an external productive input a constant stream of expenditures financed through consumption taxes and a balanced-budget rule. In order to have a constant tax on a balanced growth path, the tax rate needs to depend on de-trended consumption and thus becomes a state variable with a given initial condition. We also consider a representative household characterized by a CRRA utility function and inelastic labor. Such a formulation is known to rule out the existence of endogenous fluctuations in a standard stationary framework (see Giannitsarou 2007).

We have proved that there exists a unique Balanced Growth Path (BGP) along which the common growth rate of consumption, capital, GDP, and government spending is constant. Moreover, as in the Barro (1990) model, there is no transitional dynamics with respect to this unique BGP. However, we have shown that the BGP is not the unique long run solution of our model. Indeed, if the tax rule is counter-cyclical with respect to consumption, for any arbitrary initial value of the tax rate, close enough to its initial condition, there exists a corresponding value for the tax rate, consumption, capital, and the constant growth rate that can be an asymptotic equilibrium of our economy, namely an Asymptotic Balanced Growth Path (ABGP). An ABGP is not itself an equilibrium as it does not respect the initial conditions. However, some transitional dynamics exist with a unique equilibrium path converging toward this ABGP, and we prove that there exists a continuum of such ABGP and of equilibria with each of them converging over time to a different ABGP.

The existence of an equilibrium path converging to an ABGP is associated with the existence of consumers' beliefs that are different from those associated with the BGP. Indeed, they may believe that the consumption tax profile will not remain constant but rather change over time and eventually converge to a positive value different from the initial condition. Based on this property, we prove the existence of sunspot equilibria and thus that endogenous sunspot fluctuations may arise under a balanced-budget rule and consumption taxes although there exists a unique underlying BGP equilibrium. Moreover, a simple numerical exercise shows that our conclusions are compatible with empirically realistic values of the main structural parameters and tax rates for many OECD countries. We have finally proved that all our results do not depend on the consideration of a consumption tax but are also fully compatible with an income tax.

References

Abad, N., T. Seegmuller, and A. Venditti (2017), "Non-separable preferences do not rule out aggregate instability under balanced-budget rules: A note," *Macroeconomic Dynamics* **21**, 259–77.

Angeletos, G.-M., and J. La'O (2013), "Sentiments," *Econometrica* **81**, 739–79.

Bambi, M., and A. Venditti (2018), "Time-varying consumption tax, productive government spending, and aggregate instability," AMSE Working Paper 2018-29, Aix–Marseille School of Economics, available at https://www.amse-aixmarseille.fr/en/research/working-papers.

Barro, R. (1990), "Government spending in a simple model of endogeneous growth," *Journal of Political Economy* **98**, 103–25.

Benhabib, J., and R. Farmer (1994), "Indeterminacy and increasing returns," *Journal of Economic Theory* **63**, 19–41.

Benhabib, J., P. Wang, and Y. Wen (2015), "Sentiments and aggregate demand fluctuations," *Econometrica* **83**, 549–85.

Benhabib, J., and Y. Wen (1994), "Indeterminacy, aggregate demand, and the real business cycle," *Journal of Monetary Economics* **51**, 503–30.

Benhabib, J., K. Nishimura, and T. Shigoka (2008), "Bifurcation and sunspots in the continuous time equilibrium model with capacity utilization," *International Journal of Economic Theory* **4**, 337–55.

Campbell, J. (1999), "Asset prices, consumption and the business cycle," J. B. Taylor, and M. Woodford, eds, *Handbook of Macroeconomics*, 1231–1303, Amsterdam: North-Holland.

Cazzavillan, G. (1996), "Public spending, endogenous growth, and endogenous fluctuations," *Journal of Economic Theory* **71**, 394–415.

Dos Santos Ferreira, R., and T. Lloyd-Braga (2008), "Business cycles with free entry ruled by animal spirits," *Journal of Economic Dynamics and Control* **32**, 3502–19.

Drugeon, J.-P., and B. Wigniolle (1996), "Continuous-time sunspot equilibria and dynamics in a model of growth," *Journal of Economic Theory* **69**, 24–52.

Farmer, R. (2013), "Animal spirit, financial crises and persistent unemployment," *Economic Journal* **123**, 317–40.

Giannitsarou, C. (2007), "Balanced-budget rules and aggregate instability: The role of consumption taxes," *Economic Journal* **117**, 1423–35.

Grimmett, G., and D. Stirzaker (2009), *Probability and Random Processes*, Third Edition, New York: Oxford University Press.

Gruber, J. (2013), "A tax-based estimate of the elasticity of intertemporal substitution," *Quarterly Journal of Finance* **3**, 1–20.

Jaimovich, N., and S. Rebelo (2008), "Can news about the future drive business cycles?" *American Economic Review* **99**, 1097–118.

Jones, L., and R. Manuelli (1997), "The sources of growth," *Journal of Economic Dynamics and Control* **21**, 75–114.

Kocherlakota, N. (1996), "The equity premium: It's still a puzzle," *Journal of Economic Literature* **36**, 42–71.

Lane, P. (2003), "The cyclical behaviour of fiscal policy: Evidence from the OECD," *Journal of Public Economics* **87**, 2661–75

Linnemann, L. (2008), "Balanced-budget rules and macroeconomic stability with non-separable utility," *Journal of Macroeconomics* **30**, 199–215

Lloyd-Braga, T., L. Modesto, and T. Seegmuller (2008), "Tax rate variability and public spending as sources of indeterminacy," *Journal of Public Economic Theory* **10**, 399–421.

Mendoza, E., G. M. Milesi-Ferretti, and P. Asea (1997), "On the ineffectiveness of policy in altering long-run growth: Harberger's superneutrality conjecture," *Journal of Public Economics* **66**, 99–126.

Mendoza, E., A. Razin, and L. Tesar (1994), "Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption," *Journal of Monetary Economics* **34**, 297–323.

Mulligan, C. (2002), "Capital interest and aggregate intertemporal substitution," NBER Working Paper 9373.

Nishimura, K., C. Nourry, T. Seegmuller, and A. Venditti (2013), "Destabilizing balanced-budget consumption taxes in multi-sector economies," *International Journal of Economic Theory* **9**, 113–30.

Nishimura, K., and T. Shigoka (2006), "Sunspots and Hopf bifurcations in continuous time endogenous growth models," *International Journal of Economic Theory* **2**, 199–216.

Nourry, C., T. Seegmuller, and A. Venditti (2013), "Aggregate instability under balanced-budget consumption taxes: A re-examination," *Journal of Economic Theory* **149**, 1977–2006.

Schaechter, A., T. Kinda, N. Budina, and A. Weber (2012), "Fiscal rules in response to the crisis," IMF Working Paper 12/187.

Schmitt-Grohe, S., and M. Uribe (1997), "Balanced-budget rules, distortionary taxes, and aggregate instability," *Journal of Political Economy* **105**, 976–1000.

Shigoka, T. (1994), "A note on Woodford's conjecture: constructing stationary sunspot equilibria in a continuous time model," *Journal of Economic Theory* **64**, 531–40.

Vissing-Jorgensen, A. (2002), "Limited asset market participation and the elasticity of intertemporal substitution," *Journal of Political Economy* **110**, 825–53.

Vissing-Jorgensen, A., and O. Attanasio (2003), "Stock-market participation, intertemporal substitution and risk aversion," *American Economic Review Papers and Proceedings* **93**, 383–91.

Volkerink, B., and J. De Haan (2001), "Tax ratios: A critical survey," *Empirica* **29**, 209–24.