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Non-linear Control based on State Estimation for the Convoy of Autonomous Vehicles

M.Mahmoud Mohamed Ahmed, Nacer K. M'Sirdi and Aziz Naamane

Abstract—In this paper, a longitudinal and lateral control approach based on a nonlinear observer is proposed for a convoy of autonomous vehicles to follow a desired trajectory. To authors best knowledge, this topic has not yet been sufficiently addressed in the literature. The modeling of the vehicles convoy is revisited using robotic method, for simulation purposes and control design. With these models, a sliding mode observer is proposed to estimate the states of each vehicle in the convoy from the available sensors, then a sliding mode control based on this observer is used to control the longitudinal and lateral movement. The validation and performance evaluation are done using the well-known driving simulator Scanner-Studio. The results are presented for different maneuvers of 5 vehicles.

Keywords—Autonomous Vehicles, Convoy, Non-linear Control, Non-linear Observer, Sliding Mode.

I. Introduction

In recent decades, research on the fleet of autonomous vehicles is the subject of particular attention and is of interest to more and more laboratories, researchers, manufacturers and automotive suppliers. Indeed, several research projects, national or international, have been set up to respond to the problems related to the evolution of convoy transport means, their control approaches and their safety. Examples include projects, we can mention the project of AutoNet2030, SARTE and Chauffeur [1], [2].

Several research works have developed models and controls to control a convoy of vehicles to follow a leader who is driven by a driver or automatic. The double integrator linear model is the most used model for the longitudinal control of the convoy. It neglects many parameters of the system; the vehicle is represented by a dual integrator [3]. All vehicles are considered independent and it is only the control that links the controlled vehicle to the previous one by trying to follow it. Another model in [4], the vehicles are modeled in longitudinal by a system of the second order (double integrator) moving on a straight line and their environment (neighbors) reacted, on each side, by the application of a damping force and a stiffness force. A proposal in [5] to transform the kinematic and dynamical model coupled with a transformation of the chains which transforms this nonlinear model to a linear double integrator model. Several laws of lateral and longitudinal control of the convoy based on a bicycle model are proposed in [6]. A control approach based on the kinematic model has been proposed in [7], [8] the motion of the vehicle is controlled by a distribution algorithm based on the relative error of the previous vehicle and the position.

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In the literature most Leader-Follower control approaches belong to the local control category [1], [9], the driven vehicle slaves to the data of its predecessor. This control approach is easy to implement and requires very little information exchange between the different vehicles in the convoy. For this approach, the stability of the movements of the convoy defeated by the accumulation of errors along the convoy chain, which causes oscillations due to accumulated errors [2], [10]. Another overall global control architecture uses the information of all vehicles of the convoy or part, for example, the control referenced on the previous vehicle and the leader [11]. This control approach is divided into two categories, using either a centralized or decentralized architecture [5]. For the centralized architecture, the control law applied to each vehicle of the fleet is based on the data of all the vehicles of the convoy [12]. On the other hand, the decentralized architecture [13] is based on the data of a part of the convoy, to minimize the numbers of the sensors used [14].
In the field of nonlinear observation, we can mention the work of [15], [16] who estimated the contact forces for a vehicle using the sliding mode observer. These theoretical results have been validated experimentally with practical demonstrations of its robustness.

In this work, we propose a new control approach for the convoy of autonomous vehicles that solves the problem of trajectory tracking for the longitudinal and lateral movements of the fleet. This approach coupled the longitudinal and lateral control and based on the states estimated by a nonlinear observer consider that not all states are accessible.

First we will define the model used to estimate the state and control the convoy. This model represents the longitudinal and lateral movement and the yaw angle. In the second part, we will propose a battery of local observer by sliding mode approach to estimate the states of each vehicle of the convoy. Finally, we use a Sliding Mode Control based on the states estimates by the previous observer to compute the control laws for each vehicle. The control law of the leader vehicle targets to follow a reference trajectory and the other vehicles of the convoy, the control follows a trajectory deduced from the one of the leader and neighbors with a safety distance between the vehicles to avoid collisions.
The validation of our proposed is done with the driving simulator 'Scanner Studio' [17] coupled with Matlab Simulink.

The paper is organized as follows. Section II represents the dynamic and kinematic model of a vehicle and a convoy. The Sliding Mode Observer and the study of its convergence are presented in section III. Section IV. presented the

longitudinal and lateral control of the leader in order to follow a desired trajectory, then the control of the convoy to follow the trajectory of the leader and the study of the convergence and the attractiveness of the sliding surface. The results of simulations and conduction are presented in section V and VI.

II. MODELING

A dynamic description of the fleet is considered in this part. The vehicle fleet Model to be used is related to the control approach to be applied. The control approach may require some type of models, with specific features.

A. Dynamic model

The vehicle model can be determined by the fundamental principle of dynamics or the Lagrangian method. This leads to a set of dynamic equations to describe the vehicle motion [8] [18]. The dynamic model used to control the convoy is proposed in [19] with assumptions.

The vehicle is represented in the Fig. 1 with the following variables in (G, x, y) the vehicle reference frame. G is the gravity center.

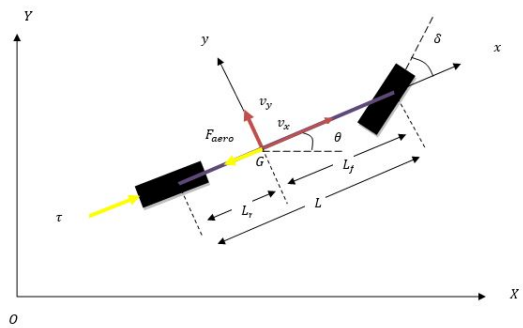

Fig. 1. The Vehicle Description

L_f is distance from the front wheel to G .
L_r: is the distance from the rear wheel center to G.
m, I_z: the mass and Inertia Moment of the vehicles.
m_w, I_w :the mass and the rotational inertia of the wheel.
$\dot{x}, v_x$: longitudinal vehicle velocity along x axis.
$\dot{y}, v_y$: lateral velocity (axis y).
θ: yaw angle and $\dot{\theta}$: yaw rate.
$a_x = \ddot{x} - \dot{y}\dot{\theta}$: longitudinal acceleration.
$a_y = \ddot{y} + \dot{x}\dot{\theta}$: lateral acceleration.
$C_{\alpha f}, C_{\alpha r}$: are respectively the cornering stiffness of the front and the rear wheels.
τ: driving/braking wheels torque.
δ : steering wheel angle.
$F_{aero} = \frac{1}{2}\rho c s \dot{x}^2$: aerodynamic force, where ρ, s and c : are the air density, the vehicle frontal surface and the aerodynamic constant.
R_t :radius of the tire and E: Vehicle's track.
L_3, I_3 : the interconnection between the different bodies composing the vehicle.

