ON/OFF Control Trajectory Computation for Steady State Reaching in Batches Petri Nets

Ruotian Liu, Rabah Ammour, Leonardo Brenner, Isabel Demongodin

ON/OFF control trajectory computation for steady state reaching in batches Petri nets

Ruotian Liu, Rabah Ammour, Leonardo Brenner, and Isabel Demongodin

Aix-Marseille University, University of Toulon, CNRS, LIS, Marseille, France
{ruotian.liu, rabah.ammour, leonardo.brenner,
isabel.demongodin}@lis-lab.fr

Abstract. This paper, dedicated to controlled generalized batches Petri nets without discrete nodes, presents a method for computing a control trajectory for reaching a steady state from a given initial marking. A steady state is characterized by a state in which the marking and the firing flow vector are constant. By controlling the firing flow vector of transitions, the proposed control strategy is an event-based one and relies on an algorithm solving a linear programming problem. This new control strategy, called in this paper as maximal flow based ON/OFF control, exploits the maximal firing flows and reduces the delay of the transient behavior for reaching the steady state. A practical communication system is provided to illustrate the relevance of such a control strategy for Cyber-Physical Systems.

Keywords: Petri nets · hybrid systems · event-driven control · steady state.

1 Introduction

The prevalence of Cyber-Physical Systems (CPSs) in a wide range of critical applications requires the development of efficient control and monitoring methods for such systems. Since the communication between entities of a CPS is carried out through network and internet, this type of system is vulnerable to attacks and threats [6]. To deal with the modeling, control and analysis of CPSs, the use of Discrete Event Systems (DESs) formalisms, such as Automata and Petri Nets (PNs), has been widely considered [10].

Some important works have been devoted to the control problems of such models with continuous Petri nets under infinite server semantics, applying discrete-time control such as ON/OFF controllers [11] or Model Predictive Control [9]. As studied by [1], the event-driven control strategies provide an alternative to time-driven ones in hybrid systems where the control action is updated when an event occurs. In this spirit, [7] have proposed an event-driven control of timed hybrid Petri nets.

In this work, we consider controlled Generalized Batches Petri Nets (cGBPNs) [3] with the transition firing flows as control inputs. Such models enrich the class of hybrid Petri nets [2] by introducing a new kind of nodes, called batch nodes.

Batch transitions act as continuous ones with finite server semantics while batch places are hybrid ones defined by three continuous characteristics: a length, a transfer speed and a maximal density. Based on the concept of batches as marking, i.e., a group of entities moving through a batch place at a certain speed, cGBPNs allow variable delays on continuous flows to be represented. cGBPNs behavior is based on a switching continuous-time and event-driven dynamics.

A control strategy of cGBPNs was presented in our prior work [8] which is steady flow based ON/OFF control strategy. It assumes that the controlled firing flow of each transition cannot exceed its steady firing flow. Although the convergence to the steady state using this strategy was proved, the main drawback of using steady flows rather than the maximal values is the impact on the time performance. The main contribution of this work is to exploit the maximal firing flows when it is possible to improve the convergence delay. Since a certain configuration of the moving entities characterized by a steady state density should be reached, the use of the maximal firing flows is restricted and is only feasible when some conditions that are developed are satisfied.

The remainder of this paper is structured as follows. In Section 2, some preliminaries on cGBPNs and their steady states are presented. A running example is introduced and used throughout the paper to illustrate the different notions. Section 3 formalizes the control problem and the steady flow based ON/OFF control is recalled. In Section 4, the new proposed control strategy and the computation of the control trajectory are detailed. A communication system illustrates the presented approach in Section 5. Finally, some conclusions and future works are presented in Section 6.

2 Preliminaries

In this section, we introduce some basic definitions on controlled generalized batches Petri nets (cGBPNs) and identify the steady states.

2.1 Generalized Batches Petri nets

We assume the reader to be familiar with hybrid Petri nets such as defined by [2]. For more details on batches Petri net formalisms, we refer readers to [3, 5].

Definition 1. *A* Generalized Batches Petri net *(GBPN) is a 6-tuple* $N = (P, T, Pre, Post, \gamma, Time)$ *where:*

- $P = P^D \cup P^C \cup P^B$ *is finite set of places partitioned into the three classes of* discrete, continuous *and* batch *places.*
- $T = T^D \cup T^C \cup T^B$ *is finite set of transitions partitioned into the three classes of* discrete, continuous *and* batch *transitions.*
- $Pre, Post : (P^D \times T \to \mathbb{N}) \cup ((P^C \cup P^B) \times T \to \mathbb{R}_{\geq 0})$ [1] *are, respectively, the pre-incidence and post-incidence matrices, denoting the weight of the arcs from places to transitions and transitions to places.*

[1] We denote $\mathbb{R}_{\geq 0}$ (resp., $\mathbb{R}_{>0}$) the set of nonnegative (resp., positive) real numbers and $\mathbb{N}$ the set of natural numbers.

– $\gamma : P^B \to \mathbb{R}^3_{>0}$ *is the* batch place function. *It associates to each batch place* $p_i \in P^B$ *the triple* $\gamma(p_i) = (V_i, d_i^{\max}, s_i)$ *that represents, respectively,* maximal transfer speed, maximal density *and* length *of* p_i.
– $Time : T \to \mathbb{R}_{\geq 0}$ *associates a nonnegative number to every transition:*
  - *if* $t_j \in T^D$, *then* $Time(t_j) = \psi_j$ *denotes the* firing delay *associated with the discrete transition;*
  - *if* $t_j \in T^C \cup T^B$, *then* $Time(t_j) = \Phi_j$ *denotes the* maximal firing flow *associated with the continuous or batch transition.* ■

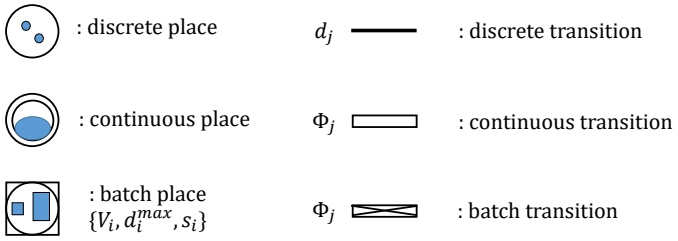


Fig. 1: Nodes of a GBPN.

The cardinality of the sets of places and transitions is denoted as $n^Y = |P^Y|$ and $h^Y = |T^Y|$, respectively, with $Y \in \{D, C, B\}$. Note that n and h are respectively the total number of places and transitions. Each node of a GBPN has a graphic representation, as shown in Fig. 1. The marking (i.e., net state) at time τ is a vector $\boldsymbol{m}(\tau) = [m_1(\tau)\ m_2(\tau) \dots m_n(\tau)]^T$ that assigns to each discrete place a nonnegative integer, to each continuous place a nonnegative real number and assigns to each batch place p_i, a series of ordered batches, $m_i(\tau) = \left\{\beta_i^1(\tau), \dots, \beta_i^r(\tau)\right\}$. A batch is a group of discrete entities, characterized by three continuous variables, a length (space unit), a density (number of entites/space unit) and a head position (space unit).

