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# **Collaborative research as an institution that incorporates research systems Case study of a mathematical problem-solving project**

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## **Abstract**

*Our chapter presents a way of modelling the factors, working methods and processes at work in an educational research project involving a collaboration between teachers and researchers. This modelling process takes as its touchstone research carried out in the Academy of Aix-Marseille over a two year-period: 2019-2020 and 2020-2021. The theoretical model falls within the scope of the anthropological theory of the didactic (ATD) and is based on the concepts of the institution, the relationship of an entity to an object and praxeology. Modelling the collaboration process enabled us to characterise the emerging research system and its functioning, as well as the conditions and constraints of the evolution of relationships to the object being studied: mathematical problem solving (MPS).*

## **Keywords**

*Modelling • collaborative processes • teachers • researchers • anthropological theory of the didactic (ATD) • research system • mathematical problem solving (MPS)*

## **Introduction**

Since 2017, a local initiative launched by an educational research federation (SFERE-Provence) and the Aix-Marseille education authority via the academic delegation for training and educational innovation has been underway in the Academy of Aix-Marseille, aiming to support teachers in priority education networks through a professionalisation process, facing the needs expressed by schools. The aim of the project is to develop cooperation between the various actors in these networks (teachers, management staff, and other members of the educational community: supervisory staff, activity leaders, educators, parents, students, etc.), and researchers interested in the problems of the field of education and training. This proactive policy, developed at the local level, has witnessed the emergence of four 'waves' of projects since 2017, each one lasting two years. The policy's stated objectives are 'on the one hand, [to] bring the educational teams of priority education networks<sup>1</sup> together

with the research teams of the laboratories of the SFERE-Provence research federation and, on the other hand, [to] strengthen and develop the role of the institution in initial and continuing education' (Feedback seminar on research-training-field projects, 2019). The aim of the projects is to bring researchers and teachers together around an object designated by the requesting institution, and to question this object in order to better understand it.

Within this framework, we have been responsible for and co-leaders<sup>2</sup> of a project since September 2019 (year 1 of the project), carried out in collaboration with three schools: a middle school and two state schools in the same school sector, located in a priority education network in an urban area of southern France. This school sector faces many social and

<sup>1</sup> The schools that belong to these networks educate students from disadvantaged socio-economic backgrounds. They have additional human, material and financial resources to work towards the academic success of these students.

<sup>2</sup> Four teacher-researchers directly involved in the project and authors of this text (a temporary teaching and research associate, three university lecturers) were designated as co-leaders of the project, as well as several management and supervisory staff from the French education system: a middle school principal, an inspector and two district educational advisors (primary level), as well as an academic inspector attached to the sector.

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academic difficulties regarding students, which are reflected in part by the students' low grades in national assessments at the beginning of Year 7. It was this problem that triggered the request for researchers to intervene in mathematical problem solving (MPS), request issued by the head of the middle school. Thus, the collaborative working arrangement with the researchers was set up in June 2019, based on an analysis and an adjustment<sup>3</sup> of the initial proposition, linked to the practical and scientific feasibility of the project. The modalities were thus co-designed and negotiated between the co-leaders.

Our system is structured around four main areas, which breaks with traditional research: 1) a prolonged period of time and presence in schools (two years); 2) an aim to change and improve professional practices (linked to the improvement of student results); 3) the joint commitment of the actors; 4) the joint planning of the intervention process 'with' the actors (and not imposed 'on' them). Therefore, the dynamic at the heart of the system is above all pragmatic: it consists in solving a problem that arises within the schools. However, the heuristic dimension is not excluded with regard to the production of knowledge generated by the research. This concerns both scientific advances and the professional development of the actors.

The collaboration is thus organised between researchers and practitioners, and it implies a strong contribution on the researchers' part in the construction of the project (since they are institutionally designated as co-leaders), it being clear that the benchmarks and orientations given initially are to be adjusted and negotiated along the way in interaction with the practitioners. The general approach consists in 'supporting' the teacher teams in the field, and reflects the unique position of the researcher in his/her intervention methods: 'that of a companion, a facilitator who encourages reflection and exchange between the actors concerned by the problem; he/she organises the context, chairs meetings, provides various resources such as conceptual frameworks to support the participants' reflection, etc.' (Morrissette, 2013, p. 45). It is therefore not a matter of administering content, but of organising a structure, a mode of operation, and promoting possibilities to help the actors decide on certain paths to take in order to cope with specific work situations (and improve them). Involvement in the system also commits the participants (teachers and researchers alike) to transforming their praxeologies together (Chevallard, 1998), and even to creating new ones, thanks to their experience, enlightened and nourished by the practical and theoretical knowledge in progress, from the perspective of their professional

development. In light of this double objective, which is namely to produce knowledge and to promote the professional development of the practitioners involved in the research, the system falls under the heading of 'collaborative' research (Desgagné, 1997).