Before presenting the dynamic model, we have the following assumptions :

Assumption 1. The vehicle is considered to be a rear engine (only the rear wheels are motorized).
Assumption 2. The steering angle of the rear wheels is assumed to be low and the steering angles of two front wheels are assumed equal.
Assumption 3. The ground/pneumatic contact forces are estimated by a linear model.
Assumption 4. The longitudinal slip rate is assumed to be negligible.

The dynamic model of a vehicle is presented as follows:

$$\begin{cases} m_e\ddot{x} - m\dot{y}\dot{\theta} + L_3\dot{\theta}^2 + \delta(2C_{\alpha f}\delta - 2C_{\alpha f}\frac{\dot{x}(\dot{y}+L_f\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2}) + \\ F_{aero} = \frac{\tau}{R_t} \\ m\ddot{y} - L_3\ddot{\theta} + m\dot{x}\dot{\theta} + 2C_{\alpha f}\frac{\dot{x}(\dot{y}+L_f\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2} + 2C_{\alpha r}\frac{\dot{x}(\dot{y}-L_r\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2} \\ = (2C_{\alpha f} - 2\frac{I_w}{R_t^2}\ddot{x})\delta \\ I_3\ddot{\theta} - L_3\ddot{y} + 2L_fC_{\alpha f}\frac{\dot{x}(\dot{y}+L_f\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2}) - 2L_rC_{\alpha r}\frac{\dot{x}(\dot{y}-L_r\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2} \\ -L_3\dot{x}\dot{\theta} = L_f(2C_{\alpha f} - 2\frac{I_w}{R_t^2}\ddot{x})\delta - (\frac{E}{2}C_{\alpha f}\frac{E\dot{\theta}(\dot{y}+L_f\dot{\theta})}{\dot{x}^2-(\dot{\theta}E/2)^2})\delta \end{cases} \quad (1)$$

where : $m_e = m + 4\frac{I_w}{R_t^2}$, $L_3 = 2m_w(L_r - L_f)$ and $I_3 = I_z + m_wE^2$.
The model represents the longitudinal and lateral movement of the fleet and the movement of the yaw angle in the vehicle frame (G,x,y). In the following we will write this model in the robotic form.
Take position vector $q_i = [q_{1i}, q_{2i}, q_{3i}]^T = [x_i, y_i, \theta_i]^T$, this model can be written for the ith vehicle in the following form:

$$M_i(q_i).\ddot{q}_i + H_i(\dot{q}_i, q_i) = U_i \quad (2)$$

where the inertia Matrix $M_i(q_i)$ is:

$$M_i(q_i) = \begin{pmatrix} m_{e_i} & 0 & 0 \\ 0 & m_i & -L_{3i} \\ 0 & -L_{3i} & I_{3i} \end{pmatrix}$$

and the vector $H_i(\dot{q}_i, q_i)$ is equal to:

$$\begin{pmatrix} -m_i\dot{q}_{2i}\dot{q}_{3i} + L_{3i}\dot{q}_{3i}^2 + \delta_i(2C_{\alpha fi}\delta_i - 2C_{\alpha fi}\frac{\dot{q}_{1i}(\dot{q}_{2i}+L_{fi}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2}) + F_{aero_i} \\ m_i\dot{q}_{1i}\dot{q}_{3i} + 2C_{\alpha fi}\frac{\dot{q}_{1i}(\dot{q}_{2i}+L_{fi}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2} + 2C_{\alpha ri}\frac{\dot{q}_{1i}(\dot{q}_{2i}-L_{ri}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2} \\ 2L_{fi}C_{\alpha fi}\frac{\dot{q}_{1i}(\dot{q}_{2i}+L_{fi}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2}) - 2L_{ri}C_{\alpha ri}\frac{\dot{q}_{1i}(\dot{q}_{2i}-L_{ri}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2} - L_{3i}\dot{q}_{1i}\dot{q}_{3i} \end{pmatrix}$$

and the input vector $U_i = (u_{1i}, u_{2i}, u_{3i})^T$:

$$U_i = \begin{pmatrix} \frac{\tau_i}{R_{t_i}} \\ (2C_{\alpha fi} - 2\frac{I_{wi}}{R_{t_i}^2}\ddot{q}_{1i})\delta_i \\ L_{fi}u_{2i} - (\frac{E_i}{2}C_{\alpha fi}\frac{E_i\dot{q}_{3i}(\dot{q}_{2i}+L_{fi}\dot{q}_{3i})}{\dot{q}_{1i}^2-(\dot{q}_{3i}E_i/2)^2})\delta_i \end{pmatrix}$$

The input U_i controls the longitudinal movement of the fleet by the driving/braking wheels torque ($u_{1i} = \frac{\tau_i}{R_{t_i}}$) and the lateral movement by the steering wheel angle ($u_{2i} = (2C_{\alpha fi} - 2\frac{I_{wi}}{R_{t_i}^2}\ddot{q}_{1i})\delta_i$).