Definition 2. *A batch* β_k *at time* τ *is defined by a triple* $\beta_k(\tau) = (l_k(\tau), d_k(\tau), x_k(\tau))$, *where* $l_k(\tau) \in \mathbb{R}_{\geq 0}$ *is the length,* $d_k(\tau) \in \mathbb{R}_{\geq 0}$ *is the density and,* $x_k(\tau) \in \mathbb{R}_{\geq 0}$ *is the head position.* ■

Note that the initial marking is denoted by $\boldsymbol{m}_0$ and time τ will be omitted in the rest of paper when there is no ambiguity.

Each batch place p_i is characterized by a maximal capacity given by $Q_i = s_i \cdot d_i^{\max}$. The output density of a batch place p_i, denoted d_i^{out}, is the density of the batch $\beta_i^r(\tau)$ whose head position is equal to the length of the batch place (i.e., $x_i^r(\tau) = s_i$), then $d_i^{\text{out}}(\tau) = d_i^r(\tau)$, else $d_i^{\text{out}}(\tau) = 0$. The batch $\beta_i^r(\tau)$ is called *output batch* of p_i. A batch with a density that equals the maximal density of a batch place, i.e., $d_i^r(\tau) = d_i^{max}$, is called a *dense batch.*

Due to the existence of batches inside a batch place, we define the marking quantity as the total quantity of a place. The marking quantity of a continuous or discrete place is equal to its marking while it corresponds for a batch place to the sum of the quantities of all batches contained inside. The marking quantity vector is formally defined as follows:

Definition 3. *The* marking quantity vector $\boldsymbol{q} = \mu(\boldsymbol{m}) \in \mathbb{R}^n_{\geq 0}$ *associated with a marking* $\boldsymbol{m}$ *is defined as follows:*

$$q_i = \begin{cases} m_i & \text{if } p_i \in P^D \cup P^C, \\ \sum\limits_{\beta_i^k \in m_i} l_i^k \cdot d_i^k & \text{if } p_i \in P^B. \end{cases}$$

■

Remark 1. $\mu(\boldsymbol{m})$ is an injective mapping which associates a given marking $\boldsymbol{m}$ to a single marking quantity vector $\boldsymbol{q}$. Its inverse is not, since more than one marking $\boldsymbol{m}$ may correspond to a given marking quantity vector $\boldsymbol{q}$.

The dynamics of a GBPN, ruled by the firing of enabled transitions and the hybrid dynamics inside batch places, uses the notion of *instantaneous firing flow vector* (IFF). More precisely, the IFF $\varphi_j(\tau) \leq \Phi_j$ of a transition $t_j \in T^C \cup T^B$ is used to represent the firing quantity by time unit. The IFF vector at time τ is denoted by $\boldsymbol{\varphi}(\tau) \in \mathbb{R}^{h^C + h^B}$. The *input* and *output flow* of a batch or continuous place p_i at time τ are the sum of all flows entering and leaving the place, respectively, denoted by: $\phi_i^{\text{in}}(\tau) = \text{Post}(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$ and $\phi_i^{\text{out}}(\tau) = \text{Pre}(p_i, \cdot) \cdot \boldsymbol{\varphi}(\tau)$.

The behavior of a GBPN is based on a timed discrete event dynamics. Between two timed events, the net state is characterized by an invariant behavior state (IB-state) [4] with linear or constant continuous evolution. Thus, the marking in discrete places, the firing flow of continuous and batch transitions and the output density of batch places are constant within an IB-state. Note that, as $\boldsymbol{\varphi}$ is constant in an IB-state, $\boldsymbol{\phi}^{\text{in}}$ and $\boldsymbol{\phi}^{\text{out}}$ are also constants between timed events.

The evolution of a GBPN is characterized by a fundamental equation expressed by:

$$\boldsymbol{q}(\tau) = \boldsymbol{q}(\tau_0) + \boldsymbol{C} \cdot \boldsymbol{z}(\tau) \tag{1}$$

where $\boldsymbol{C} = \text{Post} - \text{Pre}$ and $\boldsymbol{z}(\tau) \in \mathbb{R}^h_{\geq 0}$, called *firing quantity vector*, denotes the firing count of each transition during time interval $[\tau_0, \tau]$. Note that $\boldsymbol{z}(\tau_0) = \boldsymbol{0}$.

Example 1. Let us consider the GBPN represented in Fig. 2 composed of two continuous places and two batch places. The characteristics of each batch place (maximal transfer speed, maximal density and length) are depicted on the figure. The initial marking is $\boldsymbol{m}_0 = [8\ 8\ \emptyset\ \emptyset]^T$ which means that both batch places are empty. The maximal capacity of batch places are $Q_3 = 8$ and $Q_4 = 10$. Since the only enabled transition is t_1, the IFF vector is given by $\boldsymbol{\varphi}(\tau_0) = [2\ 0\ 0]^T$ at date $\tau_0 = 0$. Consequently, in the batch place p_3, a new batch is created as $\beta_3^1(\tau_0) = (0 \cdot V_3, \phi_3^{\text{in}}(\tau_0)/V_3, 0 \cdot V_3) = (0, 2, 0)$. At date $\tau_1 = 4$, this batch becomes

an output batch as $\beta_3^1(\tau_1) = (4 \cdot V_3, 2, 4 \cdot V_3) = (4, 2, 4)$ and a new IFF vector $\boldsymbol{\varphi}(\tau_1) = [2\ 2\ 0]^T$ is applied since transition t_2 is also enabled. Thus the time interval $[\tau_0, \tau_1]$ corresponds to an IB-state. Note that, whatever the evolution, it holds : $q_1 + q_3 = 8$ and $q_2 + q_4 = 8$.

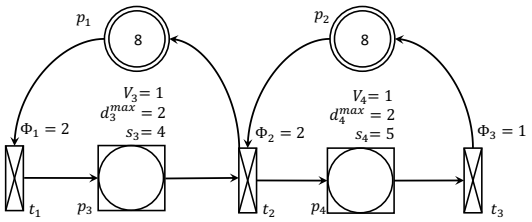


Fig. 2: A cGBPN system with initial marking $\boldsymbol{m}_0$.

2.2 Controlled GBPN and steady state

In our work, we consider the instantaneous firing flow of continuous and batch transitions as control input. This leads to the following definition.

Definition 4. *A controlled Generalized Batches Petri net (cGBPN) is a GBPN N for which a control input $\boldsymbol{u}(\tau) = \boldsymbol{\varphi}(\tau)$ is defined with $0 \leq \boldsymbol{u}(\tau) \leq \boldsymbol{\Phi}$.* ■

The following definition, taken from [3], gives the definition of a steady state of cGBPN.

