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► To cite this version:

Pierre Perrier, Mustafa Hadj-Nacer, J. Gilbert Meolans, Irina Martin Graur. Measurements and modeling of the gas flow in a microchannel: influence of aspect ratios, surface nature, and roughnesses. *Microfluidics and Nanofluidics*, 2019, 23 (97), 10.1007/s10404-019-2259-1 . hal-02904989

HAL Id: hal-02904989

<https://amu.hal.science/hal-02904989>

Submitted on 22 Jul 2020

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18 the review papers [1], [2]. However, only a few experimental works explored the
19 gas/wall interaction by varying the nature and the roughness of the walls [3],
20 [4], [5], [6]. The authors of Refs. [3], [6] studied the roughness influence and
21 reported smaller values of the Tangential Momentum Accommodation Coeffi-
22 cient (TMAC) for rough surfaces compared to that of smooth surfaces for both
23 monoatomic and polyatomic gases. However, the opposite trend was reported
24 in Ref. [4], where a large value of TMAC and substantial backscattering with
25 surfaces of large roughness (copper, $5\mu m$ grooves) were measured, as compared
26 to sapphire, glass, and electronically polished gold surfaces. The authors of Ref.
27 [5] also reported larger values of TMAC for rough surfaces compared to pol-
28 ished steel spheres surfaces. From an experimental point of view, the difficulty
29 of this kind of studies lies notably in the correct consideration of the fabrication
30 process and in the precise geometrical characterization of studied systems.

31 Recently, several numerical studies have been performed, where the surface
32 roughness was simulated by regular structures of various shapes. These sim-
33 ulations were carried out in the frame of different approaches: the continuum
34 modeling (Navier-Stokes equations), [7], [8], the DSMC modeling [9] and the
35 kinetic modeling (S-model) [10]. The influence of the random roughness was
36 studied by using the Molecular Dynamics (MD) simulations in Refs. [11], [12].
37 The random roughness was also simulated by Weierstrass-Mandelbrot fractal
38 function by applying a linearized BGK equation and the generalized Reynolds
39 equation [13]. A significant influence of the surface roughness was found in the
40 case of the Poiseuille and Couette flows [13]. A different approach was proposed
41 by the authors of Ref. [14]: new scattering kernel allowed to incorporate the
42 roughness effect directly in the boundary conditions. Despite numerous studies,
43 the problem is far to be completely resolved, and new experimental data are
44 needed to characterize gas-surface interaction.

45 Extending the topics of previous studies, [15], [16], [17], experimental and
46 theoretical analysis of the influence of the surface roughness and the surface na-
47 ture on gas flow properties at small scale is realized in this work. This analysis
48 allows us to enhance the understanding of the gas-solid surface interaction by
49 considering the behaviors of the same gases on different surfaces. The rectangu-
50 lar microchannels used in this work are fabricated by silicon etching technology.
51 The internal surfaces of these channels are covered by a gold layer of different
52 roughnesses or by a layer of silica. The relatively moderate roughness ($\approx 0.1\%$
53 of the channel high) was chosen to avoid affecting the rectangular shape of chan-
54 nel cross-section. In this work, we analyze and quantify the influence of various
55 factors on the flow properties: the gas itself, the wall materials, the solid surface
56 roughness and the height/wide ratio, characterizing the geometry of the channel
57 cross-sections.

58 Finally, we present a brief summary of a complete analytical modeling of flow
59 in the slip regime that takes into account the two lateral channel dimensions. By
60 comparing experimental values and analytical results, we deduce various values
61 of the tangential momentum accommodation coefficient, by assuming that it is
62 not the same for the horizontal and vertical walls. Indeed, in the fabrication
63 process, the roughness of the vertical channel walls is not measured. Therefore,

64 we extend our bi-dimensional treatment by introducing different accommodation
65 coefficients for the vertical and horizontal walls.

66 **2. Microchannel Properties and Experimental Setup**

67 In this section, short description of the method used to fabricate the mi-
68 crochannels is provided. Then, the experimental technique used for the mea-
69 surements of the mass flow rate through the microchannels is briefly introduced.

70 *2.1. Microchannels Fabrication Summary*

71 The details of the microchannel fabrication process are shown in Figure 1.
72 First, a silicon wafer is prepared and cleaned to remove any metal or organic
73 contaminations from the surface. Then, a drop of photoresist resin solution
74 (SPR 220) is deposited in the center of the silicon wafer. The wafer is then spun
75 at high speed between 500 to 5000 *rpm* for 30 to 60 seconds. The rotation
76 speed and the viscosity of the resin solution determine the resulting thickness
77 of the photoresist resin layer. After spinning, the silicon wafer is heated to
78 evaporate the solvent from the resin solution to form a solid resin layer (step *a*).
79 Second, the photoresist resin is exposed to UV light through a mask to define the
80 shape of the channels. This procedure allows to obtain two parts: exposed and
81 unexposed regions. The exposed resin is then removed (step *b*). The next step
82 (*c*) is the Deep Reactive-Ion Etching (DRIE) [18], [19], [20] in the parts where
83 the resin is removed. During etching the silicon wafer is attacked with Argon
84 ions (Ar^+) jet that strikes the surface perpendicularly, removing the material
85 until achieving the desired half depth ($h/2$) of the microchannel. After etching
86 step a layer of gold (Au) or silica (SiO_2) is generated on the wafer surfaces,
87 by cathodic sputtering for gold deposition and by oxidation for silicon dioxide
88 deposition (step *d*). Then, the resin remaining from the first development is
89 stripped off and the layer of gold or silica is attacked with ions to obtain the
90 desired roughness of the surface (step *e*). Another layer of photoresist resin is
91 deposited again on the wafer by sputtering technique (step *f*). The photoresist
92 resin is exposed to the UV light and the resin layer is removed in the non-
93 etched regions (step *g*). Finally, a layer of gold of 100 *nm* is deposited on the
94 wafer to be used in the bonding step (step *h*) and the remaining photoresist
95 resin is stripped away (step *i*). The last step is the wafer bonding: the wafers
96 fabricated with the same width (w) and depth ($h/2$) are assembled together
97 using the thermocompression technique at the eutectic point (636K, 2.85% Si
98 and 97.15% Au) by applying a moderate pressure on the wafers at a temperature
99 of 573 K (step *j*). After the fabrication steps the assembled wafers are cut by
100 a precise saw in order to obtain the microchannels.

101 The rectangular microchannels, used in the present study, were fabricated in
102 two different ways. First, for the channels of A group and S1 channel, two iden-
103 tical silicon wafers with a cavity of $h/2$ depth, etched using the DRIE technique,
104 are bounded together, Fig. 1(j) Wafers bonding channels A and S1). Second,
105 for the E and S2 – S4 channels, a wafer with a cavity of h depth, etched using

Channel	Material	Roughness [$10^{-9} m$]	h [$10^{-6} m$]	w [$10^{-6} m$]	L [$10^{-3} m$]
A1	Au	0.9	27.8	52.2 ± 0.5	15.07
A2		0.9	27.6	107.6 ± 0.5	15.00
A3		0.9	27.9	504.0 ± 1.5	15.06
A4		1.1	25.8	1005.5 ± 3.0	14.87
E1	Au	12.0	33.5	55.5 ± 0.5	15.02
E2		12.0	35.2	103.8 ± 0.5	15.19
E3		12.0	34.9	505.0 ± 1.5	15.22
E4		12.0	34.2	1001.3 ± 3.0	15.06
S1	SiO_2	1.1	24.3	50.1 ± 0.5	13.68
S2		1.1	42.3	100.0 ± 0.5	15.06
S3		1.1	42.0	500.0 ± 1.5	15.06
S4		1.1	41.5	1000.0 ± 3.0	15.06

Table 1: Dimensions of the rectangular microchannels. The uncertainty of the microchannel height h measurements is $\pm 0.5\mu m$, the uncertainty on the measurements of the microchannel width, w , depends on the channel width value and it is provided in Table; the uncertainty on the measurements of the channel length L is equal to ± 0.1 mm.

106 the DRIE technique, is bounded with a flat wafer, Fig. 1((j) Wafers bonding
107 channels $S2 - S4$ & E).

108 2.2. Microsystems Description

109 All the microchannels, fabricated using the technique, described in the pre-
110 vious Section, have rectangular cross-sections of different aspect ratios. They
111 were divided into three groups noted by letters A, E and S according to the
112 surface coating and surface roughness:

- 113 • The first group, referenced with letter A, corresponds to the microchan-
114 nels coated using a gold layer (Au) on all internal surfaces with a mean
115 roughness of $1nm$ ¹.
- 116 • The second group, referenced with letter E, corresponds to the microchan-
117 nels coated also with a layer of gold on all internal surfaces, but with a
118 mean roughness of $12nm$.
- 119 • The third group, referenced with the letter S, corresponds to the mi-
120 crochannels coated with silica (SiO_2) layer on all internal surfaces with a
121 mean roughness of $1.1nm$.

122 The microchannels that are in the same group differ only by their aspect ratio.
123 Table 1 summarizes the properties and dimensions of the microchannels.

¹Root Mean Square (RMS)

124 The uncertainty on the microchannel height and width measurements are
 125 related essentially to the uncertainty on the micro-ruler used to measure the
 126 cross-section dimensions and to some eventual other errors related to the quality
 127 and sharpness of the pictures taken with the microscope. The uncertainty on
 128 the microchannel height is estimated as $\pm 0.5 \mu\text{m}$, see Table 1. The uncertainty
 129 on the microchannel width depends on the average width of the microchannel
 130 and it is provided in Table 1. This uncertainty variation is related to the zoom
 131 used to take the pictures of the microchannels cross-section. The uncertainty
 132 on the microchannels length is only related to the caliper uncertainty and it is
 133 equal to $\pm 0.1 \text{ mm}$.

134 The roughness of the microchannels was measured using a profilometer
 135 (Alpha-Step IQ) before the bonding step of the two wafers. The measurements
 136 are made in three sections: beginning, center and end of the microchannel. Fig-
 137 ure 2 shows an example of roughness measurement made for the microchannel
 138 S3. The typical Root Mean Square (RMS) roughness is given in Table 1 for each
 139 group of microchannels. One can remark that the roughness of the microchan-
 140 nel A4 is not equal to that of the other microchannels of this group, because the
 141 microchannel A4 was fabricated separately, not at the same fabrication condi-
 142 tions as the other microchannels. However, it was maintained in the group A
 143 because its roughness remains close to those of the other microchannels of that
 144 group.

145 2.3. Mass Flow Rate Measurement Technique

146 The mass flow rate through a microchannel can be calculated from the equa-
 147 tion of state for an ideal gas

$$pV = MRT, \quad (1)$$

148 by assuming steady flow conditions. Here V is the volume, p and T are, respec-
 149 tively, the pressure and temperature of a gas, M is the mass of a gas and \mathcal{R}
 150 is the specific gas constant. Using the constant volume technique, the mass flow
 151 rate can be derived from Eq. (1) as it was shown in [21] and [15]

$$\dot{M} = \frac{dM}{dt} = \frac{V}{\mathcal{R}T} \frac{dp}{dt} (1 - \theta), \quad \text{with} \quad \theta = \frac{dT/T}{dp/p}. \quad (2)$$

152 In above equation, θ represents the ratio of fractional change of the temperature
 153 to the fractional change of pressure. If θ is small enough compared to 1 then
 154 it can be neglected. Below we estimate the order of θ in our experimental
 155 conditions.

156 In this work, we consider an isothermal flow through a microchannel con-
 157 necting two large tanks maintained at pressures p_{in} and p_{out} , which remain close
 158 to constant values. The pressure variations in each tank during a measurement
 159 are less than 1%, thus the flow may be considered as a steady flow. Conse-
 160 quently, the relative pressure variation dp/p in a tank is of the order of 10^{-2} .
 161 The relative variation of the temperature dT/T is calculated from the standard

162 deviation ($dT/T = s$)

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (T_i - \bar{T})^2}, \quad (3)$$

163 where \bar{T} is the average temperature during the experimental time, T_i is the
 164 instantaneous temperature measurement and n is the number of measurements
 165 during the experimental time. This standard deviation s is smaller than 2×10^{-4}
 166 for all the experiment series, so dT/T is of the order of 10^{-4} . θ in Eq.(2) is
 167 clearly less than 2×10^{-2} , so this term can be neglected.

168 Thus, from Eq. (2) the mass flow rate \dot{M}^{exp} expression can be simplified as

$$\dot{M}^{exp} = \frac{V}{\mathcal{R}T} \frac{dp}{dt}. \quad (4)$$

169 To calculate the mass flow rate using Eq. (4), the registered data of the pressure
 170 p_i at different time instants t_i are used. The stationary assumption can justify
 171 physically the implementation of first order polynomial expression in t_i

$$p(t_i) = at_i + b, \quad (5)$$

172 where the slope a of the function $p(t_i)$ is equal to dp/dt .

173 The global uncertainty on the mass flow rate measurements is around 5%.
 174 This includes the uncertainty of the volume measurements, which is of the order
 175 of 2%. The error on the coefficient a determination, Eq. (5), is of the order of
 176 1%. The relative error related to the neglecting of term θ in Eq. (2) is of the
 177 order of 2% and it is included also in the global uncertainty estimation of the
 178 mass flow rate measurements.