B. Kinematic Equations

The transformation of the velocity, from the absolute vehicle frame (G, x, y) to the velocity in the reference frame R(0,X,Y) is defined by:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} \tag{3}$$

Such as we get the kinematics of the ith vehicle:

$$\begin{cases} \dot{X}_i = \dot{x}_i \cos\theta_i - \dot{y}_i \sin\theta_i \\ \dot{Y}_i = \dot{x}_i \sin\theta_i + \dot{y}_i \cos\theta_i \end{cases} \tag{4}$$

C. Convoy Motion In The Geometric Model

The curvilinear abscissa of the ith vehicle (point M is the gravity center of each vehicle) and of the preceding vehicle is defined by S_i and S_{i-1} with S_i: the curvilinear abscissa. This abscissa is located with respect to a desired trajectory (figure 2). Let us note l_{d_i}, the desired curvilinear distance between 2 vehicles and therefore e_{s_i} is curvilinear spacing error, or difference between 2 vehicles. $e_{s_i} = S_{i-1} - S_i - l_{d_i}$.

$$e_{s_i} = \int_0^T (\dot{x}_{i-1}^2 + \dot{y}_{i-1}^2)^{\frac{1}{2}} dt - \int_0^T (\dot{x}_i^2 + \dot{y}_i^2)^{\frac{1}{2}} dt - l_{d_i} \tag{5}$$

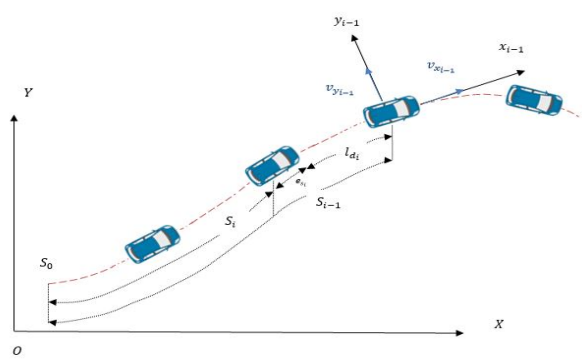

Fig. 2. Geometric description of the convoy motion

III. The Vehicles States observation

A. Vehicle State Space Model

We have as a state vector, the position and speed of each vehicle:

$$z_i = \begin{pmatrix} z_{1i} \\ z_{2i} \end{pmatrix} \tag{6}$$

with positions: $z_{1i} = [x_i, y_i, \theta_i]^T$
and velocities: $z_{2i} = [\dot{x}_i, \dot{y}_i, \dot{\theta}_i]^T$

The dynamic model of a ith vehicle of the convoy is represented in canonical forms, assuming that all the vehicle positions z_{1_i} are measurable. The dynamic model of the fleet in the equation (2) can be written in the following form:

$$\begin{cases} \dot{z}_{1i} = z_{2i} \\ \dot{z}_{2i} = f(z_{1i}, z_{2i}) + g(z_{1i})U_i \end{cases} \tag{7}$$

where : $f(z_{1i}, z_{2i}) = -M^{-1}(z_{1i})H_i(z_{1i}, z_{2i})$
and $g(z_{1i}) = M^{-1}(z_{1i})$
This canonical representation of the model will be used in the following for the observation and the control of a vehicle then the global control of the convoy.

B. First Order Sliding Mode Observer

To define the observer and estimate the states of each vehicle with the available information, we have the following assumptions:

Assumption 5. The positions ($z_{1i} = [x_i, y_i, \theta_i]^T$) are available in real-time.

Assumption 6. Model parameters $f(z_{1i}, z_{2i})$ and $g(z_{1i})$ are measurable.

The observed state noted $\hat{z}_i$ is estimated by the following block (Fig. 3), where we assume that vehicle positions z_{1i} are measurable. The observation error of the state of the system is noted $\tilde{z}_i$.

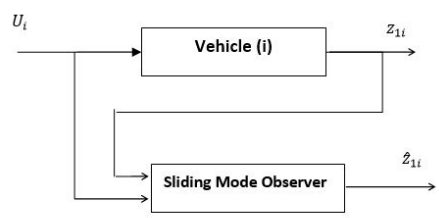

Fig. 3. Principal Diagram of an observer

The observer model is presented as follows :

$$\begin{cases} \dot{\hat{z}}_{1i} = \hat{z}_{2i} - \lambda_{1i} sign(\hat{z}_{1i} - z_{1i}) - \Lambda_{1i}(\hat{z}_{1i} - z_{1i}) \\ \dot{\hat{z}}_{2i} = f(z_{1i}, \hat{z}_{2i}) + g(z_{1i})U_i - \lambda_{2i} sign(\hat{z}_{1i} - z_{1i}) \\ -\Lambda_{2i}(\hat{z}_{1i} - z_{1i}) \end{cases} \tag{8}$$

where: $f(z_{1i}, \hat{z}_{2i}) = -M^{-1}(z_{1i})H(z_{1i}, \hat{z}_{2i})$.
$\lambda_{1i}, \lambda_{2i}, \Lambda_{1i}$ and Λ_{2i} : are positive definite diagonal gain matrices.

1) Observer Convergence Analysis: To study the convergence of this observer let us write the error dynamic equations $\tilde{z}_{1i} = \hat{z}_{1i} - z_{1i}$ and $\tilde{z}_{2i} = \hat{z}_{2i} - z_{2i}$ as follows:

$$\begin{cases} \dot{\tilde{z}}_{1i} = \tilde{z}_{2i} - \lambda_{1i} sign(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i} \\ \dot{\tilde{z}}_{2i} = \Delta f_i - \lambda_{2i} sign(\tilde{z}_{1i}) - \Lambda_{2i}\tilde{z}_{1i} \end{cases} \tag{9}$$

with: $\Delta f_i = f(z_{1i}, \hat{z}_{2i}) - f(z_{1i}, z_{2i})$
To study the convergence of this observer in finite time, we choose as Lyapunov function:

$$V_i = V_{1i} + V_{2i}$$

The first part of this function (V_{1i}) is to converge the first equation of the error ($\dot{\tilde{z}}_{1i} = 0$). V_{1i} is defined as follows:

$$V_{1i} = \frac{1}{2}\tilde{z}_{1i}^T \tilde{z}_{1i}$$

Therefore the sliding surface is attractive if the derivative of this function is negative:

$$\dot{V}_{1i} = \tilde{z}_{1i}^T(\tilde{z}_{2i} - \lambda_{1i} sign(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i})$$