Definition 5. *(Steady state) Let $\langle N, \boldsymbol{m}_0 \rangle$ be a cGBPN with $P^D = T^D = \emptyset$. The net is in* a steady state *at time τ_s if for $\tau \geq \tau_s$ the marking $\boldsymbol{m}^s$ and the instantaneous firing flow vector $\boldsymbol{\varphi}^s$ remain constant. Thus a steady state is defined by a pair $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$. The marking quantity vector associated with a steady marking $\boldsymbol{m}^s$ is denoted as $\boldsymbol{q}^s = \mu(\boldsymbol{m}^s)$.* ■

For a cGBPN with $P^D = T^D = \emptyset$, with an initial marking quantity vector $\boldsymbol{q}^0$ and a given steady marking quantity vector $\boldsymbol{q}^s$, we denote by $\boldsymbol{z}^s$ the *steady firing quantity vector*, i.e., the minimum firing quantity vector, that satisfies: $\boldsymbol{z}^s \geq \boldsymbol{0}$, and $\min \boldsymbol{z}^s$, s.t. $\boldsymbol{q}^s = \boldsymbol{q}^0 + \boldsymbol{C} \cdot \boldsymbol{z}^s$.

In a steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ where $\boldsymbol{\varphi}^s > 0$, it has been proved in [3] that the marking of a batch place p_i has one of the following regular forms:

1. A single batch : $m_i^s = \{\beta_i^o\}$ with $\beta_i^o = (s_i, d_i^o, s_i)$ and density $d_i^o \leq d_i^{\max}$. In other terms, the steady state marking of p_i is composed by a single (dense or not) output batch whose length is equal to the length of p_i.
2. Two batches : $m_i^s = \{\beta_i^e, \beta_i^o\}$ with $\beta_i^e = (l_i^e, d_i^e, l_i^e)$, $\beta_i^o = (l_i^o, d_i^{\max}, s_i)$ and $l_i^e + l_i^o = s_i$. The steady state marking of p_i is composed by a dense output batch in contact with one input batch.

In [3], ($\boldsymbol{q}^s$, $\boldsymbol{\varphi}^s$) of a cGBPN without discrete nodes, has been computed by solving a linear programming problem that only considers the net structure and the initial marking (see Proposition 4.5 in [3]). From ($\boldsymbol{q}^s$, $\boldsymbol{\varphi}^s$), the steady state marking $\boldsymbol{m}^s$ could be characterized (see Proposition 4.4 in [3]). Note that, in a steady state, the input flow and output flow of place p_i are equal and denoted as ϕ_i^s.

Definition 6. *Let $\langle N, \boldsymbol{m}_0\rangle$ be a cGBPN with $P^D = T^D = \emptyset$ and $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ a reachable steady state. The* event-driven control problem *for reaching $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ deals with the computation of a timed control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^s, \tau_s)$ feasible in the cGBPN from $\boldsymbol{m}_0$ such that $\boldsymbol{m}(\tau_f) = \boldsymbol{m}^s$ and $\boldsymbol{u}(\tau_f) = \boldsymbol{u}^s = \boldsymbol{\varphi}^s, \forall \tau_f \geq \tau_s$.* ■

Note that the controlled firing flow vector $\boldsymbol{u}^i$ is applied at date τ_i and remains constant until τ_{i+1}, corresponding to a timed interval of an IB-state. When a cGBPN is without discrete nodes, the fundamental equation Eq.(1) for the control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots, (\boldsymbol{u}^i, \tau_i), \cdots, (\boldsymbol{u}^s, \tau_s)$ becomes:

$$\boldsymbol{q}(\tau_s) = \boldsymbol{q}^0 + \boldsymbol{C} \cdot \left(\int_{\tau_0}^{\tau_1} \boldsymbol{u}^0 \cdot d\rho + \cdots + \int_{\tau_{s-1}}^{\tau_s} \boldsymbol{u}^{s-1} \cdot d\rho\right). \tag{2}$$

where $\int_{\tau_{i-1}}^{\tau_i} \boldsymbol{u}^{i-1} \cdot d\rho$ denotes the sum of firing quantity of continuous or batch transitions during time interval $[\tau_{i-1}, \tau_i]$.

Example 2. A reachable steady state for the net presented in Example 1 is given by : $\boldsymbol{m}^s = [4\ 0\ \{(4,1,4)\}\ \{(2,1,2),(3,2,5)\}]^T$ (see Fig. 3) and $\boldsymbol{\varphi}^s = [1\ 1\ 1]^T$. Batch place p_3 is in the first regular form with only one batch inside $\beta_3^o(\tau \geq \tau_s) = (4,1,4)$ while batch place p_4 has two batches: a dense output batch $\beta_4^o(\tau \geq \tau_s) = (3,2,5)$ in contact with an input batch $\beta_4^e(\tau \geq \tau_s) = (2,1,2)$. The marking remains constant since the input and output flow of each place are equal.

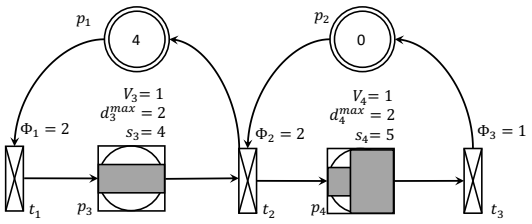


Fig. 3: The cGBPN of Example in a steady state.

3 Steady flow based ON/OFF control

The problem addressed in this paper is the control of the transient behavior of a cGBPN. More specifically, the objective is to compute a control trajectory that drives the net from a given initial marking $\boldsymbol{m}_0$ to a reachable steady

state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$. Such a control has been proposed in [8] denoted as steady flow based ON/OFF control, in this paper. This section aims to present this previous proposed control strategy. In the rest of the paper, we consider the following assumptions already done in [8].

A1 No discrete nodes ($P^D = T^D = \emptyset$).
A2 The steady firing flow vector is positive ($\boldsymbol{\varphi}^s > \mathbf{0}$).
A3 The net is conservative.

Assumption (A1) preserves the restricted class of cGBPNs for which the steady state analysis has been studied [3]. Assumption (A2) ensures that the net is consistent, i.e., there exits a positive T-semiflow. Assumption (A3) imposes that each place is contained in the support of a P-semiflow.

The basic idea of steady flow based ON/OFF control is, from the initial marking, to firstly reach the steady marking quantity vector and then to reach the steady state marking. More precisely, enabled transitions that at least one of their input places has less marking quantity than its steady marking quantity, are blocked. For the other enabled transitions, their controlled firing flows are maximized to their steady flows. For any place that reaches its steady marking quantity, an additional constraint imposes that its input flow equals its output flow. This control law is applied until the target steady state marking is reached. One can remark that the firing flow of a transition cannot exceed its steady flow. To illustrate this approach, the introduced example is considered again.