## Theoretical framework

In the usual model of collaborative research, research with a critical purpose which aims at changing practices within a group of teachers experiencing a problematic situation, is labelled as 'participatory action'. The work that we carried out belongs to this type of research but exceeds its ambition since the main problem is a non-marginal transformation of the logos (Chevallard, 2007; Artaud, 2019) and the researchers are *a priori*, like the teachers, involved in the same process of changing their praxeologies. However, it is also a form of 'collaborative' research since it adopts a transformative approach, and the reconfigured situations can be used in training. It is also 'design-oriented': indeed, the problems addressed are those that arise for practitioners; practitioners participate in the analyses; the design is based on scientific knowledge—mainly regarding the didactics of mathematics—and there is a possible generalisation of the results by the identification of invariants. So, it does not fit into a pre-existing category, and we did not envisage it that way.

The theoretical framework that allowed us to conceive, and then to model the collaboration and the elements at work in this system comes from the anthropological theory of the didactic [ATD] (Chevallard, 1992, 2007, 2019). The qualifier 'anthropological' testifies to the intention of the theory of the didactic to be attentive to all the dimensions of social reality that influence the didactic. To put it differently, a didactic phenomenon cannot be explained by considering only the institution that houses it. This qualifier also reflects the intention to analyse the didactic everywhere around us, in the past, present and future, and in any situation with a didactic purpose (a situation experienced in the street, in a company, at the university, etc.).

This theory aims to better understand human activity. It is about studying the emergence, life and dissemination of knowledge among people and institutions, in any area of society where something is studied, taught or learned.

### The concept of institution

The concept of institution is thus a fundamental concept of ATD. Yves Chevallard explains it as follows:

An institution *I* is a 'total' social device, which may certainly have only a very limited extension in social space (there are

<sup>3</sup> This adjustment concerns a methodological change to the protocol initially proposed by the head teacher.

'micro-institutions'), but which allows—and imposes—on its subjects, i.e. on the persons  $x$  who come to occupy the different positions  $p$  offered in  $I$ , the implementation of their own ways of doing and thinking. Thus, the class is an institution (whose two essential positions are those of teacher and pupil), as is the school (where other positions appear: those of education advisers, the health nurse, etc.), and likewise this institution, which encompasses classes and schools and which abounds in all kinds of positions, the education system. (Chevallard, 2003, p. 2)

Hence, an institution is an organisation that enables and imposes an operating system within its area of operation. Outside, other institutions govern. We will soon see that a single institution can contain several sub-institutions, themselves implementing an operating system that is permitted and/or imposed on the people who occupy a position  $p$  in it.

### **The concept of the Herbartian schema**

In the ATD, the study of a question is modelled by the 'Herbartian schema':

$$[S(X; Y; Q) \rightarrow M] \rightarrow A^*$$

This articulates the 'the work of the didactic system' (Chevallard, 2011, p. 21, p. 141). In its developed form, reproduced below (Chevallard, 2019), the Herbartian schema incorporates the components of  $M$ , the milieu of the study of the question  $Q$ .

$$[S(X; Y; Q) \rightarrow M = \{A_1, A_2, \dots, A_m, W_{m+1}, W_{m+2}, \dots, W_n, Q_{n+1}, Q_{n+2}, \dots, Q_p, D_{p+1}, D_{p+2}, \dots, D_q\}] \rightarrow A^*$$

In this formalism,  $S(X; Y; Q)$  designates the didactic system formed around the study of the question  $Q$ ,  $X$  representing the student entity,  $Y$  the entity of study assistants. This system allocates a didactic milieu  $M$  to itself with a view to constructing an answer  $A^*$  to the question  $Q$ . This is composed of several already existing answers  $A^\diamond$ , in culture or literature; of several works  $W$  of various kinds, such as theories, experiments, historiographical narratives, etc. allowing the collection of  $A^\diamond$ , or their analysis for the construction of  $A^*$ ; of data  $D$  of various kinds as well; of questions produced by the study of the initial question  $Q$  and which can be relative to the answers  $A^\diamond$ , to the works  $W$  or to the data  $D$ .