179 3. Analytical Method in the Slip Regime

180 In this section we present analytical results related to the influence of the
 181 lateral wall on gas flow in rectangular cross-section microchannels. In the fol-
 182 lowing we continue to use the same notations, as those already introduced in
 183 Section 2.2, namely, we denote the channel height h , the channel width w and
 184 the channel length L , see Fig. 3. The main results are presented in Ref. [17].
 185 We recall here only some important parts that are indispensable to go forward
 186 with the new analytical calculations that will permit to take into account the
 187 different properties of the channel walls.

188 3.1. Governing Equations and Slip Boundary Conditions

189 As it is well known, for an isothermal gas flow through long microchannel,
 190 the system of the Navier-Stokes (NS) equations may be simplified at the zero
 191 order of the small channel aspect ratio parameter $\varepsilon_{hL} = h/L \ll 1$. Then, the
 192 momentum conservation equation appears under the form of the Stokes equation
 193 as

$$\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} = \frac{1}{\mu} \frac{dp}{dz}. \quad (6)$$

194 Here u_z is the gas velocity along the z axis, and x, y are the directions that
 195 correspond to the channel width and height, respectively, see Fig. 3. The
 196 gas viscosity μ in Eq. (6) is calculated in the frame of the Variable Hard
 197 Sphere (VHS) model [22] as $\mu = \mu_{ref}(T/T_{ref})^\omega$, where ω is the viscosity index,
 198 which depends on the gas nature, μ_{ref} is the reference viscosity at the reference
 199 temperature T_{ref} .

200 As mentioned previously, we consider here the slip flow regime characterized
 201 by a Knudsen number varying in the range $0.01 < Kn < 0.3$. The Knudsen
 202 number, $Kn = \lambda/h$, is defined as the ratio of the molecular mean free path λ to
 203 the characteristic channel cross-section dimension h , see Fig. 3(b). As it is well
 204 known, in the slip regime, the Stokes equation (6) must be supplemented with
 205 slip boundary conditions. Moreover, it is important to remember that, since
 206 we have an axis (or a plane) of symmetry, the second order terms with respect
 207 to the Knudsen number are negligible in the conservation equation system and
 208 they are important only when they result from the slip boundary condition [23].
 209 Therefore, in the geometry considered here, according to the results given in
 210 Ref. [23], a pertinent slip velocity boundary condition may be written at the
 211 second order following λ as

$$u_z \Big|_s = \pm A_1 \lambda \frac{\partial u_z}{\partial \vec{n}} \Big|_s - A_2 \lambda^2 \Delta u_z \Big|_s, \quad (7)$$

212 where the mean free path λ is calculated using the VHS model [22] as

$$\lambda = k_\lambda \frac{\mu}{p} \sqrt{2\mathcal{R}T}, \quad k_\lambda = \frac{(7-2\omega)(5-2\omega)}{15\sqrt{\pi}}. \quad (8)$$

213 In Eq. (7) the subscript "s" corresponds to the quantities, calculated at the sur-
 214 face, \vec{n} represents the normal to the surface, and A_1 and A_2 are the coefficients
 215 depending on the gas-surface interaction. Moreover, in a more recent study, it
 216 was shown [24] that the Knudsen layer influence does not modify the form of
 217 Eq. (7), only the analytical form of A_1 and A_2 coefficient becomes different,
 218 when the contribution of the Knudsen layer is taken into account [24].

219 Now we will resolve Eq. (6) subjected to boundary condition, Eq. (7). It
 220 results from Eq. (6) that Δu_z does not depend on the x and y coordinates
 221 whatever the point considered in the channel cross-section. Thus, $\Delta u_z \Big|_s$ in Eq.
 222 (7) is constant at any point of the section perimeter. The same comment may
 223 be applied to the molecular mean free path λ , due to the negligent variation
 224 of pressure in the cross section. Then, it is convenient, first from a calculation
 225 simplification point of view, to use the function change

$$u^* = u_z + A_2 \lambda^2 \Delta u_z \Big|_s. \quad (9)$$

226 Applying this change does not change the form of Eq. (6) and the system, Eqs.
 227 (6) and (7), can be rewritten using u^* as

$$\Delta u^* = \frac{1}{\mu} \frac{dp}{dz}, \quad (10)$$

$$u^* \Big|_s = \pm A_1 \lambda \frac{\partial u^*}{\partial n} \Big|_s. \quad (11)$$

229 Using the symmetry planes of the problem ($x \geq 0$ and $y \geq 0$), the boundary
 230 conditions (11) in a quarter of the channel cross-section may be detailed as
 231 following:

$$\begin{cases} \frac{\partial u^*}{\partial x}(0, y) = 0, \\ u^*(w/2, y) = -A_1 \lambda \frac{\partial u^*}{\partial x}(w/2, y), \end{cases} \quad (12)$$

232 and

$$\begin{cases} \frac{\partial u^*}{\partial y}(x, 0) = 0, \\ u^*(x, h/2) = -A_1 \lambda \frac{\partial u^*}{\partial y}(x, h/2). \end{cases} \quad (13)$$

233 3.2. Basic Solving Methodology

234 The present approach is mainly based on the spectral properties of the partial
 235 Laplace operators included in the Stokes equation (10). When applied to the
 236 vector space of the L_2 functions, defined on a finite spatial domain D (here the
 237 channel cross-section), the Laplacian (or the partial Laplacian) is characterized
 238 by an eigen-function discrete spectrum.

239 Expanding the solution u^* of Eq. (10) on the eigen functions $f_n(y)$ of the
 240 partial Laplacian $\frac{\partial^2}{\partial y^2}$, where functions $f_n(y)$ verify border conditions of can-
 241 cellations, similar to Eq. (13), we change Eq. (10) into a set of differential
 242 equations easy to solve. Indeed the $f_n(y)$ set provides an orthogonal basis of
 243 the space $L_2(y)$ of the y functions, defined on the channel cross-section and ver-
 244 ifying, at the limits, conditions similar to Eq. (13). Moreover, the eigenvalues
 245 associated with $f_n(y)$ form a discrete countable series of real negative numbers
 246 $-\nu_n^2$. Thus, the solution u^* has the form

$$u^* = \sum_{n=0}^{\infty} g_n(x) f_n(y), \quad (14)$$

247 where n is a natural integer. Taking into account the properties of f_n functions,
 248 we find

$$f_n(y) = a_n \cos(\nu_n y), \quad (15)$$

249

$$\cotan(\beta_n) = 2A_1 K n \beta_n, \quad (16)$$

250

$$\beta_n = \nu_n h/2. \quad (17)$$

251 The $g_n(x)$ functions in Eq. (14) have to be calculated in next Section to find
 252 the solution u^* .

253 *3.3. Search for $g_n(x)$ and for the reduced velocity*

254 Now we will obtain the explicit expression for $g_n(x)$ functions involved in
 255 Eq.(14). The second member of Eq. (10) depends only on the z space variable
 256 and may be expanded on the $f_n(y)$ basis functions. Then, from Eq. (10) and
 257 using u^* velocity expansion in form of Eq. (14), we obtain a linear second order
 258 differential equation giving $g_n(x)$. Finally, solving this equation and taking also
 259 into account Eqs. (12), (15), (16) and (17) we deduce

$$u^* = \frac{h^2}{\mu} \frac{dp}{dz} \sum_{n=0}^{\infty} \phi_n(x, y), \quad (18)$$

260 where

$$\phi_n(x, y) = \frac{\sin(\beta_n) \cos(2\beta_n y/h)}{\beta_n^2 (2\beta_n + \sin(2\beta_n))} \left(\frac{\cosh(2\beta_n x/h)}{\cosh(\beta_n w/h) + \beta_n \varepsilon \sinh(\beta_n w/h)} - 1 \right). \quad (19)$$

261 In the previous expression, the small parameter ε is defined as

$$\varepsilon = 2A_1 Kn, \quad \varepsilon \ll 1. \quad (20)$$

262 Let us remind that in Ref. [17] we explained why, when transforming Eq.
 263 (6) into Eq. (10), the precision of the results given by the NS equations is not
 264 physically modified, and, more practically, why resolving first the system, Eqs.
 265 (10) and (11), and using *in fine* Eq. (9) to attain u_z we obtain strictly the
 266 same physical result as by solving the system, Eqs. (6) and (7). Hereafter, we
 267 apply the correlative remark of Ref. [17] explaining that in u^* expression only
 268 the first order terms following the Knudsen number are physically significant.
 269 Therefore, expression (18) of u^* is expanded up to the first order according to
 270 the small parameter ε , defined in (20) as proportional to the Knudsen number.
 271 Precisely

$$u^* = u_0^* + \varepsilon \left(\frac{du^*}{d\varepsilon} \right)_0, \quad (21)$$

272 where

$$u_0^* = u^* \Big|_{\varepsilon=0}, \quad \left(\frac{du^*}{d\varepsilon} \right)_0 = \frac{du^*}{d\varepsilon} \Big|_{\varepsilon=0}. \quad (22)$$

273 Since u^* depends on ε not only explicitly through relation (20), but also implic-
 274 itly via β_n related with the local Knudsen number through Eq. (16); then by
 275 using Eq. (20), equation (16) is now written formally as:

$$F(\beta_n, \varepsilon) = \cotan(\beta_n) - \varepsilon \beta_n = 0. \quad (23)$$

276 Therefore, the calculation of the total derivative of u^* must be written as:

$$\left(\frac{du^*}{d\varepsilon} \right)_0 = \sum_{n=0}^{\infty} \left(\frac{\partial u^*}{\partial \beta_n} \frac{d\beta_n}{d\varepsilon} \right)_0 + \left(\frac{\partial u^*}{\partial \varepsilon} \right)_0. \quad (24)$$

277 The calculations of u_0^* , and the derivatives $\left(\frac{\partial u^*}{\partial \beta_n}\right)_0$ and $\left(\frac{\partial u^*}{\partial \varepsilon}\right)_0$ in Eq. (24) have
 278 been easily done by using successively Eqs. (18) and (19), and the following
 279 results deduced from Eq. (23):

$$(\beta_n)_0 = \frac{\pi}{2}(2n + 1). \quad (25)$$

280 In addition, the calculation of expression $\left(\frac{d\beta_n}{d\varepsilon}\right)_0$ requires to use again Eq. (23),
 281 where F appears as an implicit function of β_n and ε . Expressing now the partial
 282 derivative of F one obtains

$$\left(\frac{d\beta_n}{d\varepsilon}\right)_0 = -\frac{\partial F/\varepsilon}{\partial F/\beta_n} = -\frac{\beta_n}{1 + \cotan^2(\beta_n) + \varepsilon}. \quad (26)$$

283 To calculate $\left(\frac{d\beta_n}{d\varepsilon}\right)_0$ from the second member of Eq. (26), we have to take $\varepsilon = 0$
 284 and $(\beta_n)_0$ from Eq. (25), which leads to

$$\left(\frac{d\beta_n}{d\varepsilon}\right)_0 = -\frac{\pi}{2}(2n + 1). \quad (27)$$

285 Then, we finally obtain following expression for velocity

$$u^* = \frac{h^2}{\mu} \frac{dp}{dz} \left(\sum_{n=0}^{\infty} [\phi_n]_0 + \varepsilon \sum_{n=0}^{\infty} ([\varphi_n]_0 + [\chi_n]_0) \right), \quad (28)$$

286 with $(\phi_n)_0$, $(\varphi_n)_0$ and $(\chi_n)_0$ given in Ref. [17], where the velocity u_z is deduced
 287 from Eq. (A 15).

288 3.4. Mass Flow Rate

289 Considering now the mass flow rate defined as

$$\dot{M} = \frac{p}{\mathcal{R}T} \int_{-w/2}^{w/2} \int_{-h/2}^{h/2} u_z(x, y) dx dy, \quad (29)$$

290 we obtain new mass flow rate expression

$$\dot{M} = \dot{M}_P \left(1 + 6A_1 \frac{T_n}{S_n} Kn_m + \frac{A_2 \pi^4}{16S_n} \frac{\mathcal{P} + 1}{\mathcal{P} - 1} \ln \mathcal{P} Kn_m^2 \right), \quad (30)$$

291 where \dot{M}_P is the analytical expression of the Poiseuille mass flow rate in the
 292 hydrodynamic flow regime

$$\dot{M}_P = \frac{h^3 w \Delta p p_m}{12 \mu \mathcal{R} T L} V_n. \quad (31)$$

293 In Eq. (30) Kn_m is the mean Knudsen number, based on the mean pressure
 294 $p_m = 0.5(p_{in} + p_{out})$ and on the characteristic length h , $\Delta p = p_{in} - p_{out}$,
 295 $\mathcal{P} = p_{in}/p_{out}$,

$$S_n = \frac{\pi^4}{96} - \frac{2h}{\pi w} S_1, \quad \text{with} \quad S_1 = \sum_{n=0}^{\infty} \frac{\tanh(0.5\pi(2n+1)w/h)}{(2n+1)^5}, \quad (32)$$

296

$$V_n = \frac{96}{\pi^4} S_n, \quad (33)$$

297 and

$$T_n = \frac{4}{3} S_n - \frac{1}{3} \left(1 - \frac{h}{w}\right) S_3, \quad S_3 = \sum_{n=0}^{\infty} \frac{\tanh^2(0.5\pi(2n+1)w/h)}{(2n+1)^4}. \quad (34)$$

298 *3.5. Extension to Two Surface Materials*

299 It is possible to extend the method, briefly presented above and published
 300 in Ref. [17], to the case when the walls of the channel are fabricated differently,
 301 see comments in Section 2.2. But, unfortunately, it is then necessary to restrict
 302 the new approach to the first order according to the Knudsen number. Indeed,
 303 in this case, the second order term of the boundary condition, Eq. (7), does
 304 not involve the Laplacian form of constant value in the channel cross-section.
 305 Remember, that the presence of this term allowed us to make the function
 306 change written in Eq. (9).