Example 3. Let us consider again the cGBPN depicted in Fig. 2 with initial marking $\boldsymbol{m}_0 = [8\ 8\ \emptyset\ \emptyset\}]^T$. The control trajectory, obtained by the steady flow based ON/OFF approach, to reach the steady state $\boldsymbol{m}^s = [4\ 0\ \{(4,1,4)\}\ \{(2,1,2),(3,2,5)\}]^T$ showed in Fig. 3 is given by $(\boldsymbol{u}^0, 0), (\boldsymbol{u}^1, 4), (\boldsymbol{u}^2, 9), (\boldsymbol{u}^3, 12)$, with $\boldsymbol{u}^0 = [1\ 0\ 0]^T$, $\boldsymbol{u}^1 = [1\ 1\ 0]^T$, $\boldsymbol{u}^2 = [1\ 1\ 0]^T$, $\boldsymbol{u}^3 = [1\ 1\ 1]^T$. At the initial marking, only the flow of transition t_1 is maximized to the steady flow since only t_1 is enabled and its input place has greater marking quantity than the steady one. At date $\tau_1 = 4$, batch place p_3 has reached its steady marking quantity and transition t_2 is enabled. The input and output controlled flows of p_3 are imposed to be equal as order to hold the marking quantity of p_3 and to feed place p_4. At date $\tau_2 = 9$, the batch created in p_4 becomes an output batch, transition t_3 is enabled and blocked since the input place p_4 has still less marking quantity than the steady marking quantity. At date $\tau_3 = 12$, all places reach the steady marking and the steady flow vector is imposed.

The use of the steady firing flow as maximal control value allows to construct the regular marking such as for place p_3 of the example. However, for place p_4, this limitation causes much more delay to create the accumulated part of the steady marking. Thus the use of the maximal firing flow could reduce the delay for reaching the steady state. From this motivation, the following work is dedicated to developing a new control strategy that exploits the maximal firing flow to improve the time performance.

4 Maximal flow based ON/OFF control

In this section, we propose a new method that improves the time performance compared with the steady flow based ON/OFF control strategy. Before introducing the control strategy, some necessary notions are first presented hereafter.

According to the regular forms in steady state marking (see Section 2), the steady marking quantity of a batch place p_i could be divided into two components of marking quantities $q_i^{f,s}$ and $q_i^{a,s}$ defined as follows.

Definition 7. *The steady marking quantity of place p_i associated with a steady state marking $\boldsymbol{m}^s$ could be divided as:*

$$q_i^s = q_i^{f,s} + q_i^{a,s},$$

where $q_i^{f,s}$ and $q_i^{a,s}$ represent, respectively, the steady marking quantity of free part and the steady marking quantity of accumulated part. They are defined as follows:

1. *If p_i is a batch place :*
   (a) *if $m_i^s = \{\beta_i^e, \beta_i^o\}$, $q_i^{f,s} = l_i^e \cdot d_i^e$ and $q_i^{a,s} = l_i^o \cdot d_i^{\max}$.*
   (b) *if $m_i^s = \{\beta_i^o\}$,*
   - *$d_i^o = \phi_i^s / V_i$, such that $q_i^{f,s} = s_i^o \cdot d_i^o$, and $q_i^{a,s} = 0$.*
   - *$d_i^o \neq \phi_i^s / V_i$, such that $q_i^{a,s} = s_i^o \cdot d_i^o$, and $q_i^{f,s} = 0$.*
2. *If p_i is a continuous place : $q_i^{f,s} = 0$ and $q_i^{a,s} = q_i^s$.* ■

Definition 8. *For a cGBPN at marking $\boldsymbol{m}$, given its steady state marking $\boldsymbol{m}^s$ and steady firing quantity vector $\boldsymbol{z}^s$. The minimum remaining quantity that enters into a place p_i from an input transition t_j for reaching the steady marking quantity vector $\boldsymbol{q}^s$ is denoted as $q_{i,j}^{rs}(\boldsymbol{m})$ and given by $q_{i,j}^{rs}(\boldsymbol{m}) = Post(p_i, t_j)(z_j^s - z_j(\boldsymbol{m}))$.* ■

Let us remark that when $q_{i,j}^{rs}(\boldsymbol{m}) \leq 0$ holds, it is not necessary to fire transition t_j to reach the steady marking quantity of place p_i at current marking $\boldsymbol{m}$.

Example 4. Let us consider the cGBPN with the given steady state $\boldsymbol{m}^s = [4\ 0\ \{(4,1,4)\}\ \{(2,1,2),(3,2,5)\}]^T$ as shown in Fig. 3. The steady flow vector is $\boldsymbol{\varphi}^s = [1\ 1\ 1]^T$ and the steady marking quantity vector is $\boldsymbol{q}^s = [4\ 0\ 4\ 8]^T$. For continuous places, q_1^s corresponds to the steady marking quantity of accumulated part $q_1^{a,s}$, i.e., $q_1^s = q_1^{a,s} = 4$, and $q_2^s = q_2^{a,s} = 0$ while $q_1^{f,s} = q_2^{f,s} = 0$. For batch place p_3, $d_3^o = \phi_3^s / V_3 = 1\ /\ 1 = 1$, thus the steady marking quantity of free part is $q_3^{f,s} = l_3^o \cdot d_3^o = 4 \cdot 1 = 4$ and the steady marking quantity of accumulated part is $q_3^{a,s} = 0$. For batch place p_4, the steady marking quantity of free part is $q_4^{f,s} = l_4^e \cdot d_4^e = 2$, and $q_4^{a,s} = l_4^o \cdot d_4^o = 6$ for the accumulated part.

For the minimum remaining quantity, the steady firing quantity vector to reach $\boldsymbol{q}^s$ at marking $\boldsymbol{m}_0$ is $\boldsymbol{z}^s = [12\ 8\ 0]$, thus, for batch place p_3 and its input transition t_1, $q_{3,1}^{rs} = \mathrm{Post}(p_3, t_1)(z_1^s - z_1(\boldsymbol{m}_0)) = 1 \cdot (12 - 0) = 12$ corresponds to the remaining quantity entering into p_3 for reaching steady marking quantity. Similarly, $q_{4,2}^{rs} = 8$, $q_{2,3}^{rs} = 0$ and $q_{2,1}^{rs} = 8$ which means that transition t_3 does not need to be fired to reach $\boldsymbol{q}^s$ as already specified by $\boldsymbol{z}^s$.

4.1 Proposed control strategy

Due to the existence of batches in a batch place, a steady state marking is reached by guaranteeing both the steady marking quantity and the regular forms of batches in a batch place. Hence, the use of the maximal flows of transitions as the threshold to feed places improves the time performance to reach $\boldsymbol{q}^s$ but more time will be needed to generate regular forms given by $\boldsymbol{m}^s$. Therefore, a condition on the minimum remaining quantity is introduced to switch the threshold from the maximal flow into the steady flow for getting the steady marking quantity and creating the regular forms as follows:

$$q^{rs}_{i,j}(\boldsymbol{m}) \leq q^{f,s}_i, \tag{3}$$

where $q^{rs}_{i,j}(\boldsymbol{m}) = \mathrm{Post}(p_i, t_j)(z^s_j - z_j(\boldsymbol{m}))$ is the minimum remaining quantity given by Definition 8. This quantity is thus compared with the free part quantity of place p_i.