When the students in the didactic system are researchers, we will speak of a research system and note it as  $S(\Xi, Z, Q)$ , where  $\Xi$  designates the researchers and  $Z$  the research assistants. The research system is then organised around a common work object that can be co-constructed and on which the actors of the new institution decide to work; this is the question  $Q$  in the modelling above. This common work object

can be chosen or imposed, but in any case, it is problematic. It will prompt action and collaboration by and between the actors to try to solve the problem, i.e. to produce an answer  $A^*$  to  $Q$ . The study of a question by a didactic system or a research system alters the relations of the student position as well as that of the study assistant to a certain number of objects.

### **The concept of relationship to an object**

The question of relationship to an object was developed in the didactics of mathematics by Yves Chevallard (1989; 2003; 2019) to model the institutional relativity of knowledge. The relationship of an entity (person or institutional position) to an object  $o$  is composed of everything that links the entity to the object  $o$  (what it thinks about  $o$ , what it does with  $o$ , does not do, can do or cannot do with  $o$ , the 'feelings' it may have towards  $o$ , etc.). We are not interested here in persons, but in institutional positions, i.e. the positions  $p$  that persons  $x$  hold in institutions  $I$ . The existence of a relationship of an entity to an object  $o$  makes it possible to say that this entity knows  $o$ , the nature of the relationship allowing the qualification of this. In other words,  $R(p, o)$  makes it possible to model how the entity  $p$  knows  $o$ .

The evolution of an entity's knowledge can then be modelled as an evolution of its relationships to a multitude of objects. Among the conditions that influence the evolution of the relationship of the position of a student in a didactic institution, the relationship of the teacher position is important. Changing the relationship of the student position to an object therefore implies changing the relationship of the teacher position to a number of objects: it is necessary to identify these objects as well as to determine which relationship to these objects is useful, or even essential, in the teacher position. This determination is based on an observation of institutional activity using different methods (questionnaires, written records of the activity of the entities, reports of class observations, etc.) and on a didactic analysis of this activity. It is the concept of praxeology that allows the analysis of the activity (Chevallard, 1998, 2007).

When analysing the activity of a didactic system or a research system, the objective of the study—i.e. the answer  $A^*$  produced by this activity—is a praxeology or, sometimes, part of a praxeology, which must be distinguished from the praxeologies that enable its production, whether these praxeologies are part of the milieu or are involved in the implementation of the study process of the question  $Q$  (Artaud, 2019; Artaud & Cirade, 2021). This is particularly crucial when the study process, which is analysed in terms of didactic praxeologies, produces a praxeology of study direction, which is again a didactic praxeology. This may be the case with teacher training, for example. But it is also

difficult when the objective of the study is not didactic. Indeed, the milieu generally includes praxeologies of the same nature as the objective of the study, without their being part of the answer  $A^*$ : they are part of the study process because they enable it to be carried out.

we have exemplified the elements of the model through the components of the research carried out, by trying to distinguish between generic and specific aspects, and paying particular attention to the conditions and constraints that favour or hinder the emergence of certain elements.

## Methodological framework

The methodology for designing the collaboration model was based on the conducting of fieldwork followed by an analysis, which involved taking a necessary step back. We are in fact ourselves actors in and analysers of the system, something which engages our position of involvement-distancing in relationship to the project. It is the construction of the analytical frameworks and the conceptualisation outlined in this chapter that allow for a change of perspective in relation to that of the practitioner, whether teacher or researcher.

From a research point of view, the aim was that the approach not remain solely focused on the production of action knowledge (i.e. knowledge for action), which could then limit it to a strictly pragmatic posture. The challenge was to maintain a distance between reflection and action, to validate the results obtained through practice and to be able to transfer the approach to other similar situations. From our point of view as researchers, it was also a question of testing a theoretical construction devised on the basis of the ATD and of refining it in order to understand its interest in the context of a new theoretical proposal/production.

This led us to propose a theoretical model that aimed to formalise (part of) our methodology by taking into account the relationships between action and research/reflection, between practice and theory in the context of collaborations between practitioners and researchers in the field of education. Our methodological choices/principles were therefore based on the two roles that we take on: research and design (of the model).

The model can be summarised as follows: a research system is created within a commissioning institution to study a question  $Q$ . The researchers in this research system come from two external institutional positions: the position of a teacher and the position of a researcher in educational sciences and didactics. Therefore, they must modify their relationships to a number of objects in order to create a relationship consistent with the position of researcher within the research system. The functioning of the research system produces an answer to at least some of the sub-questions generated by the study of  $Q$ , or at best to the question  $Q$  on the basis of a milieu that it constitutes. This response, and its production, make it possible to change the relationship to the object of study of individuals who have participated in the research system, and thereby, external institutional relationships. In what follows,

## Analysis

### *The research system and its functioning*

The collaborative research conducted in our project is presented as being hosted by an institution, which we will call here 'academic projects', which define certain conditions and constraints. For example: the request comes from institutions, and not from the teacher position, even if the latter can support it; the duration is two years; the researchers must 'support', and 'supporting is not training'.