Take it $\lambda_{1i} > |\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i}|$. We can calculate the average sign function ($sign_e(\tilde{z}_{1i})$) such as:

$$sign_e(\tilde{z}_{1i}) = \lambda_{1i}^{-1}(\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i})$$

By replacing the sign function ($sign_e(\tilde{z}_{1i})$) with its expression, the equations (9) becomes:

$$\begin{cases} \dot{\tilde{z}}_{1i} = \tilde{z}_{2i} - \lambda_{1i} sign_e(\tilde{z}_{1i}) - \Lambda_{1i}\tilde{z}_{1i} = 0 \\ \dot{\tilde{z}}_{2i} = \Delta f_i - \lambda_{2i}\lambda_{1i}^{-1}(\tilde{z}_{2i} - \Lambda_{1i}\tilde{z}_{1i}) - \Lambda_{2i}\tilde{z}_{1i} \end{cases} \tag{10}$$

The second part of the function (V_i) is defined as:

$$V_{2_i} = \frac{1}{2}\tilde{z}_{2i}^T\tilde{z}_{2i}$$

This function is intended to converge the second error to zero ($\dot{\tilde{z}}_{2i} = 0$). By calculating the derivative of this function:

$$\dot{V}_{2i} = \tilde{z}_{2i}^T[\Delta f_i - \lambda_{2i}\lambda_{1i}^{-1}\tilde{z}_{2i} + (\lambda_{2i}\lambda_{1i}^{-1}\Lambda_{1i} - \Lambda_{2i})\tilde{z}_{1i}]$$

This function is negative when we choose $\Lambda_{2i} = \lambda_{2i}\lambda_{1i}^{-1}\Lambda_{1i}$ and $\lambda_{2i} > |\Delta f_i \lambda_1|$ with $|\Delta f_i| < \varepsilon_0$.
With these convergence conditions for the two parts of the function V_i, the convergence of this observer in finite time is guaranteed.

IV. Longitudinal and lateral control

The objective of the control is:
- Control the leader to follow a desired path.
- Control the convoy with the decentralized global approach to follow the trajectory of the leader with the available information.
- Ensure a safe distance between vehicles to avoid collision.

In this section, the model used to calculate the control laws of the leader and the convoy is defined in (7). To simplify the writing, we define the error position of the fleet as defined in the dynamic model:

$$e_i = (e_{1i}, e_{2i}, e_{3i})^T = (e_{x_i}, e_{y_i}, e_{\theta_i})^T$$

where e_{x_i}, e_{y_i} and e_{θ_i} represents the longitudinal, lateral and yaw errors positions of each vehicle of the convoy.

The sliding mode control is used to control the movement of the leader and the convoy. This nonlinear control allows fast and robust convergence of the system state. This control ensures robust stability [15] and based on a switching function in the sliding surface.

A. Control of the Leader

In this paper, leader is controlled to follow a desired trajectory. The sliding surface chosen for leader is defined as follows:

$$s_0 = \dot{e}_0 + K_1 e_0 \tag{11}$$

where s_0 represents the sliding surface and e_0 represents the tracking error such as:

$$\begin{cases} \dot{e}_{x_0} = \dot{x}_0 - \dot{x}_{0_d} \\ \ddot{e}_{y_0} = a_{y_0} - a_{y_{0_d}} \end{cases} \tag{12}$$

$\dot{x}_{0_d}$: the longitudinal speed desired for the leader
$a_{y_{0_d}}$: the desired lateral acceleration for the leader.
$\dot{e}_{x_0}$: the longitudinal velocity error and $\ddot{e}_{y_0}$: the lateral acceleration error.
K_1 and K_2 are positive definite diagonal gain matrices.

The desired lateral acceleration was chosen according to the longitudinal velocity and the radius of the desired trajectory, that is to say, that in our approach the longitudinal and lateral control are coupled. The desired lateral acceleration is defined as follows:

$$a_{y_{0_d}} = \dot{x}_0^2/R_0$$

where R_0: the radius curvature of the trajectory desired of the leader.
We recall that : $a_{y_0} = \ddot{y}_0 + \dot{x}_0\dot{\theta}_0$, we can write $\ddot{e}_{y_0} = \ddot{y}_0 - \ddot{y}_{0_d}$, with $\ddot{y}_{0_d} = -\dot{x}_0\dot{\theta}_0 + \dot{x}_0^2/R_0$.
The equivalent control bring back the state of the vehicle to the desired state, it is calculated from the derivative of the sliding surface ($\dot{s} = 0$) such as:

$$\dot{s}_0 = \ddot{e}_0 + K_1\dot{e}_0 \tag{13}$$

According to the equation (13) and using the state model define in the equation (7) , the equivalent control is defined as :

$$u_{eq_0} = g(\hat{z}_{1_0})^{-1}(\dot{z}_{2_d} - k_1(\hat{z}_{2_0} - z_{2_d}) - f(\hat{z}_{1_0}, \hat{z}_{2_0}))$$

The global control applied to the leader represents the sum of the two controls (equivalent control and robust control):

$$\begin{aligned} U_0 = g(\hat{z}_{1_0})^{-1}(\dot{z}_{2_d} - k_1(\hat{z}_{2_0} - z_{2_d}) \\ ... - f(\hat{z}_{1_0}, \hat{z}_{2_0})) - K_2 sign(s_0) \end{aligned} \tag{14}$$

where the robust control : $u_{rob_0} = -k_2 sign(s_0)$.
This control law (14) controls the longitudinal and lateral movement and allows to follow the desired trajectory.