The basic idea of the proposed control strategy is to limit the firing flow of an enabled transition to its steady value when the condition given by (3) is satisfied. For any enabled transition that does not satisfy this condition, its flow is maximized to its maximum flow value. Finally, the flow of an enabled transition is set to zero when the condition given by Eq. (3) is satisfied but at least one of its input places has less quantity than steady marking quantity. At a given marking $\boldsymbol{m}(\tau)$, the proposed control strategy could be summarized as follows[2]:

- ON_{ss}: maximize $u_j(\tau)$ to φ^s_j if t_j is enabled with $\exists\ p_k \in t^\bullet_j : q^{rs}_{k,j}(\boldsymbol{m}) \leq q^{f,s}_k$.
- $\mathrm{ON}_{\max}$: maximize $u_j(\tau)$ to Φ_j if t_j is enabled with $\nexists\ p_k \in t^\bullet_j : q^{rs}_{k,j}(\boldsymbol{m}) \leq q^{f,s}_k$.
- OFF: $u_j(\tau) = 0$ if t_j is not enabled or if t_j is enabled with $\exists\ p_i \in {}^\bullet t_j : q_i(\tau) < q^s_i$ and $\exists\ p_k \in t^\bullet_j : q^{rs}_{k,j}(\boldsymbol{m}) \leq q^{f,s}_k$.

The controlled flow values are maintained during an IB-state and updated when a particular event (or possibly several events at the same time) occurs. Compared with our previous work in [8], three more events are considered. One is that the remaining marking quantity satisfies $q^{rs}_{i,j}(\boldsymbol{m}) = q^{f,s}_i$. The two other considered events are: a continuous place becomes empty and batch place becomes full where the control actions are updated to ensure the nonnegativity of marking quantity and avoid increasing the marking quantity, respectively. All the events considered by the controller are the following:

- The marking quantity of place p_i reaches its steady quantity q^s_i;
- The remaining marking quantity $q^{rs}_{i,j}(\boldsymbol{m})$ becomes equal to $q^{f,s}_i$;
- A continuous place becomes empty;
- A batch place becomes full;
- The value of batch place output density changes.

[2] For $t \in T$, ${}^\bullet t = \{p \in P \mid \mathrm{Pre}(p,t) > 0\}$, and $t^\bullet = \{p \in P \mid \mathrm{Post}(p,t) > 0\}$.

4.2 Computation of the timed control trajectory

Before we present the algorithm that allows one to compute a control trajectory, several sets must be defined at any marking $\boldsymbol{m}$.

- $P_{\emptyset}(\boldsymbol{m}) = \{p_i \in P^C \mid m_i = 0\}$ be the subset of empty continuous places.
- $P_F(\boldsymbol{m}) = \{p_i \in P^B \mid q_i = Q_i\}$ be the subset of full batch places.
- $S_L(\boldsymbol{m}) = \{p_i \in P \mid q_i < q_i^s\}$ be the subset of places whose marking quantities are lower than their steady state values.
- $S_E(\boldsymbol{m}) = \{p_i \in P \mid q_i = q_i^s\}$ be the subset of places whose marking quantities are equal to their steady state values.
- $T_L(\boldsymbol{m}) = \{t_j \in T \mid \exists p_i \in \{S_L(\boldsymbol{m}) \cap {}^\bullet t_j\}\}$ be the subset of transitions with at least one of its input places belonging to $S_L(\boldsymbol{m})$.
- $T_{\mathcal{N}}(\boldsymbol{m})$ be the subset of transitions that are not enabled at $\boldsymbol{m}$.
- $T_Z(\boldsymbol{m}) = \{t_j \in T \mid \exists\ p_i \in t_j^\bullet : q_{i,j}^{rs}(\boldsymbol{m}) \leq q_i^{f,s}\}$ be the subset of transitions with a remaining firing quantity greater or equal than the free part steady marking quantity of one of their output place.
- $T_{ZL}(\boldsymbol{m}) = T_Z(\boldsymbol{m}) \cap T_L(\boldsymbol{m})$ be the subset of transitions that belongs to $T_Z(\boldsymbol{m})$ and at least one of their input places belongs to $S_L(\boldsymbol{m})$.
- $S_{ZE}(\boldsymbol{m}) = \{p_i \in S_E(\boldsymbol{m}) \mid \forall\ t_j \in \{{}^\bullet p_i \cup p_i^\bullet\}$ such that $t_j \in T_Z(\boldsymbol{m})\}$ be the subset of places whose marking quantities are equal to their steady state values and the firing quantities of all their input and output transitions satisfy $T_Z(\boldsymbol{m})$.

Algorithm 1 computes a timed control trajectory from an initial state to a given reachable steady state. It starts at initial marking $\boldsymbol{m}_0$ with a control firing flow vector $\boldsymbol{u}^0$ initialized to $\mathbf{0}$ at line 2. Each timed event that refers to an execution of the while-end structure (lines 3 – 8), a new marking $\boldsymbol{m}^i$ is built and compared with the steady marking $\boldsymbol{m}^s$. Line 4 determines the necessary constraint sets to solve the LPP which maximizes the controlled firing flow vector at each timed event. Constraints (a) are used to maximize the flow approaching the maximal firing value. Constraints (b) show that the maximal flow of each transition t_j cannot exceed the steady firing flow φ_j^s when the remaining marking quantity associated with one of its output places p_k is less than or equal to the free part steady marking quantity i.e., $\exists\ p_k \in t_j^\bullet : q_{k,j}^{rs}(\boldsymbol{m}) \leq q_k^{f,s}$. Constraints (c) block transition t_j when it is not enabled or when belongs to $T_{ZL}(\boldsymbol{m})$. Constraints (d) require that the marking quantity cannot increase when the batch place is full. Constraints (e) claim that the marking cannot decrease when the continuous place is empty. Constraints (f) hold the marking quantity to the steady marking quantity when place p_i belongs to $S_{ZE}(\boldsymbol{m})$. Constraints (g) ensure that the input flow of a batch place should be less than or equal to the maximal flow that can be accepted by its transfer element. Constraints (h) require that the output flow of a batch place should be lower than or equal to the flow out-coming from the place. Lines 6–7 determine the nearest event and update the new marking. The procedure is repeated until the steady marking is reached. When the marking $\boldsymbol{m}^i$ reaches the steady state value $\boldsymbol{m}^s$, the controlled

Algorithm 1: Computation of control trajectory

Input: A cGBPN $\langle N, \boldsymbol{m}_0\rangle$, a reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ and the steady firing quantity vector $\boldsymbol{z}^s$