Within this institution, a research system  $S(\Xi, Z, Q)$  is created, which can be considered as a sub-institution of the mandating 'academic projects' institution, which itself can be considered as a sub-institution of the 'collaborative research' institution. As far as we are concerned, the question  $Q$  around which the research system was established is: 'How to improve the results of cycle 3 students<sup>4</sup> in MPS?'.

The body of study assistants is *a priori* absent, and the researchers of this research system, the  $\xi$  belonging to  $\Xi$ , come from two other institutions: the research institution in educational sciences and in the didactics of mathematics, through the laboratory ADEF (*Learning, Didactics, Assessment, Training*); the institution of mathematics education, through three of its establishments: two primary schools and a middle school. It is possible to split, for the purposes of analysis, the researcher position,  $p_\xi$  of this research system into two sub-positions: one of researcher-guide and the other of researcher-teacher. The creation of this 'MPS investigation' institution presupposes common objects. Therefore, it is new relationships to these objects that must be created from the relationships of the individuals holding the two institutional positions external to the research system. The functioning of the research system will thus create a relationship of the entity  $p_\xi$  to the object 'MPS in cycle 3', a relationship that can be analysed in terms of praxeologies. It is then the subjection to this institution that will make the relationships of each  $\xi$  to this object evolve, therefore allowing the relationship of the teacher position to this same object to evolve for the institutions concerned. The process of creating these new relationships is inscribed in a certain spatiality (the institution's in our case), in a long temporality

<sup>4</sup> Cycle 3 comprises of three levels: the last two levels of primary school and the first level of lower secondary school (pupils aged 9 to 12).



(two years) and in a primary school-middle school continuity dynamic (multi-year dynamic, longitudinal follow-up). It is also determined by a full set of conditions and constraints that weigh on the constitution and evolution of these relationships. That is to say, there are conditions and constraints which, for some, facilitate or allow, and for others, hinder or prevent, the construction of knowledge by the actors in the research system, or the interactions between actors and their participation in the collective work. Among these conditions and constraints, we can mention, for example, the relationship of the teacher position to research, which can hinder or, on the contrary, promote collaboration. In our research system, the researcher-guides did not put themselves in a position of research assistants for a period of time. The researchers-teachers saw themselves as experimenters but not as co-producers, which hindered the progress of the research until the relationship of the researchers-teachers changed and they allowed themselves to produce components within the research system. It should be noted here that the decision-making process in the work carried out was not of the 'bottom-up' or 'top-down' type, but that within the same work session, elements were brought in by all parties, resulting in collegial decisions.

To produce an answer to question Q and, in this same study process, to create a relationship to MPS specific to the research system, the latter will call on a fairly rich milieu *M*. Without claiming to be exhaustive, we present below elements that were incorporated into the milieu.

Questions: What are the existing relationships to MPS in the establishments concerned in the teacher position and in the student position? Which one would the institution like to see exist? Which one would be necessary for the results of the students of the schools concerned to improve? How can the relationship of the students to the problems of proportionality be changed?

Elements of answer *A*<sup>0</sup>: MPS is not an area of mathematics for the institution, but mathematics is learned to solve problems and this through MPS. The problems set for national assessments fall under the 'classic' type of problems (proportionality over-represented, data processing and management, determination of an end time, mainly). Students' difficulties relate more to mathematical techniques for dealing with the types of problems set and their justification than to cross-cutting aspects. There is a deficit in the written form of techniques and their amalgamation.

Data: Answers to two questionnaires devised to make an in-depth diagnosis/inventory of the actors' concerns, current practices and difficulties; measuring any changes. Evidence of the activity of teachers and students, of the institution; official texts (programs, resource documents in particular), results of national assessments, observation of the 2019 test, reports of working sessions and class observation, recordings, student

work on the 2020 national assessment and on the three problems set, problems set by teachers on proportionality and its uses, etc.

Work: Conditions and constraints influencing relationships in a student position; the concept of clarification; concepts of mathematical and didactic praxeologies, as well as their mobilisation for analysis and development; the concept of magnitude and the relevant mathematical praxeology relating to this concept; a mathematical praxeology around proportionality.

A first part of the study process of question Q resulted in the emergence of a secondary question *Q<sub>p</sub>* 'How to improve student performance in MPS?'. We will examine this question more specifically in what follows.

