An RML algorithm for retrieval of sinusoids with cascaded notch filters

Nacer K M'Sirdi, Ioan Doré Landau

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AN RML ALGORITHM FOR RETRIEVAL OF SINUSOIDS WITH CASCADED NOTCH FILTERS

N.K. M'SIRDI[°*], N. K. TJOKRONEGORO* AND I.D. LANDAU*

° LABORATOIRE D'AUTOMATIQUE ET ROBOTIQUE Univ. P. M. CURIE (PARIS 6)
* LABORATOIRE D'AUTOMATIQUE DE GRENOBLE (I.N.P.G.)
et GRECO SYSTEMES ADAPTATIFS (C.N.R.S)
(E.N.S.I.E.G.) B.P. 46 - 38402 ST MARTIN D'HERES FRANCE

ABSTRACT:

To provide the frequencies estimation , Adaptive Notch Filters [1-4] can be used by implementation in a cascad blocks of second order cells. Some strategies to track time varying parameters can be used [6]. Then, in a second step the estimated frequencies are used to provide the estimation of amplitudes and phases in a recursive manner. The proposed recursive algorithm consists in two steps. The first step involves a Maximum Likelihood algorithm to adapt the cascaded filters parameters [1][4], which will provide the frequencies estimates. The second step uses the last estimates, and then the estimations of amplitudes and phases are given by a Recursive Least Squares algorithm[6]. The proposed algorithm is asymptotically consistent and robust faced with the neglected dynamics. In case of time varying signals, its tracking capabilities insure the goodness of the estimations. The accuracy of estimation is better with the RML than with the ELS method for which an upper limit of the debiasing parameter is crucial in order to have a convergencewithoutlocalinstabilities.

I-INTRODUCTION:

Our main interest in this contribution, is the retrieval of sinusoidal signals in noise. AR and ARMA modelling techniques have powerfull potentialities and wellsuited for signal spectrum analysis. However, when the noise level is high and / or when the signal model has poles near or on the unit circle, serious problems arise[1]. In the narrowband case, successfull applications have been realized with AR modelling [2-3]. The high order autoregressve FIR filter is shown to be asymptotically equivalent to an Adaptive Notch Filter (**ANF**). The noise effect is reduced by increasing the order of of the AR filter at the expense of computational cost. Thus the a priori knowlege of a narrowband leads to the **Notch Filter** Structure (**NFS**) with constrained poles and zeros. This has recently been proposed in extensive simulations [4-7]. A theoretical analysis have been presented in [5][7] and the applicable estimation methods are reviewed in [8].

These last structures introduce in **ParameterAdaptationAlgorithms** (**PAA**) some nolinearity which may give some problems in the transient period of the estimation. The cascade implementation of second order have been claimed possible. **Fast Least Squares** (**FLS**) estimation algorithm case have been considered and studied in simulations[9]. The main interest of the ANF Cascade form is to simplify computation of the estimated frequencies and in case of independly time varyingfrequencies, a second order filter appears faster to track variation than a higher order one.

In section II of this paper we develop an implementation of cascaded ANF. The **Prediction Error Method** (**PEM**) is applied leading to a **Recursive Maximum Likelihood** (**RML**) algorithm. This PAA allows the computation of the estimated frequencies. In a second step, using these estimates, a **Recursive Least Squares** (**RLS**) algorithm is employed for the estimation of amplitudes and phases. The analysis and performance evaluation of these algorithms is presented in section III.

II-Recursive estimation algorithm and filter structure.

2.1 Frequency estimation whit cascaded ANF

The recursive estimation procedure consists of two stages. The first one involves an ANF in cascade form for the estimation of the frequencies. The other one uses an adaptive algorithm to provide the amplitudes and phases estimations. The involved signals may be modeled as follows where v_k is the noise disturbance:

$$y_k = \sum_{i=1}^{p} c_i \sin(\omega_i . k + \beta_i) + v_k \tag{1}$$

Adaptive Notch Filters are very well suited for estimation of the sinusoidal component frequencies. Let us consider the notch filter transfert function in cascade form:

$$H(z^{-1}) = \prod_{i=1}^{p} H_i(z^{-1}) = \prod_{i=1}^{p} [\, (1+a_i z^{-1} + z^{-2}) / (1+a_i r z^{-1} + r^2 z^{-2})] \tag{2}$$

With no loss of generality we assume identical bandwidth

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for the notches. If the frequencies are all independant we can write for all j=1...p:

$$\tilde{y}_k^j = \prod_{\substack{i=1 \\ i \neq j}}^{p} H_i(q^{-1}) \cdot y_k \qquad (3)$$

and the prediction error is: $\varepsilon_k = H_j(q^{-1})\, \tilde{y}_k^j$ (4)