1 . **Output:** Control trajectory $(\boldsymbol{u}^0, \tau_0), (\boldsymbol{u}^1, \tau_1), \cdots$

2 Initialize : $\boldsymbol{q}^0 = \mu(\boldsymbol{m}_0)$, $\boldsymbol{m}^0 = \boldsymbol{m}_0$, $\boldsymbol{u}^0 = \boldsymbol{0}$, $\boldsymbol{z}^0 = \boldsymbol{0}$, $\tau_0 = 0$, $i = 0$;

3 **while** $\boldsymbol{m}^i \neq \boldsymbol{m}^s$ **do**

4 Determine $T_{\mathcal{N}}(\boldsymbol{m}^i)$, $T_L(\boldsymbol{m}^i)$, $T_Z(\boldsymbol{m}^i)$, $T_{ZL}(\boldsymbol{m}^i)$, $P_{\emptyset}(\boldsymbol{m}^i)$, $P_F(\boldsymbol{m}^i)$ $S_L(\boldsymbol{m}^i)$, $S_E(\boldsymbol{m}^i)$, $S_{ZE}(\boldsymbol{m}^i)$;

5 Solve the following LPP: $\max \boldsymbol{1}^T \cdot \boldsymbol{u}^i$ *s.t.*

$$\begin{cases} \text{(a) } 0 \leq u_j^i \leq \Phi_j & \forall t_j \in T \\ \text{(b) } 0 \leq u_j^i \leq \varphi_j^s & \forall t_j \in T_Z(\boldsymbol{m}^i) \\ \text{(c) } u_j^i = 0 & \forall t_j \in T_{\mathcal{N}}(\boldsymbol{m}^i) \cup T_{ZL}(\boldsymbol{m}^i) \\ \text{(d) } C(p_k, \cdot) \cdot \boldsymbol{u}^i \leq 0 & \forall p_k \in P_F(\boldsymbol{m}^i) \\ \text{(e) } C(p_k, \cdot) \cdot \boldsymbol{u}^i \geq 0 & \forall p_k \in P_{\emptyset}(\boldsymbol{m}^i) \\ \text{(f) } C(p_k, \cdot) \cdot \boldsymbol{u}^i = 0 & \forall p_k \in S_{ZE}(\boldsymbol{m}^i) \\ \text{(g) } \mathrm{Post}(p_k, \cdot) \cdot \boldsymbol{u}^i \leq V_k \cdot d_k^{\max} & \forall p_k \in P^B \\ \text{(h) } \mathrm{Pre}(p_k, \cdot) \cdot \boldsymbol{u}^i \leq V_k \cdot d_k^{\mathrm{out}} & \forall p_k \in P^B \end{cases}$$

6 Determine all the next timed events, select the nearest in time and deduce time τ_{i+1} ;

7 Determine the new marking $\boldsymbol{m}^{i+1}$, the marking quantity vector $\boldsymbol{q}^{i+1}$ and the current firing quantity vector $\boldsymbol{z}^{i+1}$;

8 i = i + 1;

9 $\boldsymbol{u}^i = \boldsymbol{\varphi}^s$;

10 Return $(\boldsymbol{u}^0, \tau_0), \cdots, (\boldsymbol{u}^i, \tau_i)$

firing flow vector is set to its steady firing flow vector $\boldsymbol{\varphi}^s$ as shown in line 9. The complexity of this algorithm is polynomial since it is obtained by solving a LPP at each timed step.

Remark 2. When transitions are in structural conflicts that are effective ones, the proportional policy is used to solve these conflicts by adding the following constraints : $u_a^i \cdot \varphi_b^s = u_b^i \cdot \varphi_a^s$ if $\forall t_a, t_b \in T, {}^\bullet t_a \cap {}^\bullet t_b \neq \emptyset$ and $t_a, t_b \notin T_{\mathcal{N}} \cup T_{ZL}$.

4.3 Transient behavior of the controlled net

In this part, we focus on the convergence of the proposed algorithm for reaching a give steady state from an initial one. Different from the previous work that all the places converge and maintain the steady marking quantities, here we consider all the transitions should be included in the set of T_Z, then this set will hold on until the steady state marking is reached.

Lemma 1. *Consider a cGBPN $\langle N, \boldsymbol{m}_0\rangle$ that satisfies assumptions (A1) – (A3). By Algorithm 1, a transition included in T_Z remains in T_Z whatever the net evolution.*

Proof. A transition t_j is included in T_Z if $\exists\ p_i \in t_j^\bullet : q_{i,j}^{rs}(\boldsymbol{m}) \leq q_i^{f,s}$, which is equivalent to $\exists\ p_i \in t_j^\bullet : z_j(\boldsymbol{m}) \geq z_j^s - q_i^{f,s}/\mathrm{Post}(p_i, t_j)$. Since $z_j^s - q_i^{f,s}/\mathrm{Post}(p_i, t_j)$ is a constant value and the firing quantity z_j is monotonically increasing, any new reachable marking $\boldsymbol{m'}$ will satisfy $z_j(\boldsymbol{m'}) \geq z_j(\boldsymbol{m}) \geq z_j^s - q_i^{f,s}/\mathrm{Post}(p_i, t_j)$ for place p_i which means that t_j will remain in T_Z. ♢

Lemma 2. *Consider a cGBPN $\langle N, \boldsymbol{m}_0\rangle$ that satisfies assumptions (A1) – (A3). By Algorithm 1, all the transitions in the net N will be included in T_Z in finite time.*

Proof. Based on Lemma 1, the transitions that are already included in T_Z at $\boldsymbol{m}_0$ remain in T_Z. In case that transition t_j is not initially included in T_Z at $\boldsymbol{m}_0$, which means $\forall p_i \in t_j^\bullet : q_{i,j}^{rs}(\boldsymbol{m_0}) > q_i^{f,s}$, let us assume that this transition will never be included in T_Z and show that this leads to a contradiction.

The transition t_j cannot be included in T_Z in finite time means that it is blocked before satisfying the condition $\exists p_i \in t_j^\bullet : q_{i,j}^{rs}(\boldsymbol{m_0}) \leq q_i^{f,s}$ and remains blocked whatever the net evolution. According to Algorithm 1, a transition that does not belong to T_Z is blocked only when it is not enabled. More precisely, one of input places p_k of transition t_j should be empty. Due to assumption (A3), the empty place p_k must belong to a conservative component. This means that there must exist at least one upstream place p_l that has a marking quantity which is greater than its steady one. By Algorithm 1, the output transition of place p_l cannot be blocked for infinite time. Thus, place p_k cannot remain empty which contradicts the initial assumption. Consequently, transition t_j can be included in T_Z in finite time.