To study the existence of the surface, we can choose a Lyapunov function $V_0 = \frac{1}{2}s_0^T s_0$. The sliding surface is attractive if the derivative of this function is negative:

$$\dot{V}_0 = s_0^T\dot{s}_0 = s_0^T[\ddot{e} + K_1\dot{e}] = s_0^T[\dot{z}_{2_0} - \dot{z}_{2_d} + K_1\dot{e}]$$

Replacing $\dot{z}_{2_0}$ with its expression:

$$\dot{V}_0 = s_0^T[f(z_{1_0}, z_{2_0}) + g(z_{1_0})U_0 - \dot{z}_{2_d} + K_1\dot{e}]$$

Replacing the expression of the U_0 control in the previous equation:

$$\dot{V}_0 = s_0^T[f(z_{1_0}, z_{2_0}) - g(z_{1_0})K_2 sign(s_0) + \dot{z}_{2_d} - K_1(\hat{z}_{2_0} - z_{2_d}) - f(\hat{z}_{1_0}, \hat{z}_{2_0}) - \dot{z}_{2_d} + K_1\dot{e} \quad (15)$$

After the calculation we find:

$$\dot{V}_0 = s_0^T[-\Delta f_0 - M^{-1}(z_{1_0})K_2 sign(s_0)] \quad (16)$$

The matrix $M(z_{1_0})$ is positive definite and reversible. To ensure the convergence and the existence of this sliding surface, we can increase the gains of the matrix K_2 in the following: $|\Delta f_0| < \varepsilon_0 < K_2$.
The derivative of this function (16) is negative:

$$\dot{V}_0 < -\eta_0|s_0| < 0$$

where ε_0 and η_0 : positive constants.
By respecting the conditions of the convergence of the sliding surface, the stability of the path tracking for the leader is ensured.

B. Control of the Convoy

The decentralized global approach is considered in this paper. That is, the control applied to each vehicle of the convoy is based on the information of the previous vehicle and the leader.

The longitudinal control is intended to impose a reference speed on the convoy and ensure a safe distance between vehicles to avoid collisions (Fig. 4) and the lateral control makes it possible to follow the leader trajectory and to minimize the error and the lateral deviation with respect to the desired path by the steering angle (Fig. 5) [20]. Both controls are coupled by longitudinal velocity.

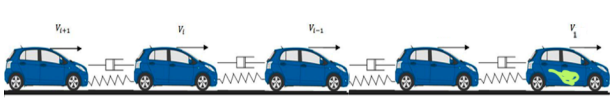

Fig. 4. The longitudinal movement of the convoy

The following assumptions are essential for calculating the longitudinal and lateral control laws for the convoy:
Assumption 7. The information of the leader and the previous vehicle are available for each vehicle.
Assumption 8. The characteristic of the leader's trajectory is available for the convoy (the radius or curvature of the path).

Let the sliding surface defined for the control applied to ith vehicle based on the leading vehicle and the previous vehicle information as follows:

$$s_i = \dot{e}_i + K_3\dot{e}_{i,0} + K_4 e_i \quad (17)$$

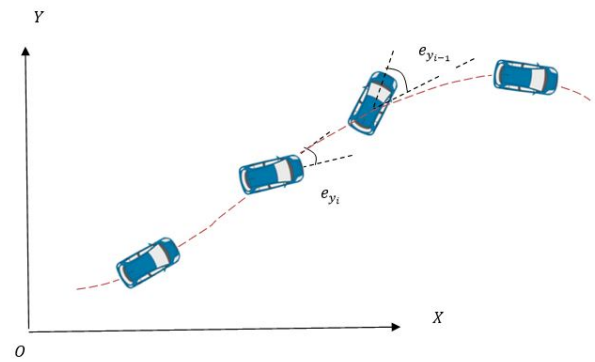

Fig. 5. Lateral deviation of the convoy

where s_i represents the sliding surface for the control of the ith vehicle, $\dot{e}_{i,0}$ the speed error between the ith vehicle and leader, e_i represents the position errors between the ith vehicle and the previous (i-1)th vehicle . K_3, K_4 and K_5 are positive definite diagonal gain matrices.

We define the curvilinear spacing error between the vehicles of the convoy:

$$e_{x_i} = S_i - S_{i-1} + l_d$$

where: l_d : the safety distance. S_i: the curvilinear abscissa.
The error of the longitudinal velocity is defined as follows:

$$\dot{e}_{x_i} = \dot{x}_i - \dot{x}_{i-1}$$

The speed error between the ith vehicle and the leader is:

$$\dot{e}_{x_{i,0}} = \dot{x}_i - \dot{x}_0$$

where $\dot{x}_0$, $\dot{x}_i$ and $\dot{x}_{i-1}$ are longitudinal speed of leader, ith vehicle and (i-1)th vehicle.

The lateral movement of the fleet (Fig. 5) is controlled based on the lateral acceleration. The objective of this control is to follow the reference trajectory of the leader and to cancel the lateral deviation, that is to say $e_{y_i} = 0$.
The lateral acceleration error is defined as follows:

$$\ddot{e}_{y_i} = a_{y_i} - a_{y_{i_d}} \quad (18)$$

where a_{y_i}: the lateral acceleration and $a_{y_{i_d}}$: the desired lateral acceleration.
The desired lateral acceleration is calculated according to the desired trajectory and the longitudinal speed:

$$a_{y_{i_d}} = \dot{x}_i^2/R_i$$

where R_i : the radius curvature of the trajectory desired of the ith vehicle. In order to calculate the law of the control we replace $a_{y_{i_d}}$ by its expression: $a_{y_i} = \ddot{y}_i + \dot{x}_i\dot{\theta}_i$.

The lateral acceleration error defined in (18) can be written in the form:

$$\ddot{e}_{y_i} = \ddot{y}_i - \ddot{y}_{i_d}$$

where $\ddot{y}_{i_d} = -\dot{x}_i\dot{\theta}_i + \dot{x}_i^2/R_i$.