307 Thus, we consider hereafter, at the first order only, the case of a channel,
 308 where both vertical channel walls are made from a first material corresponding
 309 to a first order slip coefficient, A_{1w} , and both horizontal walls are made from
 310 another material, characterized with the slip coefficient A_{1h} . In the following,
 311 to avoid cumbersome expressions we use simplified notations A_w and A_h for the
 312 slip coefficients related to the vertical and horizontal walls, respectively. The
 313 boundary conditions on the wall reduce now to an equation similar to Eq. (11),
 314 where u^* is now changed into u_z and where the slip coefficient changes following
 315 the wall orientation. Of course, the Stokes equation directly solved here is Eq.
 316 (6) involving u_z instead of Eq. (10) involving u^* . Detailing now the boundary
 317 condition in the (x, y) coordinates we obtain equations similar to Eqs. (12)
 318 and (13). Again, u^* is changed into u_z and A_1 is successively changed into A_w
 319 and A_h which were respectively related to two dimensionless numbers, small
 320 compared to one and associated to the Knudsen number:

$$\varepsilon_w = A_w Kn, \quad \varepsilon_h = A_h Kn. \quad (35)$$

321 Consequently, Eqs. (14) - (17) are maintained, nothing however that the coef-
 322 ficient A_1 is here changed into A_h because deriving from the new forms of Eqs.
 323 (13) and (23), namely:

$$F(\beta_n, \varepsilon_h) = \cotan(\beta_n) - \varepsilon_h \beta_n = 0. \quad (36)$$

324 Then, the search of u^* is now replaced by the search of u_z . We use practically
 325 the same equations, but as previously noticed, in Eq.(12), A_1 is changed into
 326 A_w . Consequently, in the new form of Eq. (19) only ε_w would appear explicitly,
 327 replacing ε . Thus, the u_z expansion takes the form

$$u_z = (u_z)_0 + \varepsilon_w \left(\frac{du_z}{d\varepsilon_w} \right)_0. \quad (37)$$

328 However, in expression $\left(\frac{du_z}{d\varepsilon_w}\right)_0$ the parameter β_n is involved, but β_n is calculated
 329 from Eq. (36) and so related to ε_h , thus we obtain

$$\left(\frac{du_z}{d\varepsilon_w}\right)_0 = \sum_{n=0}^{\infty} \left(\frac{\partial u_z}{\partial \beta_n} \frac{d\beta_n}{d\varepsilon_h} \frac{d\varepsilon_h}{d\varepsilon_w} \right)_0 + \varepsilon_w \left(\frac{\partial u_z}{\partial \varepsilon_w} \right)_0. \quad (38)$$

330 In previous expression the derivative $\frac{d\beta_n}{d\varepsilon_h}$ is obtained from Eq. (36) in the same
 331 way as in Eq. (26), and where $\frac{d\beta_n}{d\varepsilon_w} = \frac{d\beta_n}{d\varepsilon_h} \frac{d\varepsilon_h}{d\varepsilon_w}$ with $d\varepsilon_h/d\varepsilon_w = A_h/A_w$. Finally,
 332 we obtain an expression close to Eq. (28), where ε is changed into ε_w and where
 333 appears the ratio A_h/A_w coming from the new derivative $d\beta_n/d\varepsilon_w$:

$$u_z = \frac{h^2}{\mu} \frac{dp}{dz} \left(\sum_{n=0}^{\infty} (\phi_n)_0 + \varepsilon_w \sum_{n=0}^{\infty} \left[\left(\frac{A_h}{A_w} \varphi_n \right)_0 + (\chi_n)_0 \right] \right). \quad (39)$$

334 Naturally the functions ϕ_n , φ_n , χ_n are exactly the same as the corresponding
 335 quantities given in [17]. To provide the complete analytical development these
 336 functions are also given in Appendix A.

337 Then, calculating the mass flow rate, we find expressions close to those pre-
 338 sented in [17]. Considering the group of the equations, allowing the calculation
 339 of the mass flow rate, Eqs. (30) - (34), the second order term vanish, and A_1
 340 becomes A_w , so new mass flow rate expression reads formally:

$$\dot{M} = \dot{M}_P \left(1 + 6 \frac{A_w}{S_n} T_n(A_w, A_h) Kn_m \right). \quad (40)$$

341 Then, parameter $T_n(A_w, A_h)$ in Eq. (34), is slightly modified and it becomes:

$$T_n(A_w, A_h) = \frac{4}{3} \frac{A_h}{A_w} S_n - \frac{1}{3} \left(\frac{A_h}{A_w} - \frac{h}{w} \right) S_3, \quad (41)$$

342 where S_3 is determined by Eq. (34). The other quantities remain exactly the
 343 same as previously.

344 4. Implementation of the Methodology

345 The two analytical methods, discussed in Section 3, which are based on
 346 two different assumptions, are applied hereafter in order to model the gas
 347 flow through rectangular channel, taking into account the effect of its two-
 348 dimensional cross-section area.

349 Following the first method, we suppose that for each channel all the walls
 350 (vertical and horizontal) are characterized by the same material and the same
 351 roughness. In this method we use a second order approach: for each channel we
 352 determine a global coefficient A_1 at the first order and another global coefficient
 353 A_2 at the second order according to the Knudsen number.

354 The second method is based on a different assumption. In this approach,
 355 we suppose that the etching process used to fabricate the channels can induce

356 a roughness difference between the vertical and horizontal walls. In this case,
 357 we use a first order approach, which leads to two first order coefficients to be
 358 determined, A_w and A_h , for the vertical and horizontal walls, respectively. But
 359 to find these two coefficients we need an additional hypothesis, namely, in each
 360 group of channels (A, E and S) all the vertical walls are considered similar, *i.e.*
 361 characterized with a same A_w , while all the horizontal walls are supposed to
 362 have the same A_h coefficient. Finally, considering the experimental values of the
 363 mass flow rate, we calculate the coefficients, following each method described
 364 above, then, we determine in each case what is the more pertinent method to
 365 retain.

366 4.1. Slip coefficient extraction with a single surface material method

367 It is assumed here that the horizontal and vertical walls material is the same
 368 for each channel. The analytical expression for the mass flow rate, obtained
 369 in Section 3.4, see Eq. (30), is used here to extract the "experimental" slip
 370 coefficient. As it was obtained from the theoretical analysis, see Section 3.4, the
 371 analytical mass flow rate can be expressed in the second order polynomial form
 372 according to the Knudsen number. By dividing the dimensional mass flow rate,
 373 Eq. (30), by the analytical expression of the Poiseuille mass flow rate in the
 374 hydrodynamic flow regime, \dot{M}_P , Eq. (31), we obtain the following dimensionless
 375 analytical expression:

$$S^T = B_0^T + B_1^T Kn_m + B_2^T Kn_m^2, \quad (42)$$

376 where

$$B_0^T = 1, \quad B_1^T = 6A_1 \frac{T_n}{S_n}, \quad \text{and} \quad B_2^T = \frac{A_2 \pi^4}{16S_n} \frac{\mathcal{P} + 1}{\mathcal{P} - 1} \ln \mathcal{P}. \quad (43)$$

377 We can also assume that the measured values of the mass flow rate can be rep-
 378 resented by an analogous dimensionless second order polynomial form according
 379 to the Knudsen number:

$$S_f^{exp} = B_0^{exp} + B_1^{exp} Kn_m + B_2^{exp} Kn_m^2. \quad (44)$$

380 Comparing expressions (42), (43) and (44), the coefficients A_1 and A_2 are ex-
 381 pressed as

$$A_1 = \frac{\sigma_p}{k_\lambda} = \frac{B_1^{exp} S_n}{6T_n}, \quad \text{and} \quad A_2 = \frac{\sigma_{2p}}{k_\lambda^2} = \frac{16B_2^{exp} S_n}{\pi^2 \ln \mathcal{P}} \frac{\mathcal{P} - 1}{\mathcal{P} + 1}. \quad (45)$$

382 Therefore, the "experimental" slip coefficients of first and second order, σ_p and
 383 σ_{2p} , respectively, are found as

$$\sigma_p = \frac{B_1^{exp} k_\lambda S_n}{6T_n}, \quad \text{and} \quad \sigma_{2p} = \frac{16B_2^{exp} k_\lambda^2 S_n}{\pi^2 \ln \mathcal{P}} \frac{\mathcal{P} - 1}{\mathcal{P} + 1}. \quad (46)$$

384 It is clear from the previous expressions that the lateral wall influence is present
 385 in both coefficients B_1^{exp} and B_2^{exp} through the coefficients S_n and T_n , see Eqs.

386 (32) - (34). The dependence of velocity slip coefficients, σ_p and σ_{2p} , on the
 387 molecular interaction model, here VHS model [22], comes from the coefficient
 388 k_λ , see Eqs. (8), (46).

389 By solving numerically the linearized BGK kinetic equation and using the
 390 Maxwellian diffuse-specular scattering kernel for various values of the accom-
 391 modation coefficient α , the authors of [25] have proposed the relation between
 392 the first order slip coefficient σ_p and the accommodation coefficient α . This
 393 expression was further improved in [26] and it becomes:

$$\sigma_p(\alpha) = \frac{2 - \alpha}{\alpha}(\sigma_p(1) - 0.1211(1 - \alpha)), \quad (47)$$

394 where $\sigma_p(1) = 1.016$ is the velocity slip coefficient for $\alpha = 1$ calculated in [27].

395 For all gases the second order treatment appeared us the most pertinent
 396 in slip regime using the same criteria as those used in [15]. The values of the
 397 main parameters used as criteria appear in Tables 8, 10 and 12. For each Knud-
 398 sen number range we calculated the determination coefficient, r^2 , the squared
 399 residual sum $s_r = \sqrt{(1/(n-p) \sum_{i=1}^n e_i^2)}$, and the standard error on the fit curves
 400 $E_s = \sqrt{1/n \sum_{i=1}^n e_i^2 / S^{exp}}$, where $e_i = S_i^{exp} - S_{f_i}^{exp}$ is the local difference be-
 401 tween the measured and fitting values and it represents the local fitting error,
 402 n is the number of measurements, p is the number of the unknown coefficients
 403 of the fitting model, and $\overline{S^{exp}} = 1/n \sum_{i=1}^n S_i^{exp}$ is the averaged value of the
 404 measured values. Analyzing the values of these coefficients given in Tables 8, 10
 405 and 12, respectively, for first and second order approaches, we found that the
 406 determination coefficient of second order is slightly more closer to one than that
 407 of first order. However, the squared residual sum and the standard error are
 408 of the same order for the both fitting. Therefore, we can conclude that for the
 409 studied cases both first and second fittings reproduce well the measurements.

410 4.2. Slip coefficient extraction with two surface materials method

411 It is assumed here that for each channel the horizontal and vertical walls
 412 roughnesses are different. By analogy with the previous methodology we can
 413 express the analytical mass flow rate, Eq. (40), in the following dimensionless
 414 first order polynomial form, by dividing expression (40) by \dot{M}_P :

$$S^T = C_0^T + C_1^T K n_m, \quad (48)$$

415 where

$$C_0^T = 1, \quad C_1^T = 6A_w \frac{T_n(A_w, A_h)}{S_n}, \quad (49)$$

416 here $T_n(A_w, A_h)$ and S_n are given by Eqs. (41) and (32), respectively. We
 417 assume also here that the measured values of the mass flow rate can be presented
 418 using a first order dimensionless polynomial form:

$$S_f^{exp} = C_0^{exp} + C_1^{exp} K n_m. \quad (50)$$

419 Then, by comparing expressions (48) and (50) we obtain:

$$A_w T_n(A_w, A_h) = \frac{1}{6} C_1^{exp} S_n. \quad (51)$$

420 However, practically we cannot calculate both values of A_w and A_h simultane-
 421 ously, because we have only one equation (51) to determine them. Therefore,
 422 as it was mentioned previously, we will consider that all the channels belonging
 423 to the same group have the same coefficients A_w and A_h . Doing so, we can
 424 obtain, for each group of channels sufficient quantity of data to extract these
 425 two coefficients.