Proposition 1. *Given a cGBPN $\langle N, \boldsymbol{m}_0\rangle$ that satisfies assumptions (A1) – (A3) and a given reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$. The control trajectory computed by Algorithm 1 drives the system from its initial state to the given steady state in finite time.*

Proof. According to Lemmas 1 and 2, all the transitions of the net will be included in T_Z in finite time. When $T_Z = T$, the controlled firing flows are maximized to the steady ones by constraint (b) in Algorithm 1 and the sets T_{ZL} and S_{ZE} will satisfy $T_{ZL} = T_L$ and $S_{ZE} = S_E$. In this case, the algorithm is equivalent to the one presented in our previous work in [8] where the convergence of the marking to the steady state marking has been proved (see Proposition 12 in [8]). ♢

Example 5. Let us consider again the cGBPN with an initial marking $\boldsymbol{m}_0 = [8\ 8\ \emptyset\ \emptyset\}]^T$ in Fig. 2 and the reachable steady state $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ with $\boldsymbol{m}^s = [4\ 0\ \{(4,1,4)\}\ \{(2,1,2),(3,2,5)\}]^T$ as shown in Fig. 3, and $\boldsymbol{\varphi}^s = [1\ 1\ 1]^T$. The steady marking quantity vector is $\boldsymbol{q}^s = [4\ 0\ 4\ 8]^T$, and the steady firing quantity vector is $\boldsymbol{z}^s = [12\ 8\ 0]^T$. Under the proposed strategy, a control trajectory that drives the net from its initial state to the steady state is obtained as follows.

- At $\tau_0 = 0$, $T_{\mathcal{N}}(\boldsymbol{m}_0) = \{t_2, t_3\}$. As $\boldsymbol{q}^0 = [8\ 8\ 0\ 0]^T$, the subsets of places are: $S_E(\boldsymbol{m}_0) = \emptyset$, $S_L(\boldsymbol{m}_0) = \{p_3, p_4\}$, $P_F(\boldsymbol{m}_0) = \emptyset$ and, $T_L(\boldsymbol{m_0}) = \{t_2, t_3\}$. The firing quantity vector is $\boldsymbol{z}(\boldsymbol{m}_0) = [0\ 0\ 0\ 0]$, by checking the condition $q^{rs}_{i,j}(\boldsymbol{m}_0) \leq q^{f,s}_i$, $T_Z(\boldsymbol{m}_0) = \{t_3\}$, $T_{ZL}(\boldsymbol{m}_0) = \{t_3\}$ and, $S_{ZE}(\boldsymbol{m}_0) = \emptyset$. Consequently, transition t_1 is enabled and $\text{ON}_{\max}$ since it is not in $T_Z(\boldsymbol{m}_0)$ while transitions t_2 and t_3 are not enabled, thus $u^0_2 = u^0_3 = 0$. Algorithm 1 returns the controlled firing flow vector $\boldsymbol{u}^0 = [2\ 0\ 0]^T$.
- At $\tau_1 = 2$, places p_1 and p_3 reach steady marking quantities so $S_E(\boldsymbol{m}^1) = \{p_1, p_3\}$ where $\boldsymbol{m}^1 = [4\ 8\ \{(2,2,2)\}\ \emptyset]^T$ and $\boldsymbol{q}(\tau_1) = [4\ 8\ 4\ 0]^T$. This implies $T_{\mathcal{N}}(\boldsymbol{m}^1) = \{t_2, t_3\}$, $P_F(\boldsymbol{m}^1) = \emptyset$, $S_L(\boldsymbol{m}^1) = \{p_4\}$ and $T_L(\boldsymbol{m}^1) = \{t_3\}$. The firing quantity vector is $\boldsymbol{z}(\boldsymbol{m}^1) = [4\ 0\ 0]$, $T_Z(\boldsymbol{m}^1) = \{t_3\}$, $T_{ZL}(\boldsymbol{m}^1) = \{t_3\}$, and $S_{ZE}(\boldsymbol{m}^1) = \emptyset$. Consequently, transition t_1 is enabled and $\text{ON}_{\max}$ since it is not in $T_Z(\boldsymbol{m}^1)$. Transitions t_2 and t_3 are not enabled, thus $u^1_2 = u^1_3 = 0$. The obtained controlled firing flow vector is $\boldsymbol{u}^1 = [2\ 0\ 0]^T$.

By continuing the Algorithm execution, the resulting control trajectory is given by $(\boldsymbol{u}^0, 0), (\boldsymbol{u}^1, 2), (\boldsymbol{u}^2, 4), (\boldsymbol{u}^3, 6), (\boldsymbol{u}^4, 7), (\boldsymbol{u}^5, 8), (\boldsymbol{u}^6, 9), (\boldsymbol{u}^7, 10)$ with $\boldsymbol{u}^0 = [2\ 0\ 0]^T$, $\boldsymbol{u}^1 = [2\ 0\ 0]^T$, $\boldsymbol{u}^2 = [0\ 2\ 0]^T$, $\boldsymbol{u}^3 = [1\ 2\ 0]^T$, $\boldsymbol{u}^4 = [1\ 0\ 0]^T$, $\boldsymbol{u}^5 = [1\ 1\ 0]^T$, $\boldsymbol{u}^6 = [1\ 1\ 0]^T$, $\boldsymbol{u}^7 = [1\ 1\ 1]^T$.

One can remark that the delay $\tau_s = 10$ for reaching the steady state is improved compared with the steady flow based ON/OFF control $\tau_s = 12$ obtained in Example 3 although the number of steps is greater (7 generated events with the proposed strategy and 3 events using the steady flow strategy). The time improvement is due to the use of the maximal firing flow (vectors $\boldsymbol{u}^0$, $\boldsymbol{u}^1$, $\boldsymbol{u}^2$, and, $\boldsymbol{u}^3$).

5 Study case

The considered system depicted in Fig. 4 is a communication system composed of three buffers and four routers. Its corresponding cGBPN model is represented in Fig. 5(a) where three buffers are respectively modeled by three batch places p_4, p_5, p_6 with $\gamma(p_4) = (V_4, d^{\max}_4, s_4) = (8, 4, 4)$, $\gamma(p_5) = (2, 3, 5)$, $\gamma(p_6) = (2, 4, 4)$, and the routers are modeled with five controlled transitions such that the processing rates of the routers are controllable variables. The maximal flow of the transitions are given by: $\Phi_1 = \Phi_2 = \Phi_4 = \Phi_5 = 4$, $\Phi_3 = 8$. Data arrived in buffer B_1 from router R_1 are processed by the router R_2 which allows to direct the data flows throughout two different channels represented by (B_2, R_3) and (B_3, R_4). The desired (nominal) behavior of the system is represented by the steady state showed in Fig. 5(a) and characterized by $(\boldsymbol{m}^s, \boldsymbol{\varphi}^s)$ with $\boldsymbol{m}^s = [10\ 0\ 0\ \{(4, 0.5, 4)\}\ \{(5, 1, 5)\}\ \{(2, 1, 2), (2, 4, 4)\}]^T$ and $\boldsymbol{\varphi}^s = [4\ 2\ 2\ 2\ 2]^T$. It means that only buffer B_3 has an accumulation while data in buffers B_1 and B_2 are processed as soon as they are received by the routers. The steady marking quantity vector is $\boldsymbol{q}^s = [10\ 0\ 0\ 2\ 5\ 10]^T$.