To calculate the equivalent control, we derive the equation of

the sliding surface (17) and canceling ($\dot{s}_i = 0$) we find : $u_{eq_i} = g(\hat{z}_{1_i})^{-1}[(I + K_3)^{-1}(\dot{z}_{2_{i-1}} + K_3\dot{z}_{2_0} - K_4\dot{e}_i) - f(\hat{z}_{1_i}, \hat{z}_{2_i})]$

The global control applied to each vehicle of the convoy is defined as follows:

$$U_i = -K_5 sign(s_i) + g(\hat{z}_{1_i})^{-1}[(I + K_3)^{-1}(\dot{z}_{2_{i-1}} + K_3\dot{z}_{2_0} \\ ... - K_4\dot{e}_i) - f(\hat{z}_{1_i}, \hat{z}_{2_i})] \quad (19)$$

This control (19) makes it possible to control the longitudinal and lateral movement of the convoy and to follow the path of the leader. The longitudinal movement is controlled by the driving/braking wheels torque and the lateral movement controlled by the steering wheel angle.

To study the convergence of the sliding surface defined to control the convoy, we choose the Lyapunov function :

$$V_i = \frac{1}{2}s_i^T s_i \quad (20)$$

The sliding surface is attractive if the derivative of this function is negative. We derive (20) and replace $\dot{s}_i$ by its expression:

$$\dot{V}_i = s_i^T(\ddot{e}_i + K_3\ddot{e}_{i,0} + K_4\dot{e}_i)$$

We replace the acceleration error ($\ddot{e}_i, \ddot{e}_{i,0}$) by its expression in (7):

$$\dot{V}_i = s_i^T[(I+K_3)(f(z_{1i}, z_{2i})+g(z_{1i})U_i)-\dot{z}_{2_{i-1}}-K_3\dot{z}_{2_0}+K_4\dot{e}_i]$$

Replacing U_i by its expression (19) :

$$\dot{V}_i = s_i^T[-(I + K_3)(\Delta f_i + g(z_{1_i})K_5 sign(s_i) - \dot{z}_{2_{i-1}} - \\ K_3\dot{z}_{2_0} + K_4\dot{e}_i + \dot{z}_{2_{i-1}} + K_3\dot{z}_{2_0} - K_4\dot{e}_i] \quad (21)$$

After the calculation:

$$\dot{V}_i = s_i^T[-(I + K_3)(\Delta f_i + g(z_{1_i})K_5 sign(s_i)]$$

The inertia matrix ($g(z_{1_i}) = M^{-1}(z_{1i})$) is defined as strictly positive and reversible. If the parameters and the states are well estimated, Δf_i converges quickly to zero ($\Delta f_i \Rightarrow 0$). We have I represents the identity vector, let K_3 be a matrix of positive defined gains, so $(I + K_3) > 0$.
Let K_5 defined as follows: $|\Delta f_i| < \varepsilon_i < K_5$.
With these criteria, the derivative of the function (20):

$$\dot{V}_i < -\eta_i|s_i| < 0$$

where ε_i and η_i : positive constants.
The convergence and attractiveness of the sliding surface defined in (17) are proven.
The stability of the convoy is always checked and ensured, provided that the conditions of convergence of the sliding surface are respected.

V. SIMULATIONS

To validate these results we used two software; Matlab Simulink and the SCANeR Studio driving platform . The 5 vehicles of the convoy Fig. 6 are controlled to follow the imposed trajectory represented in the Fig. 7. The leading vehicle moves on this trajectory and the convoy follows it with a safety distance between vehicles to avoid a collision.
We choose the parameters of a vehicle in SCANeR Studio:

Parameter	Value	Parameter	Value
m	1500 kg	m_w	$23.2kg$
I_z	$1652.7kg.m^2$	I_w	$2kg.m^2$
C_{α_f}	$67689N/rad$	$C_{\alpha r}$	$69253N/rad$
Lr	$1.441m$	Lf	$1.099m$
s	$2m^2$	E	$1.5m$
c	0.3	ρ	1.3

TABLE I
VEHICLE PARAMETERS VALUES SCANER-STUDIO

This trajectory (Fig. 7) was chosen to validate the longitudinal and lateral control of the fleet. In the first step, we assume that we have data (Position, speed, and acceleration) of a vehicle moving in SCANeR Studio to validate the observer define in the section III.

Fig. 6. Convoy 5-vehicle SCANeR Studio

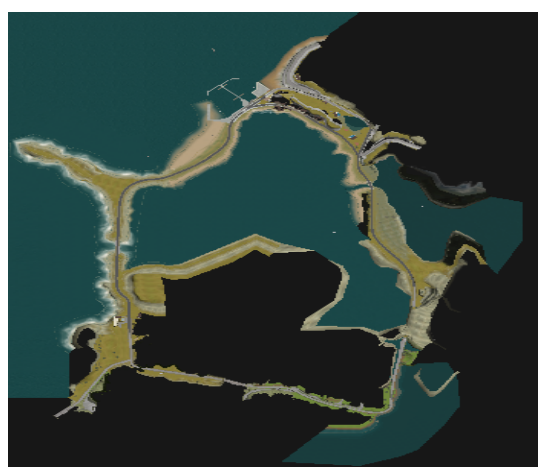

Fig. 7. Trajectory-SCANeR Studio

We present the results of the leader simulation to compare the reel states provided by SCANeR Studio and the states