426 Practically we consider the following approach. The theoretical expression
 427 of the dimensionless mass flow rate, Eqs. (48), (49), can be written for a channel
 428 by equalizing it to the experimental expression, S^{exp} , as following:

$$A_0 + A_h(8 - S_0)Kn_m + A_w \frac{h}{w} S_0 Kn_m = S^{exp}, \quad S_0 = 2 \frac{S_3}{S_n}. \quad (52)$$

429 The previous expression is then written for all measured values of the dimen-
 430 sionless mass flow rates S^{exp} and for all channels in each group. We write so
 431 a large number of equations involving the same unknown coefficients A_0 , A_w
 432 and A_h . Here A_0 coefficient is a free parameter, which theoretically has to be
 433 equal to 1. However, we decided to calculate its value from the fitting procedure
 434 (multiple linear regression), as we have done for two other coefficients, A_w and
 435 A_h . We obtain here an overdetermined system of equations taking a matrix
 436 form, involving a vector of unknown coefficients, where each line corresponds
 437 to a measurement. This system is classically resolved by minimizing the norm
 438 of its residual vector, where the coefficients A_0 , A_w and A_h are included, then,
 439 their convenient values are extracted.

440 5. Results and Discussion

441 The mass flow rates through all the microchannels considered in this study
 442 were measured for three gases, Helium, Nitrogen and Argon. Then, we extract
 443 the velocity slip and the accommodation coefficients from the mass flow rate
 444 measured for the series of channels type A , E and S by considering first the
 445 same roughness on the vertical and horizontal walls, Section 5.1, and then,
 446 by considering the second approach, Section 5.2, which allows us to consider
 447 different roughnesses on the vertical and horizontal walls.

448 The geometrical characteristics for the channels of group A are provided
 449 in Table 1. The experimental conditions for the microchannels of this group
 450 are summarized in Tables 2 and 3. To accomplish one of the goals fixed in
 451 this article, namely the study of the surface roughness influence on the momen-
 452 tum transfer between a gas and a surface, we conducted an additional series
 453 of mass flow rate measurements through the microchannels of group E which
 454 have similar rectangular cross-section aspect ratios and the same material (Au)
 455 of the internal surfaces coating as the microchannels of group A . However,

456 the microchannels of group E have a surface roughness 12 times greater than
 457 that of the microchannels A, see Table 1. The experimental conditions for the
 458 microchannels of group E are summarized in Tables 4 and 5.

459 To study the influence of the surface nature on the momentum transfer
 460 between a gas and a surface, the microchannels of the group S were fabricated
 461 by coating the internal surfaces with a layer of silica (SiO_2). The channel of
 462 this group were etched in a wafer to a depth h , than covered by an other plate
 463 wafer. This technology was different from the fabrication of A and E channels,
 464 where two half parts of depth ($h/2$) were etched in a wafer. However, the surface
 465 roughness of group S channels is similar to that of microchannels of the group
 466 A. Therefore, a comparison of the TMAC results between these two groups
 467 can clarify the influence of the surface materials on the momentum exchange
 468 between a gas and a surface. The experimental conditions for microchannels of
 469 the group S are given in Tables 6 and 7.

470 5.1. Results for the single surface material method

471 In this Section we present the results of the first method, where the horizon-
 472 tal and vertical walls of each channel are considered to have the same properties
 473 (material and roughness). The experimental data of the mass flow rate in two
 474 Knudsen number ranges: $[0, 0.1]$ and $[0, 0.3]$ are fitted using the first and second
 475 order polynomial forms, respectively, see Section 4.1. Then, the first order slip
 476 coefficient in the case of the first order fit, and first and second order coefficients
 477 in the case of the second order fit, are extracted. Finally, the tangential mo-
 478 mentum accommodation coefficients are calculated for each channel. All data
 479 of the experimental fitting coefficient and the slip and tangential momentum
 480 accommodation coefficients, are provided in Tables 8 - 13. Examples of the
 481 first and second order polynomial fits of the experimental data are provided on
 482 Figures 4 - 6. By analyzing these data, we will conclude on the validity of the
 483 proposed methodology, as well as on the influence of the surface nature and the
 484 surface roughness on the gas-surface interaction.

485 5.1.1. Influence of the channel cross-section aspect ratio

486 The influence of the microchannel aspect ratio on the gas flow through mi-
 487 crochannels is assumed to be taken into account by the factors S_n , T_n and V_n
 488 in the expression for the mass flow rate, see Eqs. (30) - (34). Considering
 489 the results, reported in Tables 8, 10 and 12, we found that the values of the
 490 fitting coefficients, B_0^{exp} for all groups of channels are close to the theoretical
 491 value $B_0^T = 1$, Eqs. (30) and (43). If we check carefully the values of the
 492 coefficient B_0^{exp} for the S and A channels, they are different from 1 by 1.6%
 493 and 2.7%, respectively. However, for the channels of group E the difference
 494 increases to 6.6%, see Table 10. The detailed analysis of the possible nature of
 495 this discrepancy, see Appendix B, allows us to conclude that the probable fac-
 496 tor influencing the results of the microchannels E can be the gold layer takeoff
 497 inside the microchannels, which causes the change of their cross-section shape.

498 The fact that the fitting coefficients B_0^{exp} remains close to 1, when the chan-
 499 nel aspect ratio changes, confirms the correctness of the theoretical expression

500 of the mass flow rate, Eq. (30). In addition, by analyzing Tables 8, 10 and 12,
501 it can be seen that for each group of channels and for each gas, the extracted
502 coefficients (first and second order velocity slip and accommodation coefficients)
503 have very close values, regardless the channel cross-section changes.

504 Furthermore, in our bi-dimensional modeling, we supposed that all channel
505 walls are homogeneous and that the gas-surface interaction is the same for the
506 upper, bottom and lateral walls. Therefore, when a remaining influence of the
507 cross-section aspect ratio appears in the behavior of the first and second order
508 velocity slip coefficients, it may be explained by the difference of the gas-surface
509 interaction between the vertical and horizontal walls or by the deformation of
510 the cross-section shape.

511 5.1.2. Influence of the surface nature

512 We are going to focus our analysis on the microchannels of groups A and S .
513 The wall roughness of the channels in these groups is very low, of the order of one
514 nanometer and the aspect ratio ranges are comparable, see Table 1. It is clear
515 that practically only the nature of the walls is different between these groups of
516 microchannels. The microchannels of group A are covered with gold (Au) and
517 the microchannels of group S are covered with Silica (SiO_2). For all channels of
518 group A , and for all the used gases, the tangential momentum accommodation
519 coefficients are found to be very close, each other, except for microchannel $A1$,
520 which has an accommodation coefficient smaller than the other microchannels
521 of this group, see Table 9.

522 For the microchannels of group S , see Figure 7, all the tangential momentum
523 accommodation coefficients are very similar and their values are around 0.92.
524 These small deviations of the tangential momentum accommodation coefficient
525 from its mean value are most of the time of the order of the uncertainty of
526 measurement, see Table 13. Therefore, in this case the TMAC seems to be
527 weakly dependent on the gas nature.

528 However, if comparing the TMAC values, obtained for all S group mi-
529 crochannels with those obtained for the microchannels $A2$, $A3$ and $A4$, one can
530 notice that lower values are globally obtained for the microchannels of group
531 S . As previously noted, both groups of microchannels A and S have almost the
532 same surface roughness, therefore, from this comparison it can be concluded
533 that the gold (Au) material is slightly more diffusive than the silica (SiO_2).

534 5.1.3. Influence of the surface roughness

535 The microchannels of both groups (A and E) differ only by roughness of
536 their internal surfaces, see Table 1. The other characteristics, like the surface
537 material, or the gas species involved in this analysis, are the same. The re-
538 spective cross-section aspect ratios are also similar. Therefore, it would be very
539 interesting to compare directly the influence of the surface roughness through
540 the respective slip and accommodation coefficients, obtained for the channels of
541 groups A and E . But, unfortunately, as it is explained in Appendix B, a prob-
542 able gold layer takeoff inside the microchannels E modified the cross sections
543 and the flows, which made them unsuitable for the extraction of both exact

544 coefficients in the frame of the models developed in this work. Nevertheless,
545 qualitative and comparative descriptions are possible notably concerning the
546 different gases behaviors. The analysis is focused here on the dependence of σ_p
547 and α coefficients on the molecular mass of the gases.

548 From Table 11 one can observe for microchannels E that higher is the molec-
549 ular mass smaller is the accommodation coefficient, even for diatomic gas like
550 N_2 . However, in some cases, this hierarchy may be perturbed due to the influ-
551 ence of the internal energy of the polyatomic gas molecules. Conversely, in the
552 case of groups A and S , we have mentioned in the previous paragraphs that for
553 the microchannels of these groups, the TMAC dependence on the gas nature,
554 hence on its molecular mass, is not significant, almost null, for several cases, see
555 Tables 9 and 13.

556 Comparing the gases behavior obtained in three groups of channels, one
557 can conclude that the wall roughness plays an important role on the TMAC
558 dependence on the molecular mass. First, the smooth surfaces of microchannels
559 A and S make the influence of the gas type, *i.e.* of the gas molecular mass, on the
560 TMAC almost nonexistent. Then, when the surface roughness increases, the gas
561 molecules stand out by their molecular mass, such as the highest TMAC value
562 is obtained for the lightest gas, at least for the monoatomic molecules. Finally,
563 the influence of the roughness on the value of the accommodation coefficient
564 itself will be discussed below, in the global frame of two theoretical approaches.

565 5.1.4. Second order velocity slip coefficient

566 When the experimental data are fitted using the second order polynomial
567 form, Eq. (44), two velocity slip coefficients, σ_p and σ_{2p} , are extracted using
568 expressions (45) and (46). The values of both coefficients are presented in Tables
569 9, 11 and 13. Most of σ_{2p} values lie in the range from 0.15 to 0.25. However, for
570 the microchannels with small aspect ratios, smaller and even negative values of
571 σ_{2p} coefficient are obtained, see Tables 9, 11 and 13.

572 The discussion about the physical significance of the second order velocity
573 slip coefficient and its theoretical values can be found in Refs. [28], [23], [29],
574 [24], where the values of σ_{2p} coefficient were found to be equal to 0.884, 0.766,
575 0.243 and 0.184, respectively. The measured here values of the second velocity
576 slip coefficient are close to the two last theoretical values. In addition, in Ref.
577 [30] the authors tested the influence of the accommodation coefficient on the σ_{2p}
578 coefficient and it was found that this coefficient varies very slightly with accom-
579 modation coefficient variations. In our experimental conditions the variation of
580 σ_{2p} coefficient is also weak.

581 The dependence of σ_{2p} coefficient on the molecular mass of a gas and on
582 the aspect ratio of the channel cross-section was weakly investigated in the
583 literature, see Refs, [31], [32], [33], [16], [34]. The authors of Refs, [33], [16], [34]
584 showed experimentally that σ_{2p} coefficient increases with the molecular mass of
585 a gas. This conclusion was derived by analyzing the gas flow in microchannels
586 of the circular, Refs. [33], [34], and rectangular, Ref. [16], cross-sections. In
587 the present study, the same behavior of the coefficient σ_{2p} increasing with the
588 gas molecular mass is also observed, particularly for the microchannel $A4$ (see

589 Table 9). However, by analyzing globally the behavior of σ_{2p} coefficient, one
 590 can notice that the influence of the gas molecular mass is not significant. The
 591 values obtained for each microchannel are very similar, almost within their
 592 uncertainties, but that could be related to the small variation of σ_{2p} depending
 593 on the accommodation of molecules. Taking into account the previous comments
 594 we would like to underline that the variation of σ_{2p} coefficient is generally small.

595 5.2. Results for non-homogenous walls method

596 In Section 4.2, we presented a new generalization of the previously devel-
 597 oped methodology [17], which allows to take into account different properties of
 598 the vertical and horizontal walls of a microchannel. This difference in the ver-
 599 tical and horizontal walls properties can appear during the fabrication process
 600 (channel etching), see Section 2.1, or just due to the wall coating with differ-
 601 ent materials. The new approach introduces two velocity slip coefficients, σ_{ph} ,
 602 and σ_{pw} , that are related to the properties of the horizontal and vertical walls,
 603 respectively.

604 In this Section, we consider the same experimental data for the channels
 605 A , E and S that were used in the previous Sections to extract the velocity
 606 slip and tangential momentum accommodation coefficients, to implement the
 607 new approach and obtain two velocity slip coefficients and two accommodation
 608 coefficients, related to the vertical and horizontal walls. As it was explained
 609 in Section 4.2, for the two coefficients A_w and A_h , which are related to the
 610 velocity slip coefficients, we have only one equation. Therefore, we have to use
 611 all the experimental data from each group of channels together to obtain the A_w
 612 and A_h coefficients, and then, the σ_{pw} and σ_{ph} coefficients, for that particular
 613 group of channels. The obtained results for all groups of channels, A , E and S ,
 614 and two gases, Helium and Argon are given in Tables 14 and 15. Even if the
 615 absolute values obtained for group E channels are not sure, we try to detect a
 616 systematic relation between same parameters of A and S groups and E groups.
 617 In our analysis of the channel of S group we do not included the $S1$ channel in
 618 our analysis, because it was fabricated by different process compared to $S2$ - $S4$
 619 channels: namely the channel $S1$ was made using the half heigh ($h/2$) cavities
 620 bounding, see Section 2.2 and Fig. 1. We do not include in our analysis the
 621 results for Helium, because the determination coefficient for this gas was found
 622 very low and therefore the fitting used was not representative for these data.