The last equation presents a second order notch filter. Thus independence between frequencies yields independence between parameters a_i of each second order cell. Then each cellcan be adapted independently of the others afterprefiltering the signal by the others. If we suppose that the filters $H_i(z^{-1})$ with $i \neq j$ and i=1...p have converged then the filter $H_j(z^{-1})$ will remove the remaining component. Thus there exits a unique global minimum for the quadratic criterion. Following the Prediction Error technique for the estimation algorithm, we obtain the gradient:

$$\psi_{k-1}^j = -\frac{d\varepsilon_k}{da_j} = \frac{(1-r)(1-rq^{-2})}{(1+r.a_j q^{-1}+r^2 q^{-2})^2}\, \tilde{y}_{k-1}^j \qquad (5)$$

The RML algorithm may be summarized by the following equations:

$$\begin{aligned} \hat{a}_j(k) &= \hat{a}_j(k-1) + F_{k-1}\, \psi_{k-1}^j\, \varepsilon_k \\ F_k &= F_{k-1} \big/ (\lambda + F_{k-1} \psi_{k-1}^{j2}) \end{aligned} \qquad (6)$$

An approximation must be done to reduce computational complexity. It yields an Approximate RML algorithm. The computational complexity is essentally du to the computation of the gradient. We can then use the formula:

$$\psi_{k-1}^j = \frac{f_j(k-1) - r.\varepsilon_{k-1}}{1 + r\, a_j q^{-1} + r^2 q^{-2}} \qquad (7)$$

With $f_j(k-1)$ computed as follows:

$$\begin{aligned} y_k^j &= \prod_{\ell=j+1}^{p} H_\ell(q^{-1})\, y_k \;;\; j=p-1,\dots,1 \\ f_j(k) &= y_k^j - y_k^{j-1} \end{aligned} \qquad (8)$$

2.2 Amplitude and phase estimation with WRLS:

In this second step we assume the frequencies known and use a Weighted Recursive Least Squares algorithm to estimate the amplitudes and phases as in [10]. The frequency estimates are provided by the first step.

Model (1) of the signal may be written:

$$y_k = \sum_{\ell=1}^{p} \left(g_\ell \cos \omega_\ell . k + h_\ell \sin \omega_\ell . k \right) + v_k \qquad (9)$$

where the amplitudes C_ℓ and phases β_ℓ (ℓ=1...p) are given by:

$$C_\ell = (g_\ell^2 + h_\ell^2)^{1/2} \text{ and } \quad tg\, \beta_\ell = \frac{g_\ell}{h_\ell} \qquad (10)$$

We can define the parameter vector and the observation vector as:

$$\theta_k = [g_1, \dots, g_p, h_1, \dots, h_p]^T \qquad (11)$$

$$\text{and} \quad \phi_k = [\cos\omega_1 k, .., \cos\omega_p k, \sin\omega_1 k, .., \sin\omega_p k]^T$$

The parameter vector is estimated with the WRLS algorithm:

$$\begin{aligned} \varepsilon_k^o &= y_k - \hat{\theta}_{k-1}^T \phi_k \\ F_k &= \frac{1}{\lambda}\left[F_{k-1} - \frac{F_{k-1}\phi_k \phi_k^T F_{k-1}}{\lambda + \phi_k^T F_{k-1} \phi_k} \right] \\ \hat{\theta}_k &= \hat{\theta}_{k-1} + F_k \phi_k \varepsilon_k^o \end{aligned} \qquad (12)$$

In equation (12) the filtered version of the signal produced by the first stage, can be used.

III- Analysis and Performance Evaluation

3.1 Convergeace of the ANF

a) Second order ANF (RML)

Let us first consider the case of a single cell $H_j(z)$ (see (4)) driven by the signal defined in (3) and assume the filters $H_i(z)$ i=1..j-1,j+1,..n have converged to their optimal value. Thus $\tilde{y}_k^j$ is composed by a single frequency ω_j plus additive independant noise. The bandwidth are large enough to remove(or attenuate) the other frequencies.

Taking for the signal an AR model with poles on the unit circle and applying the RML algorithm, the a posteriori error can be written:

$$\varepsilon_k = \frac{\hat{A}(rq^{-1})}{A(rq^{-1})} (\hat{a}_j - a_j)\, \psi_{k-1}^j + \frac{v_k}{\hat{A}(rq^{-1})} \qquad (13)$$

with $\tilde{y}_k^j = -a_j \tilde{y}_{k-1}^j - \tilde{y}_{k-2}^j + v_k$

and

$$\begin{aligned} A(rq^{-1}) &= 1 + r.a_j q^{-1} + r^2 q^{-2} \\ \hat{A}(rq^{-1}) &= 1 + r.\hat{a}_j q^{-1} + r^2 q^{-2} \end{aligned} \qquad (14)$$

It is well known that the RML algorithm needs a stability monitoring in general case. For the ANF this procedure can be removed in virtu of the following Lemma.

Lemma 1: Let r by exponentially time varying, from 0 to one, according to: $r_k = r_d \cdot r_{k-1} + (1-r_0) \cdot r_f$,

Then there exit an r_d, r_0 and r_f such that $\hat{A}(r_k . q^{-1})$ is infinitely often stable.

proof : we can take $r_f <$ inverse of the maximum modulus of the unstable poles of $A(q^{-1})$.