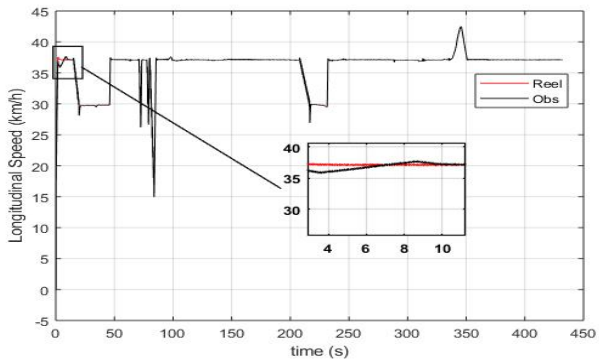

Fig. 8. Velocity v_x of the leader reel and observer

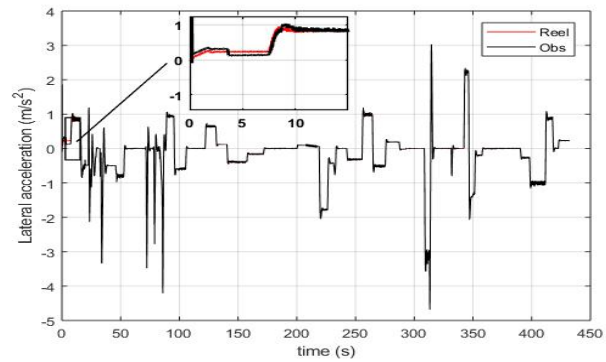

Fig. 10. Lateral acceleration a_y of the leader, reel and observer

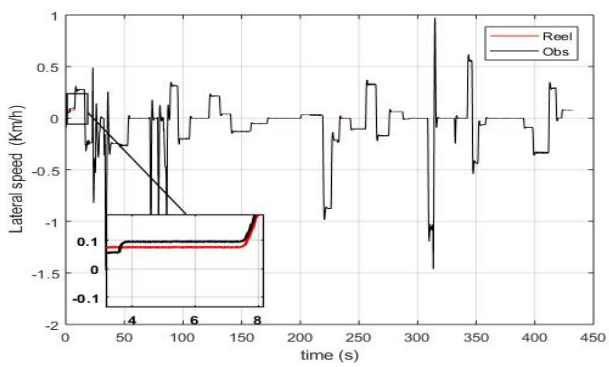

Fig. 9. Velocity v_y of the leader reel and observer

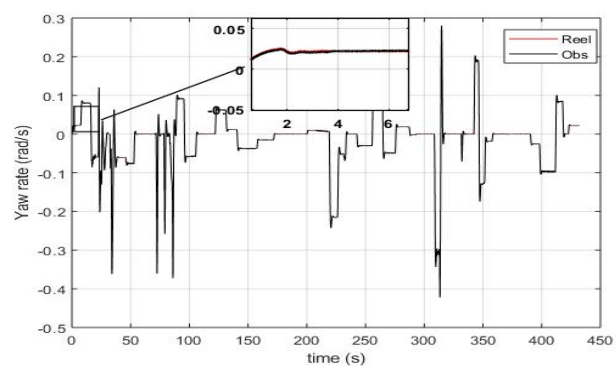

Fig. 11. Yaw rate reel and observer

estimated by the observer. As defined in the assumption 5, we consider that all positions (x_i, y_i and θ_i) are available in real-time, the objective being to estimate the remains of states (speed and acceleration). This observer has been used for each vehicle in the fleet to estimate their actual states, it represents a local observer, such that each vehicle estimates their states from available sensors (position) to calculate their own law of control.

To avoid the problem of reluctance we have replaced the $sign$ function by the sat function for the observer and the control law. To test the robustness of this observer we give initial conditions for the positions ($\hat{x}_{i_0} = +2m$ and $\hat{y}_{i_0} = +2m$) different to that of the real states.

Fig. 8 shows the reel longitudinal velocity and the velocity estimated by the observer. It can be seen that after t = 9s, the estimated states converge with the real states. The convergence time depends on the gain of the observer and the initial conditions chosen for the positions. For the non-linear observer and the control, the gains are chosen from several tests while respecting the stability conditions defined in the convergence study. The estimated lateral acceleration is calculated according to the states estimated by the observer ($\hat{a}_{y_i} = \hat{\dot{y}}_i + \hat{\dot{x}}_i\hat{\dot{\theta}}_i$) and the results Fig. 10 show their rapid convergence after t=8s. The reel and estimated lateral velocity and yaw rate are presented in Fig. 9 and Fig. 11.

After validation of the observer, we present in the following the results of the control by the sliding mode control applied on the convoy. The leading vehicle was controlled by the control

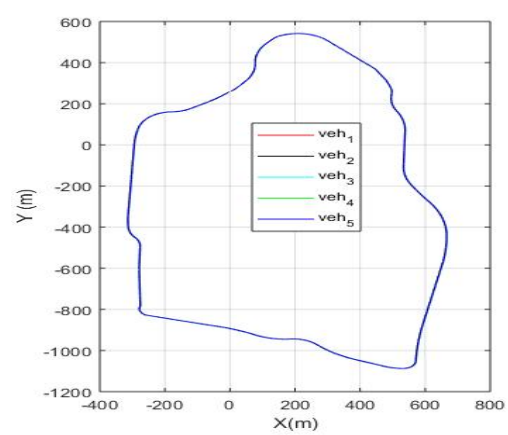

Fig. 12. Trajectory of the convoy

defined in (14) to follow the trajectory presented at Fig. 7, the convoy uses a decentralized global approach to control the longitudinal and lateral movement, follow the desired trajectory in accordance with the constraints defined in section IV. The law of the control uses the states estimated by the observer.

Fig. 12 shows the movement of the convoy in the reference trajectory, we can see that the proposed control law for a fleet of vehicles allows to control the convoy in both directions of movement (longitudinal and lateral), ensure a safe distance between vehicles to avoid a collision. Each vehicle in the