623 Before deriving some general tendencies of the accommodation coefficients
 624 behavior, we need to note that we do not have any measurements of the vertical
 625 wall roughness. However, generally the range of the vertical wall roughness
 626 created by DRIE techniques, see Ref. [35], is between $100nm$ and $1\mu m$.

627 We analyzed all groups of channels and only two gases, see Tables 14 and
 628 15, but from these results we can derive some common properties:

- 629 1. The tangential momentum accommodation coefficients of the horizontal
 630 walls, α_h , are always greater than those of the vertical walls, α_w .
- 631 2. The tangential momentum accommodation coefficients of the horizontal
 632 walls, α_h , are not very sensitive to the gas nature and these coefficients
 633 α_h are close to one (near full diffusive reflection of molecules).

- 634 3. The tangential momentum accommodation coefficients of the vertical walls,
635 α_w , of group *A* channels are always smaller than those of group *E* and
636 globally the tangential momentum accommodation coefficients of the ver-
637 tical walls, α_w , are not purely diffusive.
- 638 4. The observations related to point 1 and the fact that the roughness of the
639 vertical wall is larger than that of the horizontal wall, seem to indicate
640 that in our measurements, the accommodation coefficient increases when
641 the roughness of the surface decreases.
- 642 5. For the channels of group *A* the tangential momentum accommodation
643 coefficient of the vertical wall, α_w , depends on the nature of the gas.
644 However, in point 2, it was found that the tangential momentum accom-
645 modation coefficient of the horizontal walls, α_h , is not very sensitive to
646 the gas nature. However, the roughness of two walls is very different and
647 probably there is a specific threshold of the roughness value, above which
648 the gas nature starts to influence the accommodation coefficient.

649 *5.3. Comparison between the results of the first and second methods*

650 Let us note first that the second method was applied only for two groups of
651 channels, *A* and *E*. In addition, the quantitative comparison is possible only
652 for the channels of group *A*, because, as explained in Appendix B, the channels
653 of group *E* are probably affected by a gold layer take off modifying their cross-
654 section. Thus, when considering group *A* channels, the values of the various
655 accommodation coefficients obtained by the first method for *He* and *Ar*, see
656 Table 9, may be compared to the unique pair of values for *He* and *Ar*, given
657 by the second approach in Table 14. Let us remind the general comment given
658 above in Section 5.1.1, where the various accommodation coefficients of group
659 *A* channels are found close to each other in the first method and that the aspect
660 ratio has a weak influence. This fact brings a support for that hypothesis used
661 in the second method: an unique pair of coefficients for a group of channels.

662 Nevertheless, an apparent small contradiction seems to appear about the
663 trend of the accommodation coefficient variation with the roughness variation.
664 Indeed, in point 4, just here above, we have pointed out that the accommodation
665 coefficient increases when the surface roughness decreases. But, when detailing
666 the roughness in group *A*, we can observe using the first method, that the *A4*
667 channel has highest roughness (20% compared to the other channels in the same
668 group, see Table 1) and highest accommodation coefficient, see Table 9, which
669 seems to contradict the property given here above. In fact, the value given in
670 Table 1 for the roughness, characterizes only the horizontal walls of the channels.
671 Furthermore, we have seen in Section 5.2 that the vertical walls have a higher
672 roughness. Consequently, in the first method, all the unique accommodation
673 coefficients are decreased under this influence. But, in the *A4* channel the
674 relative vertical wall influence is the smallest because it has the largest aspect
675 ratio, see Table 1. Therefore, it is not possible to predict and compare the
676 mean roughness really acting on the coefficients of the first method. Thus, the
677 previous comparison may not be considered as significant.

678 Finally, the new second method used to analyze the result does not seem to
679 contradict the first method. Conversely, it seems to be a good generalization,
680 able to capture some physical information on the local nature of a surface.
681 The analysis of the results with two different accommodation coefficients of
682 horizontal and vertical walls of a channel, seems to be pertinent if the surface
683 ratio between horizontal and vertical walls remains moderate (weak aspect ratio
684 influence) and if the probable roughness is different between these walls. To use
685 all the advantages, offered by this last approach, we plan to apply it in the
686 future to other type of channels.

687 6. Conclusion

688 Experimental and theoretical slip regime approaches, developed previously,
689 were applied here to the gas flow through rectangular microchannels of differ-
690 ent aspect ratios, materials and roughnesses. Our measurements and model-
691 ings allowed us to extract the velocity slip coefficients using the Maxwell type
692 specular-diffuse gas-wall interaction model. We used two theoretical modelings
693 for the flow, taking into account the two-dimensional effect of the rectangular
694 cross-section. The first approach used a single accommodation coefficient for all
695 the walls (horizontal and vertical) of a channel. However, the second approach
696 employed two different accommodation coefficients: α_h for the two horizontal
697 walls and another coefficient α_w for the two vertical walls. Moreover, this pair
698 of coefficients remains the same for all channels of the same group.

699 Using the first method (single accommodation coefficient for all walls), it
700 appears that the aspect ratio does not significantly influence the values of the
701 velocity slip and tangential momentum accommodation coefficients and, in a
702 more general way, the physical properties characterizing the flow through the
703 channels of a same surface coating. This result gives credibility to the as-
704 sumption used in second method: existence of a same pair of accommodation
705 coefficients in each group of channels having the same surface coating.

706 Considering that the vertical walls have a higher roughness than the horizon-
707 tal ones, we deduced from the second method that in our roughness range (*i.e.*
708 for rather low roughness) with the present wall material (*Au*), the tangential
709 momentum accommodation coefficient decreases when the roughness increases.

710 In the present range of roughness, using the results obtained with different
711 wall materials, the calculations using the first method show that globally the
712 gold material (*Au*) is slightly more diffusive than silica (*SiO₂*).

713 We have observed, according the two methods and in our condition of low
714 roughness, that very small changes of the flow properties, notably of the accom-
715 modation coefficient occur when the molecular mass of the gas changes.

716 The previous comments show that the two modeling methods, proposed and
717 employed here, are rather consistent and complementary. The second approach
718 appears to potentially be able to detect some local properties of the gas-surface
719 interaction. Consequently, we plan in the future to develop and to extend the
720 application of this method in various directions: notably in rectangular channels

721 having horizontal and vertical walls differing by their wall material or roughness.
722 Furthermore, concerning the roughness influence on the accommodation process,
723 the problem is more general and more complex. It would be interesting to
724 investigate how varies the accommodation beyond the weak roughness studied
725 in the present article; *i.e.* in a much larger range of roughness, and it would
726 be useful to diversify the wall materials and the flow regimes to conduct these
727 investigations.

728 **7. Acknowledgments**

729 We wish to acknowledge the support of MIMENTO platform of technol-
730 ogy and especially FEMTO-ST Laboratory (<http://www.femto-st.fr>) for the mi-
731 crosystems fabrication and the measurement of the microchannels dimensions.
732 This research received funding from the European Community's Seventh Frame-
733 work Program (ITN FP7/2007-2013) under grant agreement number 215504.

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Microchannel	A1		A2	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	0.4	16 000	0.1	34 400
Inlet pressure p_{in} (Pa)	142.3	136 760	14.4	129 190
Outlet pressure p_{out} (Pa)	28.4	88 220	3.3	93 300
Average Knudsen number Kn_m	$1.7 \cdot 10^{-3}$	6.8	$1.8 \cdot 10^{-3}$	70.4

Table 2: Experimental conditions for the microchannels A1 and A2.

Microchannel	A3		A4	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	1.9	135 000	3.4	285 000
Inlet pressure p_{in} (Pa)	61.6	125 100	64.9	120 250
Outlet pressure p_{out} (Pa)	22.1	79 200	25.7	79 500
Average Knudsen number Kn_m	$1.7 \cdot 10^{-3}$	14.7	$2.1 \cdot 10^{-3}$	14.6

Table 3: Experimental conditions for the microchannels A3 and A4.

Microchannel	E1		E2	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	0.9	21 200	0.8	66 100
Inlet pressure p_{in} (Pa)	342.1	131 850	139.4	129 720
Outlet pressure p_{out} (Pa)	89.7	94 900	23.1	89 600
Average Knudsen number Kn_m	$1.1 \cdot 10^{-3}$	4.3	$1.4 \cdot 10^{-3}$	5.6

Table 4: Experimental conditions for the microchannels E1 and E2.

Microchannel	E3		E4	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	2.1	253 000	2.3	574 000
Inlet pressure p_{in} (Pa)	63.7	105 850	27.2	114 650
Outlet pressure p_{out} (Pa)	22.4	74 000	8.8	80 000
Average Knudsen number Kn_m	$6.6 \cdot 10^{-4}$	11.36	$1.6 \cdot 10^{-3}$	27.7

Table 5: Experimental conditions for the microchannels E3 and E4.

Microchannel	S1		S2	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	0.4	10 220	1.2	174 600
Inlet pressure p_{in} (Pa)	234.8	130 220	105.6	125 000
Outlet pressure p_{out} (Pa)	57.5	89 850	26.5	31 460
Average Knudsen number Kn_m	$2.0 \cdot 10^{-3}$	4.8	$1.6 \cdot 10^{-3}$	6.0

Table 6: Experimental conditions for the microchannels S1 and S2.

Microchannel	S3		S4	
Quantity	Min	Max	Min	Max
Mass flow rate \dot{M}^{exp} ($10^{-12}kg/s$)	4.3	372 400	3.9	952 700
Inlet pressure p_{in} (Pa)	66.3	131 000	25.9	124 380
Outlet pressure p_{out} (Pa)	22.9	108 650	9.1	93 300
Average Knudsen number Kn_m	$1.1 \cdot 10^{-4}$	9.3	$1.2 \cdot 10^{-4}$	24.0

Table 7: Experimental conditions for the microchannels S3 and S4.

829

830 Appendix A. Useful expressions

831 To provide the complete definition of all functions used in the analytical
832 derivation we give here the expressions for functions $(\phi_n)_0$, $(\chi_n)_0$ and $(\varphi_n)_0$,
833 which can be found also in Appendix of Ref. [17]:

$$(\phi_n)_0 = \frac{4(-1)^n}{k_n^3} \cos(k_n y/h) \frac{\cosh(k_n x/h) - \cosh(\omega_n)}{\cosh(\omega_n)}, \quad (\text{A.1})$$

834 where $k_n = \pi(2n + 1)$ and $\omega_n = k_n w/(2h)$.

$$(\chi_n)_0 = \frac{2(-1)^{n+1}}{k_n^2 \cosh(\omega_n)} \tanh(\omega_n) \cos(k_n y/h) \cosh(k_n x/h) \quad (\text{A.2})$$

835

$$(\varphi_n)_0 = \frac{2(-1)^{n+1}}{k_n \cosh(\omega_n)} (\cosh(\omega_n) - \cosh(k_n x/h)) \times \quad (\text{A.3})$$

$$\left(\frac{4}{k_n} \cos(k_n y/h) + \frac{2y}{h} \sin(k_n y/h) \right) -$$

$$\cos(k_n y/h) \left(\frac{2x}{h} \sinh(k_n x/h) - \frac{w}{h} \sinh(\omega_n) \right) -$$

$$\frac{w}{h} \tanh(\omega_n) \cos(k_n y/h) (\cosh(k_n x/h) - \cosh(\omega_n)). \quad (\text{A.4})$$

836

Channel	Gas	B_0^{exp}	B_1^{exp}	B_2^{exp}	s_r	r^2	$E_s(\%)$
A1	He	1.023± 0.008	10.625± 0.165	-	0.021	0.996	1.4
		1.019± 0.007	10.666± 0.142	1.635± 0.536	0.020	0.999	0.9
	N ₂	1.009± 0.005	12.104± 0.111	-	0.016	0.998	1.1
		1.008± 0.006	12.181± 0.171	-1.555± 0.850	0.020	0.999	1.1
	Ar	1.012± 0.009	12.621± 0.190	-	0.028	0.995	1.9
		1.027± 0.014	11.731± 0.390	3.115± 1.863	0.044	0.997	2.3
A2	He	1.001± 0.003	8.785 ± 0.056	-	0.008	0.999	0.6
		1.000± 0.008	8.681 ± 0.183	2.324± 0.729	0.024	0.999	1.3
	N ₂	0.996± 0.004	9.168 ± 0.098	-	0.015	0.997	1.1
		0.999± 0.008	8.701 ± 0.195	7.575± 0.812	0.027	0.999	1.5
	Ar	1.017± 0.005	9.580 ± 0.112	-	0.016	0.997	1.1
		1.014± 0.006	9.448 ± 0.157	6.699± 0.645	0.021	0.999	1.1
A3	He	1.011± 0.002	8.699± 0.046	-	0.006	0.999	0.5
		1.013± 0.003	8.426± 0.062	3.981± 0.237	0.009	0.999	0.5
	N ₂	1.003± 0.002	9.413± 0.042	-	0.007	0.999	0.5
		1.006± 0.002	9.047± 0.038	4.944± 0.144	0.006	0.999	0.3
	Ar	1.013± 0.003	10.078± 0.068	-	0.011	0.999	0.8
		1.017± 0.002	9.593± 0.060	5.767± 0.256	0.008	0.999	0.5
A4	He	0.988± 0.003	7.993± 0.073	-	0.008	0.998	0.6
		0.984± 0.005	7.974± 0.106	4.072± 0.442	0.014	0.999	0.8
	N ₂	0.989± 0.003	9.110± 0.080	-	0.010	0.998	0.8
		0.993± 0.005	8.660± 0.121	6.555± 0.464	0.018	0.999	1.0
	Ar	1.007± 0.003	9.326± 0.074	-	0.012	0.998	0.9
		1.008± 0.007	8.784± 0.177	9.661± 0.670	0.025	0.999	1.4

Table 8: Fitting coefficients of the dimensionless mass flow rate S^{exp} obtained with the first (first line) and second order (second line) approximations (44) in the mean Knudsen number ranges $[0, 0.1]$ and $[0, 0.3]$, respectively, for microchannels of the group A. The last three columns in this Table are: s_r is the squared residual sum, r^2 is the determination coefficient and $E_s(\%)$ is the standard error, see Section 4.1 for more details. We remind the dimensions of the microchannel A1 ($h = 27.8\mu\text{m}$, $w = 52.2\mu\text{m}$); A2 ($h = 27.6\mu\text{m}$, $w = 107.6\mu\text{m}$); A3 ($h = 27.9\mu\text{m}$, $w = 504.0\mu\text{m}$); A4 ($h = 25.8\mu\text{m}$, $w = 1005.5\mu\text{m}$), see Table 1.