Now applying the theoretical bakground in [11] and in particular Theorem 4 of [11] leads to the following:

Theorem 1: Under the assumptions

A1: is infinitely often stable

A2: for some $\hat{a}_j$ fixed $\psi_k^j, \varepsilon_k, \omega_k = \frac{v_k}{A(rq^{-1})}$ are stationary

A3: ω_k is independent of ψ_{k-1}^j

and if the following transfert function is SPR: $\frac{\hat{A}(rz^{-1})}{A(rz^{-1})} - \frac{1}{2}$

Then one has the following properties:

with $D_c = \left\{ a \;/\; (a - \hat{a}_j)\, \psi_{k-1}^j = 0 \right\}$

Convergence Domain.

$$\begin{aligned} &P1: \lim(\varepsilon_k - \omega_k) = 0 \text{ wp1} \\ &P2: \lim(\hat{a}_j) \in D_c \text{ wp1} \end{aligned} \qquad (15)$$

where D_c is the convergence domain

Lemma 2: $\forall \hat{a}_j, a_j$ there exists an r such that $\frac{\hat{A}}{A} - \frac{1}{2}$ is SPR.

The last condition on r is less restrictive than the one for the stability. Lemmas 1 and 2 allow us to assert the global stability of the ANF if r is appropriately chosen such that $\hat{A}(r.q^{-1})$ is always stable during estimation.

b) ELS parameter adaptation algorithm:

If the ELS estimation method is applied (13) becomes [5]:

$$\varepsilon_k = \frac{1}{A(rq^{-1})} (\hat{a}_j - a_j)\, \psi_{k-1}^j + \omega_k \qquad (16)$$

The SPR condition is more restrictive than the stability one on r and Theorem 1 cannot be applied in all cases. There exits an upper limit ℓ for the debiasing parameter in order to have the SPR condition. This limit ℓ depend of the signal frequency and limits the accuracy of the estimates.

Thus the local stability cannot be etablished although the global stability can be ensured.

c) Uniqueness of the RML estimates:

In theorem 1 we have asymptotically wp1 (P1):

$$\varepsilon_k = \frac{1}{A(rq^{-1})} v_k \qquad (17)$$

Equation (4) and (14) yield:

$$\varepsilon_k = \frac{\hat{A}(q^{-1})}{\hat{A}(rq^{-1})} \tilde{y}_k^j = \frac{\hat{A}(q^{-1})}{\hat{A}(rq^{-1})} \cdot \frac{v_k}{A(q^{-1})} \qquad (18)$$

It follows from (17) and (18):

$$\frac{\hat{A}(q^{-1})}{\hat{A}(rq^{-1})} = \frac{A(q^{-1})}{A(rq^{-1})} \qquad (19)$$

and then $\hat{a}_1 = a_1$ with probability one (wp1) if $r \neq 1$.

Theorem 2: If the order is correct the RML estimates for ANF are unique. ∎

The main results conserning ANF are:

i) the global convergence can be ensured with an appropriate choice of the debiasing parameter without any stability monitoring.

ii) The RML estimates are unique.

For p cells in cascade adapted along (3_6), each cell will converge near to a local minimum [5]. The filtering (3) will make these minima distinct removing, for each cell, the other frequencies.

3.2 Amplitude and phase estimates:

Exponential convergence of the amplitude and phase estimates is garanteed by the following theorem which uses the concept of persistent excitation [12].

Theorem 3: The RLS estimation scheme (12) with exponential forgetting factor is exponentially stable.

The proof of this theorem can be conducted in the same lines as in [12]. The regressors are composed by p sinusoids having different frequencies. Then the persistent excitation condition is satisfyed:

$$0 < \alpha I \leqslant \sum_{k=j}^{j+s} \phi_k \phi_k^T \leqslant \beta I < \infty \qquad (20)$$

Equation (20) yields by lemma 1 of [12]:

$$0 < \frac{\alpha(\lambda^{-1} - 1)}{\lambda^{-(s+1)} - 1} I \leqslant F_{k-1}^{-1} \leqslant \frac{\beta}{1 - \lambda^{s+1}} + o(\lambda^k) \qquad (21)$$

The adaptation gain is bounded and will never be zero (tracking capabilities). Finally reasoning as in [12] we obtain.

$$\|\hat{\theta}_k\|^2 \leqslant \frac{\lambda^{-(s+1)} - 1}{\alpha(\lambda^{-1} - 1)} \lambda^k . \lambda_{max}(F_0^{-1}) \|\theta_0\|^2 \qquad (22)$$

IV- Conclusion

The main results demonstrate the good performances observed in simulation for the ANF. The exponential convergence of the WRLS estimates of amplitudes and phases is proved by use of the theoretical backgroung on persistent excitation [12].

These results are very important for the time varying systems or in case of neglected dynamics due to the resulting robustness of the algorithms. The implementation form studied here is computationally attractive and robust also when frequency, amplitude or phase is time varying.

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