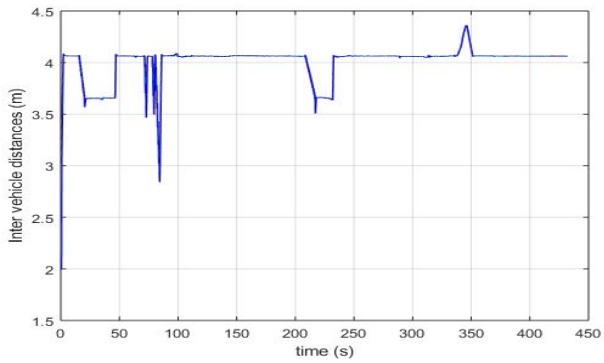

Fig. 13. Inter vehicle distance

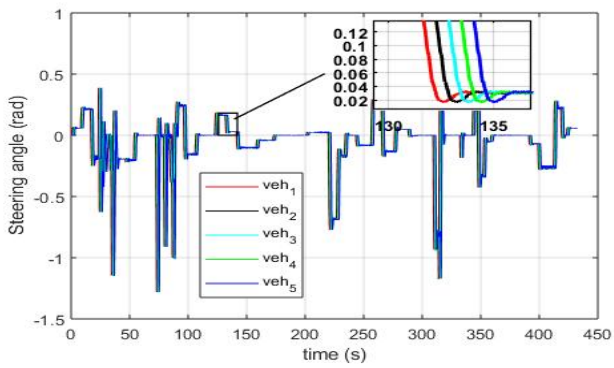

Fig. 14. Steering angle

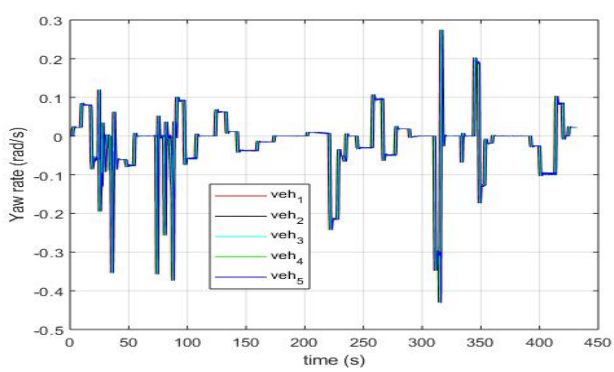

Fig. 15. Yaw rate of the convoy

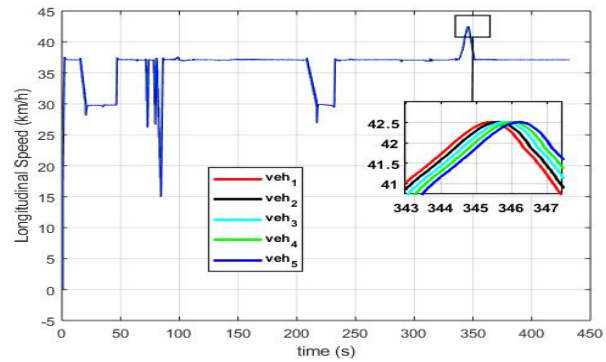

Fig. 16. Velocity v_x of the convoy

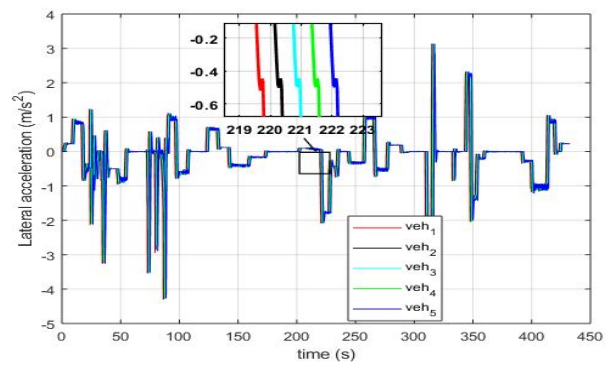

Fig. 17. Lateral acceleration a_y of the convoy

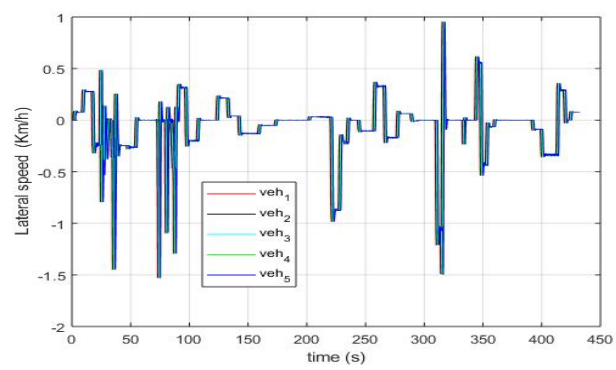

Fig. 18. Velocity v_y of the convoy

convoy uses leader information and neighbors to calculate their control law, which is part of the family of decentralized approaches. It is found that the lateral error is practically negligible, that is to say, no angular deviation between the vehicles of the convoy and the reference trajectory, the safety distance is about 4 m between each neighbor Fig. 13. The safety distance is proportional to the speed. In our case, the fleet moves at a speed ($v_{x_{max_i}} < 45km/h$), so the safety distance is minimal. On the other hand, if the convoy is traveling at a high speed, the distance between the vehicles must increase. Fig. 16 and Fig. 18 represent the longitudinal and lateral speeds of the fleet. The longitudinal movement is controlled by the driving/braking wheels torque and the lateral movement is controlled by the steering angle, the latter is calculated as follows:

$$\delta_i = u_{2_i}/(2C_{\alpha fi} - 2\frac{I_{wi}}{R_{ti}^2}\ddot{x}_i)$$

The steering angle is shown in Fig. 14. It represents the lateral movement for each vehicle in the convoy, this angle is proportional to the lateral acceleration Fig. 17 and the yaw rate Fig. 15.

The control approach that we proposed, controls the two movements of the convoy, as shown in Fig. 12, the accumulation errors are almost negligible which proves a better performance for path tracking. With the decentralized global approach we have that tracking errors are not accumulated to other vehicles.

VI. Conclusion

In this article, we controlled a convoy of autonomous vehicles using the reconstructed states by an First Order Sliding Mode Observer, to reduce the numbers of the sensors used. The model used represents a nonlinear system with bilateral couplings for a fleet of road vehicles that can be used for a large number of vehicles. The Observer were developed to estimate the position, speed and acceleration of each vehicle in the fleet. The practical results show the rapid convergence of this observer to the real state. Finally, a sliding mode control based on this observer, that solves the leader following problem for a set of coordinated unconnected cars have been presented. Two controls were calculated for the leader and convoy vehicles, in our work leading is driven autonomously. The second control for the convoy is based on the decentralized approach, as each vehicle in the convoy uses the information from the leader and the predecessor vehicle. Simulation results show that the control law ensures uniform convergence of the tracking errors.

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