Channel	Gas	molecular mass (g/mol)	σ_p	α	σ_{2p}
A1	He	4.00	1.263± 0.020	0.885± 0.008	-
			1.268± 0.017	0.883± 0.007	0.053± 0.018
	N ₂	28.02	1.338± 0.012	0.855± 0.005	-
			1.346± 0.019	0.851± 0.007	-0.043± 0.023
Ar	39.95	1.305± 0.020	0.867± 0.008	-	
		1.213± 0.040	0.906± 0.018	0.075± 0.045	
A2	He	4.00	1.109± 0.007	0.953± 0.003	-
			1.096± 0.023	0.960± 0.011	0.095± 0.030
	N ₂	28.02	1.076± 0.011	0.969± 0.006	-
			1.021± 0.023	0.997± 0.012	0.265± 0.028
Ar	39.95	1.052± 0.012	0.981± 0.006	-	
		1.038± 0.017	0.989± 0.009	0.206± 0.020	
A3	He	4.00	1.132± 0.006	0.943± 0.003	-
			1.096± 0.008	0.960± 0.004	0.192± 0.011
	N ₂	28.02	1.138± 0.005	0.940± 0.002	-
			1.094± 0.005	0.961± 0.002	0.203± 0.006
Ar	39.95	1.141± 0.008	0.938± 0.004	-	
		1.086± 0.007	0.965± 0.003	0.208± 0.009	
A4	He	4.00	1.044± 0.009	0.986± 0.005	-
			1.041± 0.014	0.987± 0.007	0.198± 0.022
	N ₂	28.02	1.106± 0.010	0.955± 0.005	-
			1.051± 0.015	0.982± 0.007	0.272± 0.019
Ar	39.95	1.060± 0.008	0.978± 0.004	-	
		0.998± 0.020	1.009± 0.011	0.351± 0.024	

Table 9: Slip and tangential momentum accommodation coefficients found experimentally from the first (first line) and second order (second line) approximations of the dimensionless mass flow rate S^{exp} (44), respectively, for microchannels of the group A. We remind the dimensions of the microchannel A1 ($h = 27.8\mu\text{m}$, $w = 52.2\mu\text{m}$); A2 ($h = 27.6\mu\text{m}$, $w = 107.6\mu\text{m}$); A3 ($h = 27.9\mu\text{m}$, $w = 504.0\mu\text{m}$); A4 ($h = 25.8\mu\text{m}$, $w = 1005.5\mu\text{m}$), see Table 1.

Channel	Gas	B_0^{exp}	B_1^{exp}	B_2^{exp}	s_r	r^2	$E_s(\%)$
<i>E1</i>	<i>He</i>	1.042± 0.001	8.133± 0.032	-	0.005	0.999	0.4
		1.042± 0.003	7.978± 0.061	3.081± 0.244	0.009	0.999	0.5
	<i>N₂</i>	1.037± 0.001	9.333± 0.033	-	0.005	0.999	0.4
		1.042± 0.003	8.926± 0.083	4.359± 0.329	0.013	0.999	0.7
	<i>Ar</i>	1.048± 0.001	10.146± 0.035	-	0.005	0.999	0.4
		1.054± 0.002	9.562± 0.053	6.682± 0.233	0.007	0.999	0.4
<i>E2</i>	<i>He</i>	1.030± 0.001	7.227± 0.025	-	0.004	0.999	0.3
		1.032± 0.001	6.972± 0.024	3.177± 0.098	0.003	0.999	0.2
	<i>N₂</i>	1.027± 0.001	8.103± 0.023	-	0.004	0.999	0.3
		1.026± 0.003	7.976± 0.066	2.880± 0.269	0.010	0.999	0.6
	<i>Ar</i>	1.039± 0.001	8.704± 0.035	-	0.006	0.999	0.4
		1.039± 0.003	8.599± 0.073	2.536± 0.302	0.011	0.999	0.7
<i>E3</i>	<i>He</i>	1.036± 0.001	6.728± 0.022	-	0.003	0.999	0.3
		1.039± 0.001	6.424± 0.022	3.810± 0.090	0.003	0.999	0.2
	<i>N₂</i>	1.033± 0.001	7.572± 0.025	-	0.004	0.999	0.3
		1.034± 0.001	7.354± 0.032	3.508± 0.130	0.005	0.999	0.3
	<i>Ar</i>	1.045± 0.001	8.220± 0.034	-	0.005	0.999	0.4
		1.046± 0.001	7.972± 0.033	3.773± 0.134	0.005	0.999	0.3
<i>E4</i>	<i>He</i>	1.054± 0.002	6.839± 0.044	-	0.007	0.999	0.5
		1.056± 0.003	6.612± 0.074	3.223± 0.290	0.011	0.999	0.7
	<i>N₂</i>	1.055± 0.003	7.614± 0.078	-	0.011	0.997	0.9
		1.055± 0.005	7.344± 0.135	4.930± 0.546	0.020	0.999	1.2
	<i>Ar</i>	1.062± 0.003	8.414± 0.071	-	0.010	0.998	0.8
		1.066± 0.006	7.980± 0.160	5.238± 0.641	0.024	0.999	1.4

Table 10: Fitting coefficients of the dimensionless mass flow rate S^{exp} obtained with the first (first line) and second order (second line) approximations in the mean Knudsen number ranges [0.0, 0.1] and [0.0, 0.3], respectively, for microchannels of the group *E*. The last three columns in this Table are: s_r is the squared residual sum, r^2 is the determination coefficient and $E_s(\%)$ is the standard error, see Section 4.1 for more details. We remind the dimensions of the microchannel *E1* ($h = 33.5\mu\text{m}$, $w = 55.5\mu\text{m}$); *E2* ($h = 35.2\mu\text{m}$, $w = 103.8\mu\text{m}$); *E3* ($h = 34.9\mu\text{m}$, $w = 505.0\mu\text{m}$); *E4* ($h = 34.2\mu\text{m}$, $w = 1001.3\mu\text{m}$), see Table 1.

Channel	Gas	molecular mass (g/mol)	σ_p	α	σ_{2p}
<i>E1</i>	<i>He</i>	4.00	0.947 ± 0.004	1.038 ± 0.002	-
			0.928 ± 0.007	1.048 ± 0.004	0.094 ± 0.007
	<i>N₂</i>	28.02	1.010 ± 0.004	1.003 ± 0.002	-
			0.966 ± 0.009	1.027 ± 0.005	0.115 ± 0.009
<i>Ar</i>	39.95	1.027 ± 0.004	0.994 ± 0.002	-	
		0.968 ± 0.005	1.026 ± 0.003	0.153 ± 0.005	
<i>E2</i>	<i>He</i>	4.00	0.899 ± 0.003	1.065 ± 0.002	-
			0.867 ± 0.003	1.084 ± 0.002	0.123 ± 0.004
	<i>N₂</i>	28.02	0.937 ± 0.003	1.043 ± 0.002	-
			0.922 ± 0.008	1.051 ± 0.004	0.095 ± 0.009
<i>Ar</i>	39.95	0.942 ± 0.004	1.040 ± 0.002	-	
		0.931 ± 0.008	1.047 ± 0.005	0.074 ± 0.009	
<i>E3</i>	<i>He</i>	4.00	0.874 ± 0.003	1.080 ± 0.002	-
			0.834 ± 0.003	1.104 ± 0.002	0.181 ± 0.004
	<i>N₂</i>	28.02	0.914 ± 0.003	1.056 ± 0.002	-
			0.888 ± 0.004	1.072 ± 0.002	0.142 ± 0.005
<i>Ar</i>	39.95	0.929 ± 0.004	1.048 ± 0.002	-	
		0.901 ± 0.004	1.064 ± 0.002	0.133 ± 0.005	
<i>E4</i>	<i>He</i>	4.00	0.892 ± 0.006	1.069 ± 0.003	-
			0.863 ± 0.010	1.087 ± 0.006	0.155 ± 0.014
	<i>N₂</i>	28.02	0.923 ± 0.009	1.051 ± 0.005	-
			0.891 ± 0.016	1.070 ± 0.010	0.203 ± 0.022
<i>Ar</i>	39.95	0.955 ± 0.008	1.033 ± 0.004	-	
		0.906 ± 0.018	1.061 ± 0.011	0.189 ± 0.023	

Table 11: Slip and tangential momentum accommodation coefficients found experimentally from the first (first line) and second order (second line) approximations of the dimensionless mass flow rate S^{exp} , for microchannels of the group *E*. We remind the dimensions of the microchannel *E1* ($h = 33.5\mu\text{m}$, $w = 55.5\mu\text{m}$); *E2* ($h = 35.2\mu\text{m}$, $w = 103.8\mu\text{m}$); *E3* ($h = 34.9\mu\text{m}$, $w = 505.0\mu\text{m}$); *E4* ($h = 34.2\mu\text{m}$, $w=1001.3\mu\text{m}$), see Table 1.

Channel	Gas	E_0^{exp}	E_1^{exp}	E_2^{exp}	s_r	r^2	$E_s(\%)$
S1	He	0.989± 0.003	9.380± 0.072	-	0.010	0.999	0.7
		0.988± 0.004	9.222± 0.085	3.497± 0.315	0.013	0.999	0.7
	N ₂	0.993± 0.003	10.203± 0.062	-	0.009	0.999	0.7
		0.984± 0.009	10.421± 0.220	3.158± 0.866	0.032	0.999	1.8
	Ar	1.000± 0.005	10.983± 0.112	-	0.017	0.997	1.2
		0.991± 0.011	11.098± 0.286	4.876± 1.138	0.042	0.998	2.2
S2	He	0.997± 0.007	9.582± 0.135	-	0.017	0.996	1.1
		0.997± 0.007	9.325± 0.147	4.825± 0.561	0.018	0.999	0.9
	N ₂	0.992± 0.004	10.941± 0.082	-	0.013	0.998	0.9
		0.989± 0.004	10.868± 0.104	4.101± 0.410	0.016	0.999	0.9
	Ar	1.007± 0.003	11.493± 0.082	-	0.013	0.998	0.9
		1.008± 0.005	11.313± 0.134	3.289± 0.537	0.021	0.999	1.1
S3	He	0.998± 0.002	9.468± 0.033	-	0.006	0.999	0.4
		1.002± 0.002	9.086± 0.042	4.918± 0.173	0.006	0.999	0.4
	N ₂	0.994± 0.003	10.085± 0.083	-	0.012	0.998	0.9
		0.993± 0.003	9.914± 0.082	5.369± 0.357	0.012	0.999	0.7
	Ar	1.006± 0.003	11.071± 0.080	-	0.011	0.998	0.8
		1.009± 0.003	10.722± 0.074	4.560± 0.315	0.011	0.999	0.6
S4	He	0.995± 0.002	9.515± 0.039	-	0.006	0.999	0.5
		0.996± 0.003	9.266± 0.080	4.015± 0.345	0.011	0.999	0.6
	N ₂	0.991± 0.002	10.096± 0.042	-	0.006	0.999	0.5
		0.993± 0.002	9.822± 0.049	4.284± 0.225	0.007	0.999	0.4
	Ar	1.002± 0.003	10.654± 0.071	-	0.011	0.999	0.8
		1.003± 0.003	10.386± 0.076	4.699± 0.330	0.011	0.999	0.7

Table 12: Fitting coefficients of the dimensionless mass flow rate S^{exp} obtained with the first (first line) and second order (second line) approximations in the mean Knudsen number ranges [0.0, 0.1] and [0.0, 0.3], respectively, for microchannels of the group S . The last three columns in this Table are: s_r is the squared residual sum, r^2 is the determination coefficient and $E_s(\%)$ is the standard error, see Section 4.1 for more details. We remind the dimensions of the microchannel $S1$ ($h = 24.3\mu\text{m}$, $w = 50.1\mu\text{m}$); $S2$ ($h = 42.3\mu\text{m}$, $w = 100.0\mu\text{m}$); $S3$ ($h = 42.0\mu\text{m}$, $w = 500.0\mu\text{m}$); $S4$ ($h = 41.5\mu\text{m}$, $w = 1000.0\mu\text{m}$), see Table 1.