Let us consider that some faults or cyber-attack causes a deviation from the normal behavior which leads the system to the state represented in Fig. 5(b)

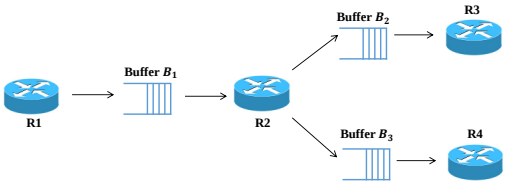
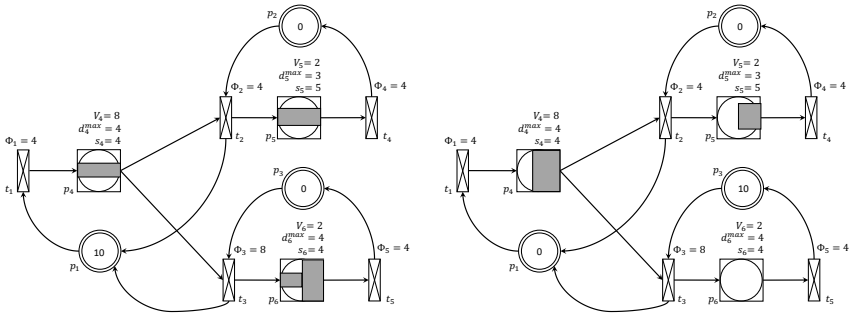


Fig. 4: The considered communication system.

with $\boldsymbol{m}_0 = [0\ 0\ 10\ \{(3,4,4)\}\ \{(2.5,2,5)\}\ \emptyset]^T$. The steady firing quantity vector for reaching the steady marking quantity vector is $\boldsymbol{z}^s = [0\ 0\ 10\ 0\ 0]^T$.



(a) A steady state. (b) The initial marking.

Fig. 5: A cGBPN of study case.

The proposed control strategy is used to compute a control trajectory that drives the net from its abnormal state given by $\boldsymbol{m}_0$ to the steady state $\boldsymbol{m}^s$. By applying Algorithm 1, the following control trajectory is obtained $(\boldsymbol{u}^0, 0)$, $(\boldsymbol{u}^1, 0.625), (\boldsymbol{u}^2, 1), (\boldsymbol{u}^3, 1.5), (\boldsymbol{u}^4, 2), (\boldsymbol{u}^5, 2.5)$ with $\boldsymbol{u}^0 = [0\ 2\ 8\ 2\ 0]^T$, $\boldsymbol{u}^1 = [0\ 2\ 8\ 2\ 0]^T$, $\boldsymbol{u}^2 = [4\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^3 = [4\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^4 = [4\ 2\ 2\ 2\ 2]^T$, $\boldsymbol{u}^5 = [4\ 2\ 2\ 2\ 2]^T$.

Consequently, 5 events are generated and the delay for reaching the steady state with the maximal flow based ON/OFF strategy is $\tau_s = 2.5$.

Under the steady flow based ON/OFF control strategy, a control trajectory that drives the net from its initial state to the steady state is $(\boldsymbol{u}^0, 0), (\boldsymbol{u}^1, 0.625)$, $(\boldsymbol{u}^2, 2), (\boldsymbol{u}^3, 2.5), (\boldsymbol{u}^4, 3), (\boldsymbol{u}^5, 5)$ with $\boldsymbol{u}^0 = [0\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^1 = [0\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^2 = [0\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^3 = [4\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^4 = [4\ 2\ 2\ 2\ 0]^T$, $\boldsymbol{u}^5 = [4\ 2\ 2\ 2\ 2]^T$.

Although the number of generated events is equal (5 events), the delay for reaching the steady state is $\tau_s = 5$ and is much higher compared with the maximal flow based ON/OFF control strategy.

From the results above, one can conclude that the proposed control method that considers the maximal firing flow of transition improves the time performance.

6 Conclusions

An algorithm that incrementally computes a control trajectory for reaching the steady state from an initial one has been proposed. The main advantage of this algorithm is to exploit the maximal firing flow of transitions compared with the previous one developed in [8]. The convergence of the algorithm has been proved and the proposed method is applied to recover a communication systems from its abnormal state to its nominal steady state. In future work, we plan to compare the two methods to quantify the improvement. We will focus on the optimality to address the minimum-time control problem.

References

1. Cassandras, C.G.: The event-driven paradigm for control, communication and optimization. Journal of Control and Decision **1**(1), 3–17 (2014)
2. David, R., Alla, H.: Discrete, continuous, and hybrid Petri nets, vol. 1. Springer (2005)
3. Demongodin, I., Giua, A.: Dynamics and steady state analysis of controlled generalized batches Petri nets. Nonlinear Analysis: Hybrid Systems **12**, 33–49 (2014)
4. Demongodin, I., Giua, A.: Linear programming techniques for analysis and control of batches Petri nets. In: 10th Int. Workshop on Discrete Event Systems, WODES'10, Berlin, Germany (2010)
5. Demongodin, I.: Generalised batches Petri net: hybrid model for high speed systems with variable delays. Discrete Event Dynamic Systems **11**(1-2), 137–162 (2001)
6. Humayed, A., Lin, J., Li, F., Luo, B.: Cyber-physical systems security—a survey. IEEE Internet of Things Journal **4**(6), 1802–1831 (2017)
7. Júlvez, J., Di Cairano, S., Bemporad, A., Mahulea, C.: Event-driven model predictive control of timed hybrid Petri nets. Int. journal of robust and nonlinear control **24**(12), 1724–1742 (2014)
8. Liu, R., Ammour, R., Brenner, L., Demongodin, I.: Event-driven control for reaching a steady state in controlled generalized batches Petri nets. In: 15th Int. Workshop on Discrete Event Systems, WODES'20, Rio de Janeiro, Brazil (2020)
9. Mahulea, C., Giua, A., Recalde, L., Seatzu, C., Silva, M.: Optimal model predictive control of timed continuous Petri nets. IEEE Trans. on Automatic Control **53**(7), 1731–1735 (2008)
10. Rashidinejad, A., Wetzels, B., Reniers, M., Lin, L., Zhu, Y., Su, R.: Supervisory control of discrete-event systems under attacks: an overview and outlook. In: 18th European Control Conference (ECC'19). pp. 1732–1739 (2019)
11. Wang, L., Mahulea, C., Júlvez, J., Silva, M.: ON/OFF strategy based minimum-time control of continuous Petri nets. Nonlinear Analysis: Hybrid Systems **12**, 50–65 (2014)