Channel	Gas	molecular mass (g/mol)	σ_p	α	σ_{2p}
<i>S1</i>	<i>He</i>	4.00	1.130± 0.009	0.944± 0.004	-
			1.111± 0.010	0.953± 0.005	0.120± 0.011
	<i>N₂</i>	28.02	1.142± 0.007	0.938± 0.003	-
			1.167± 0.025	0.927± 0.011	0.093± 0.025
<i>Ar</i>	39.95	1.151± 0.012	0.934± 0.005	-	
		1.163± 0.030	0.928± 0.014	0.125± 0.029	
<i>S2</i>	<i>He</i>	4.00	1.168± 0.016	0.926± 0.007	-
			1.137± 0.018	0.940± 0.008	0.157± 0.018
	<i>N₂</i>	28.02	1.240± 0.009	0.894± 0.004	-
			1.232± 0.012	0.898± 0.005	0.115± 0.012
<i>Ar</i>	39.95	1.219± 0.009	0.903± 0.004	-	
		1.200± 0.014	0.912± 0.006	0.081± 0.013	
<i>S3</i>	<i>He</i>	4.00	1.227± 0.004	0.900± 0.002	-
			1.178± 0.005	0.922± 0.002	0.232± 0.008
	<i>N₂</i>	28.02	1.215± 0.010	0.905± 0.004	-
			1.195± 0.010	0.914± 0.004	0.218± 0.014
<i>Ar</i>	39.95	1.248± 0.009	0.891± 0.004	-	
		1.209± 0.008	0.908± 0.004	0.161± 0.011	
<i>S4</i>	<i>He</i>	4.00	1.240± 0.005	0.894± 0.002	-
			1.208± 0.010	0.908± 0.005	0.196± 0.017
	<i>N₂</i>	28.02	1.223± 0.005	0.902± 0.002	-
			1.190± 0.006	0.916± 0.003	0.180± 0.009
<i>Ar</i>	39.95	1.208± 0.008	0.908± 0.004	-	
		1.178± 0.009	0.922± 0.004	0.172± 0.012	

Table 13: Slip and tangential momentum accommodation coefficients found experimentally from the first (first line) and second order (second line) approximations of the dimensionless mass flow rate S^{exp} , respectively, for microchannels of the group *S*. We remind the dimensions of the microchannel *S1* ($h = 24.3\mu\text{m}$, $w = 50.1\mu\text{m}$); *S2* ($h = 42.3\mu\text{m}$, $w = 100.0\mu\text{m}$); *S3* ($h = 42.0\mu\text{m}$, $w = 500.0\mu\text{m}$); *S4* ($h = 41.5\mu\text{m}$, $w=1000.0\mu\text{m}$), see Table 1.

Channels	Gas	A_0	A_h	A_w	r^2
<i>A1</i> + <i>A2</i> + <i>A3</i> + <i>A4</i>	<i>He</i>	1.029	1.411	1.945	0.999
	<i>Ar</i>	1.011	1.564	2.858	0.997
<i>E1</i> + <i>E2</i> + <i>E3</i> + <i>E4</i>	<i>He</i>	1.040	1.118	1.373	0.999
	<i>Ar</i>	1.048	1.370	1.767	0.999
<i>S2</i> + <i>S3</i> + <i>S4</i>	<i>Ar</i>	1.032	1.654	3.645	0.930

Table 14: Fitting coefficients of the dimensionless mass flow rate S^{exp} , Eq. (52), obtained with the first order approximations in the mean Knudsen number ranges [0.0, 0.1]. The non-homogenous wall method is used. r^2 is the determination coefficient. The errors on the coefficients are not provided because they are smaller than 10^{-4} .

Channels	Gas	σ_{ph}	σ_{pw}	α_h	α_w
A1 + A2 + A3 + A4	<i>He</i>	1.110	1.529	0.953	0.786
	<i>Ar</i>	1.069	1.955	0.973	0.666
E1 + E2 + E3 + E4	<i>He</i>	0.879	1.080	1.077	0.968
	<i>Ar</i>	0.937	1.209	1.043	0.908
S2 + S3 + S4	<i>Ar</i>	1.131	2.493	0.943	0.557

Table 15: Slip and tangential momentum accommodation coefficients calculated from the measured mass flow rate by using the second approach with two accommodation coefficients, α_h and α_w , for the horizontal and vertical walls, respectively. The results for two groups of microchannels de types A, E and S are presented for two gases *He* and *Ar*, of molecular masses 4.00 and 39.95, respectively.

837 Appendix B. Microchannels of group E

838 From Table 10, one can see that the values of the fitting coefficients, B_0^{exp} ,
839 obtained with the first and second order polynomial form, are different from
840 the theoretical value $B_0^T = 1$, see Eq. (43). This surprising discrepancy, greater
841 than experimental uncertainty, may be caused by an error in the measurement
842 of microchannels dimensions (h and/or w) or by an error on the mass flow rate
843 measurements. The possible reasons for these surprising results are discussed
844 below. Several assumptions are made and analyzed in order to find the causes
845 of such results.

846 1. Due to the higher value of the coefficients B_0^{exp} , compared to 1, see Ta-
847 ble 10, the first explanation on the obtained results is a possible error
848 in the measurements of the microchannels dimensions. By bringing back
849 the values of the coefficients B_0^{exp} to 1, either by increasing the value of
850 the microchannels height h (or width w), or both at the same time, the
851 values of coefficient σ_p decrease and then the values of the coefficient α
852 increase further. Thus, this hypothesis does not seem to solve the prob-
853 lem. In addition, the same method of cross-section measurements is used
854 for the other microchannels of groups *A* and *S*, where reasonable results
855 were obtained. Therefore, we eliminate the hypothesis of an error in the
856 microchannels dimensions.

857 2. The second hypothesis is a possibility of an error in the experimental mass
858 flow rate \dot{M}^{exp} , extracted from pressure measurements by using Eq. (4).
859 In this case, there are two possible sources for such kind of errors: first,
860 the measurement errors of the parameters used to calculate mass flow rate
861 (essentially tank volumes), second, a leak or an outgassing problem caused
862 by mishandling.

863 The possibility of an error in the measurements of the tanks volume was
864 checked and cancelled. Very simple and accurate water weighting techni-
865 que was used to measure the tanks volume. In addition, the measure-
866 ments were repeated several times each and the results of these measure-
867 ments were different within the experimental uncertainty of the volume

868 (less than 2.0%).
869 The second possible source of the problems is the outgassing or leak. The
870 outgassing and the leak are checked before each experiment, then these
871 two factors cannot be the sources of problems.
872 3. In the same time, multiple reasons push us to think that the gold layer
873 takeoff is the cause of B_0^{exp} coefficient and the α values to be different
874 from their theoretical values:

- 875 • First, after eliminating all the previous hypotheses, it is the only one
876 that seems to be reasonable and tangible.
- 877 • Second, during the preliminary tests of microchannels we have en-
878 countered similar problem of gold layer takeoff with other microchan-
879 nels, where the takeoff blocked completely the microchannels.

880 For the microchannels E we think that the takeoff happened inside the
881 microchannels, that is why the verification which was conducted with the
882 microscope did not reveal anything. Further verification would involve the
883 use of very expensive cutting technique and could not be realized in the
884 frame of our project.

885 4. Moreover, another verification of the theoretical nature in this time, was
886 used and it concerns the assimilation of the cross-section of the microchan-
887 nels E to a circular cross-section and to calculate the equivalent hydraulic
888 diameter. Using this technique of the hydraulic diameter calculation and
889 the analytical expression of the mass flow rate for tubes, the velocity slip
890 coefficient (σ_p) and TMAC (α) were calculated and their new values seem
891 to be more reasonable, *i.e.* in the interval determined by the Maxwell
892 model. The resulting TMAC values are in the range $[0.9, 1.0]$ for the
893 microchannels $E1$ and $E2$ and are of the order of 0.7 and 0.6 for the mi-
894 crochannels $E3$ and $E4$, respectively. Low TMAC values obtained for the
895 microchannels $E3$ and $E4$ are due to the large aspect ratio of these mi-
896 crochannels, hence, their assimilation to a circular section is not pertinent.

897 Finally, it was shown that the probable factor influencing the results of the
898 microchannels E is the gold layer takeoff inside the microchannels, which causes
899 the change of their cross-section shape. The change of the microchannels cross-
900 section induces that the theoretical model used here for rectangular channels is
901 not adapted for the analysis.

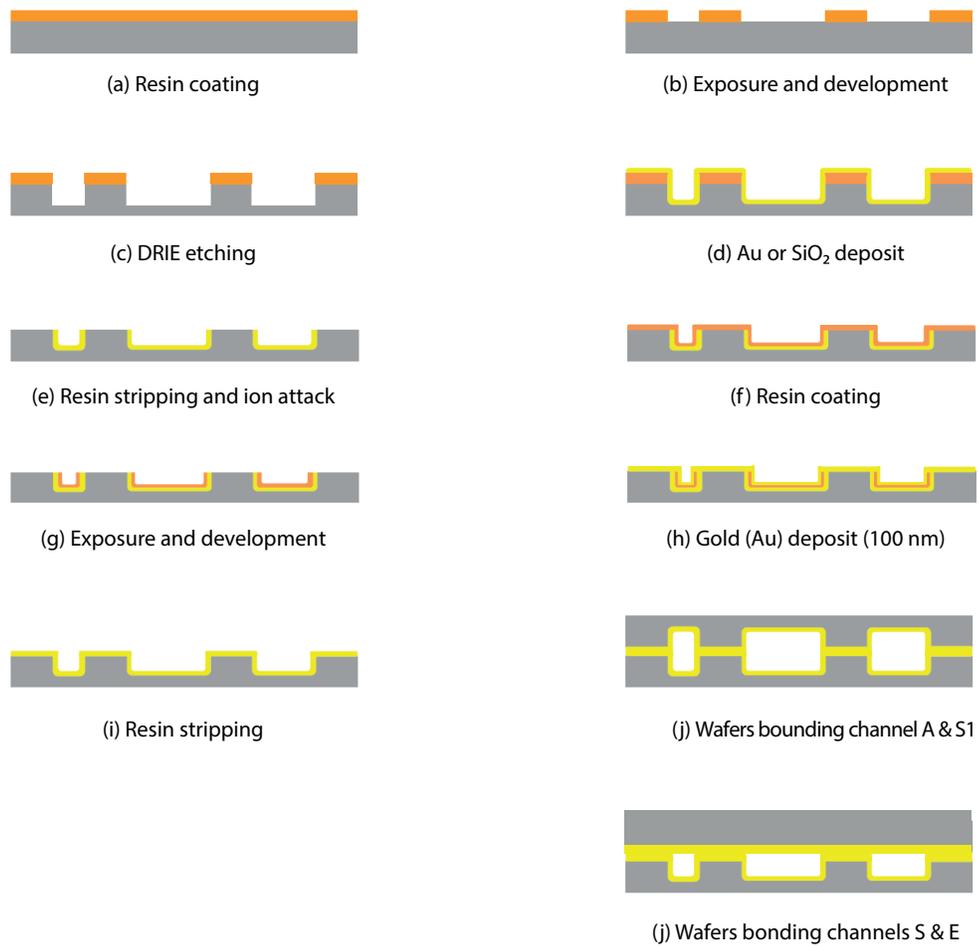


Figure 1: Rectangular microchannels fabrication process.

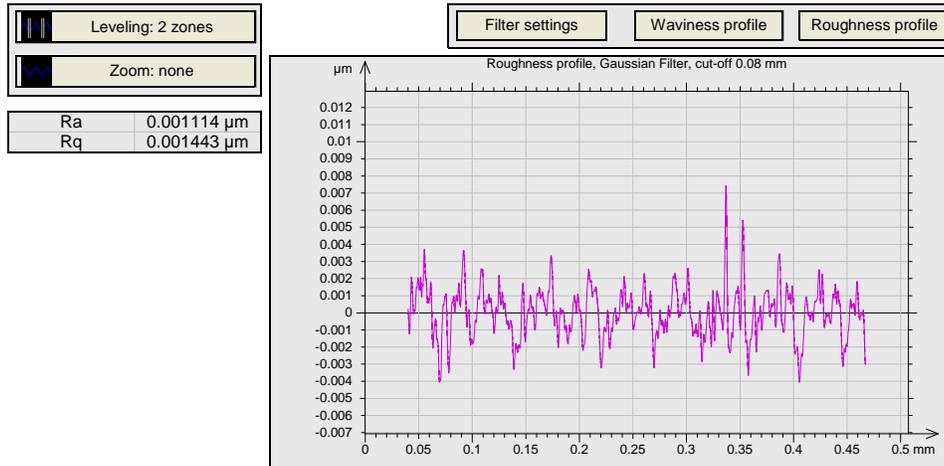


Figure 2: Graph of the roughness for the microchannel S3.

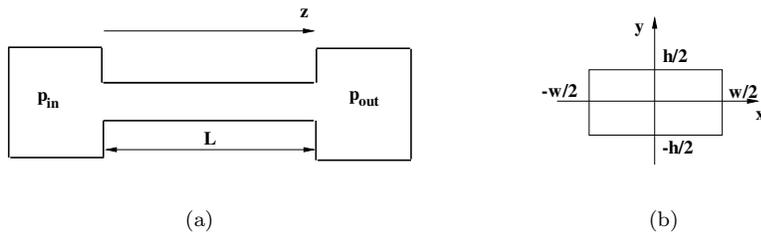


Figure 3: (a) Physical domain, (b) channel cross-section.

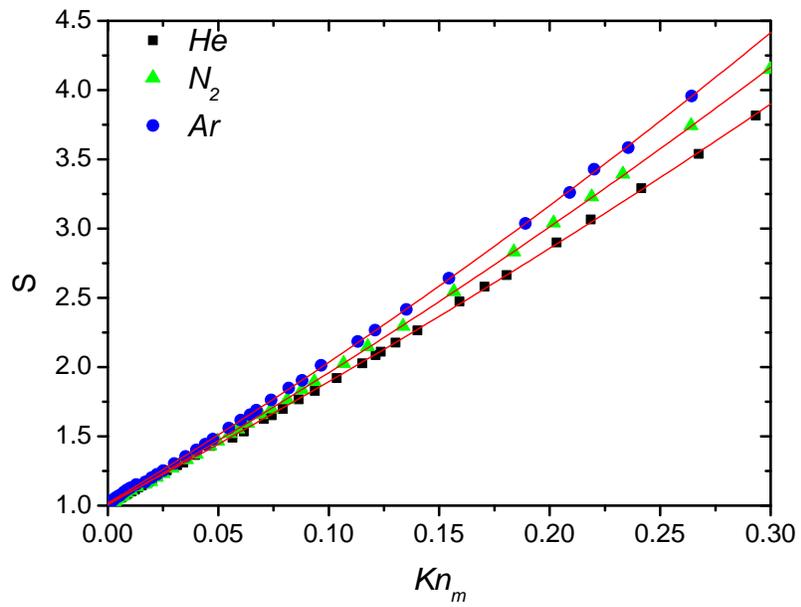
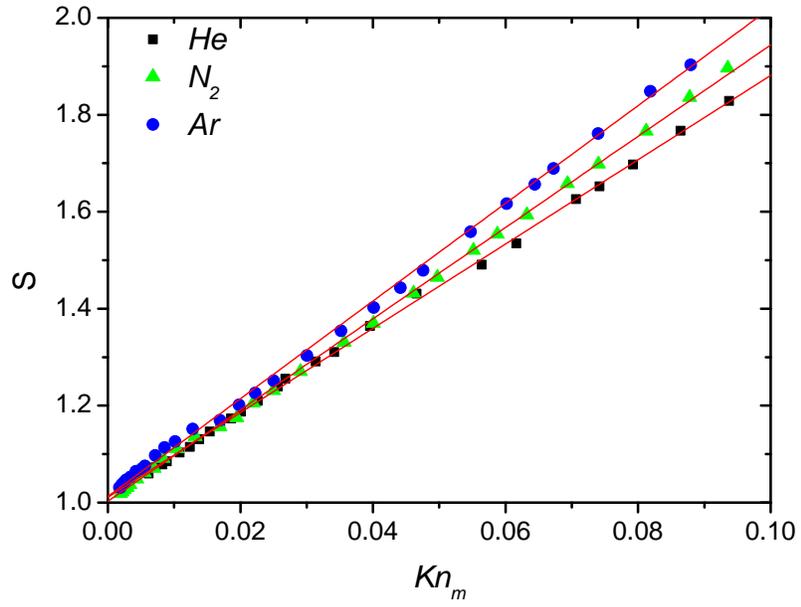


Figure 4: Dimensionless mass flow rate S^{exp} (44) plotted as function of the mean Knudsen number, fitted with first and second orders polynomial expressions for microchannel A3, upper and down figure, respectively.

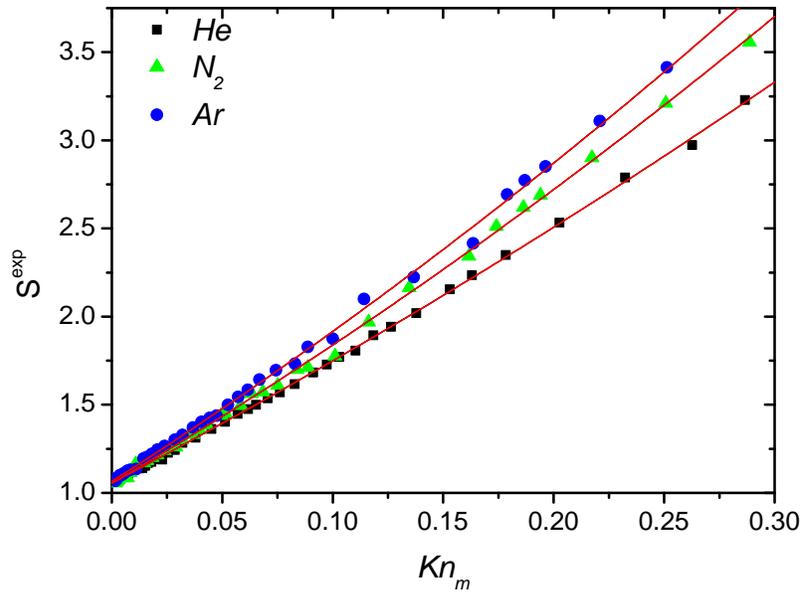
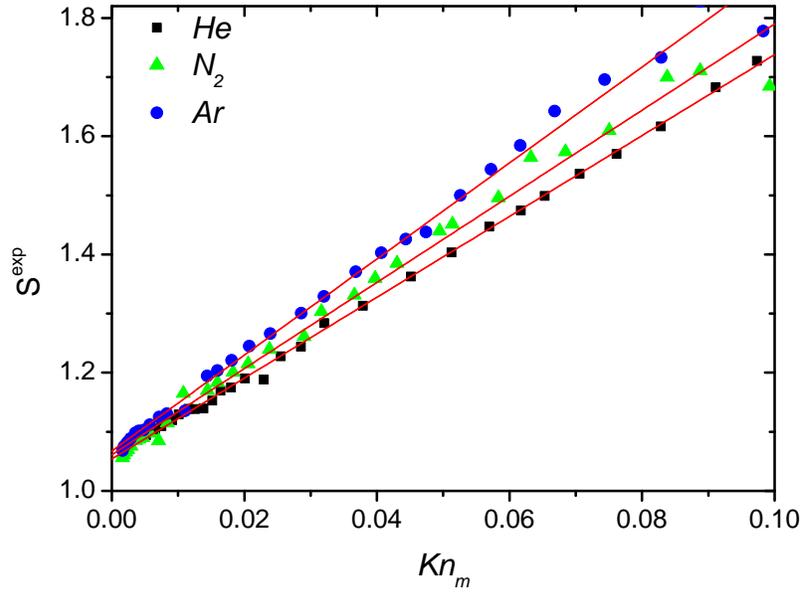


Figure 5: Dimensionless mass flow rate S^{exp} plotted as function of the mean Knudsen number, fitted with first and second order polynomial expressions on the upper and down plots, respectively, for the microchannels $E4$.

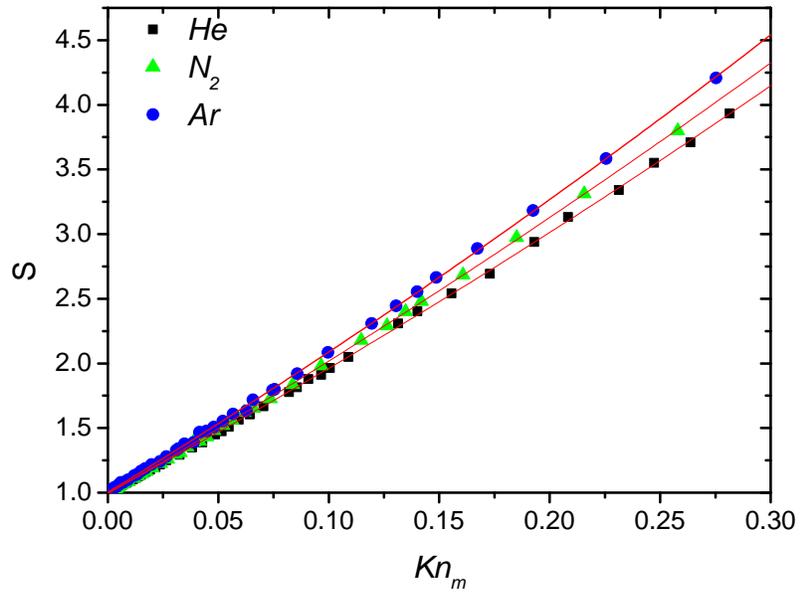
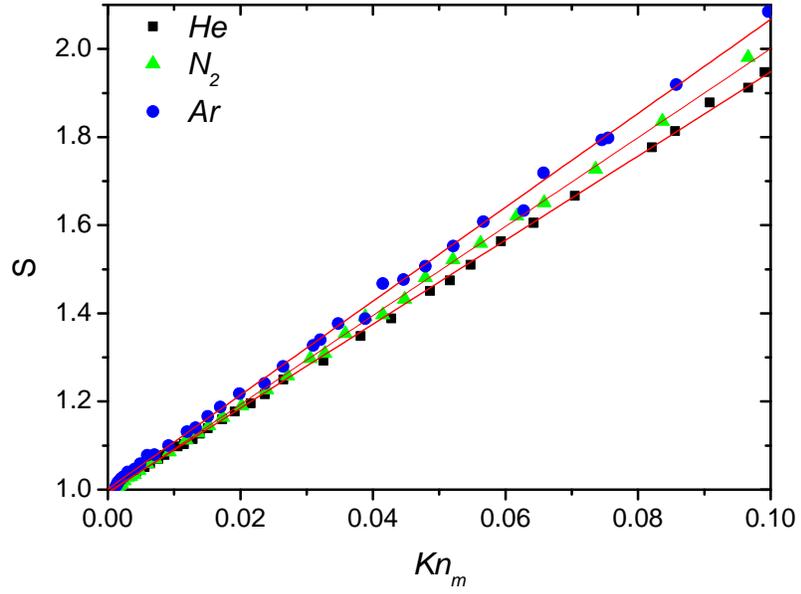


Figure 6: Dimensionless mass flow rate S^{exp} plotted as function of the mean Knudsen number, fitted with first and second order polynomial expressions, upper and down plots, respectively, for the microchannels $S4$.

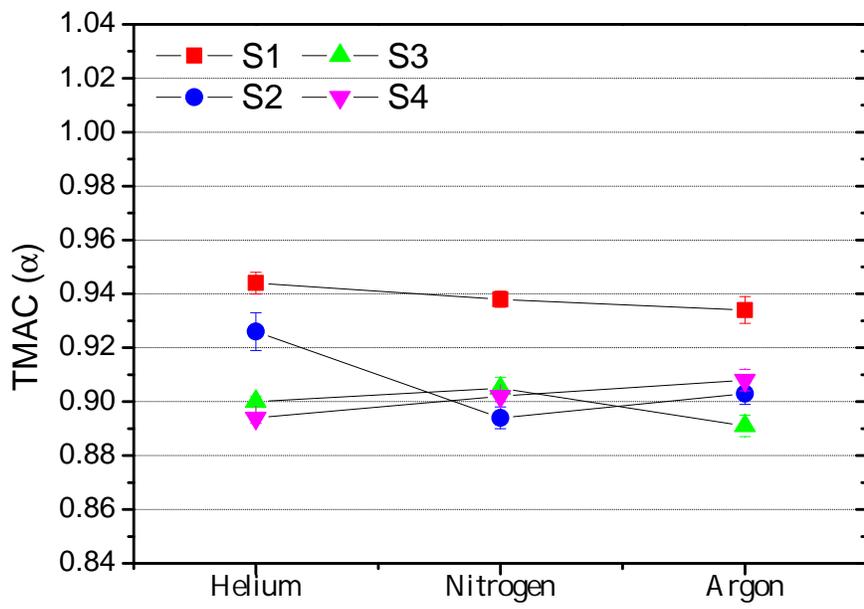


Figure 7: The TMAC for the microchannel of group S , obtained with the first order treatment.