### ***Answers given in terms of the evolution of teacher position relationships***

The work carried out in the first part of the project, in particular based on the analysis of national mathematics assessments when entering 6<sup>th</sup> grade (September 2019 and 2020) and a complementary co-constructed assessment, led the group of researchers  $\Xi$  to observe that proportionality was a source of difficulties. On the one hand, proportionality relates to magnitudes, and this concept must be apparent in the praxeologies implemented, which is not the case in the relationship to proportionality prevailing in cycle 3; on the other hand, there is a problem with the juxtaposition of techniques that hinders proportionality problem-solving. The decision was thus taken to prepare a mathematical praxeology which articulated different techniques into a single one by amalgamating them according to their occasions for use (Artaud, 2010; 2019), and which made the concept of magnitude apparent and clearer to manipulate (Artaud, in press). This decision came from the collective work carried out together: proportionality appeared in the assessments as a point of difficulty for the students; the teachers therefore set problems to work on this aspect; in the analysis of these problems, the question of the juxtaposition of techniques was highlighted by a teacher as a source of difficulties, and the question of magnitudes by a researcher; the group therefore agreed on the constitution of a praxeology that would attenuate, if not resolve, these difficulties.

Once this praxeology was developed, the research system worked on its institutionalisation in the classroom. Therefore, the relationships to several objects will be called upon and modified: specifically, we can cite the objects of proportionality, magnitude and institutionalisation. We will examine below what the response produced by the research system made possible to develop in the relationship of the teacher position in the establishments concerned by the project.

Feedback on the project organised in spring-summer 2021 provided access to statements by teachers on the change

in relationships to proportionality and institutionalisation, but also to productions supporting these statements. The new relationship to proportionality now incorporates the use of magnitudes through the automatic writing of units and the fact that the coefficient of proportionality is a quotient magnitude. This led to the work with the coefficient of proportionality being either based on quantities of the same nature, or on a small number of everyday situations that hold meaning for the students—when writing the coefficient of proportionality ‘in extension’: for example, we will write ‘3 euros per kilo’ for the price/mass quotient magnitude. Several elements systematically present in the old relationship to proportionality, which often led to the manipulation of numbers to the detriment of magnitudes, resulting in false equivalence, find their rightful place: the ‘proportionality table’, the unitary technique and that of cross-multiplication. The most significant modification concerns the proportionality table: now used sparingly, mainly with a function of synthesis, it includes magnitudes and not their only measures. The new relationship to institutionalisation, for its part, incorporates a discursive formatting of techniques and jointly, rationales for techniques, with an automatic progression to writing. The text is more present, and the different technical ingredients are amalgamated according to their occasions of use. The concepts of ‘modelling’ and of ‘mathematical formatting’ are also present in the new relationship in connection with the work carried out in the didactics of mathematics. In a more functional way, they replace some elements that were more structurally present in the initial relationship (such as ‘writing a response sentence’ or ‘making an assumption’).

This modification of the relationship to proportionality problem solving for the teacher position in the establishments concerned goes beyond this mathematical theme. Teachers verbalise the fact that it is their ‘professorial praxeologies’ that have evolved, that is to say know-how but also technological-theoretical justifications for these practices. Indeed, their discourse no longer considers MPS as a single ‘block’; they incorporate the idea that the mathematical theme is a relevant line of work; they consider that writing and preliminary development by the teacher of the mathematical praxeology relating to the topic is an important aspect of the teacher’s work, likely to improve the help he/she can give the students so that they succeed in MPS.

## Conclusion and perspectives

Each collaborative study is an investigation on a Q issue that benefits from analysis as a sub-institution of a mandating institution, whether it is institutionally identified, as here with the academic projects, or more broadly a part of the ‘invisible college’ that is collaborative research. The fact of seeing this

system as an element of an institution allows us to analyse the relationships in the researcher position that are created and modified, but also the creation conditions of these research systems. We could, for example, examine how the fact that a research system clearly belongs to a mandating institution or to an invisible college favours, allows or, on the contrary, hinders the creation of institutional relationships to certain objects. For example, here, we can think that the research systems created in the ‘academic projects’ institution have little chance of changing the relationships in the position of teacher in the academic institution, given the small number of these projects and their lability—the mandating institution will in fact cease to exist by 2023, replaced by an ‘Ampiric 2020’ institution of a different nature.

It was the study of a specific mathematical theme that enabled the emergence of elements in response to the question studied by the research system. But it was the previous work that created the conditions for this theme to emerge and for the relationships of the teachers involved in the project to evolve sufficiently for them to engage in the study of MPS on a specific theme. Of course, the particular school context (presence of pupils from special education units<sup>5</sup>, material conditions, resources allocated, presence of national education managers, etc.), the mismatch between the short time of the school year (teachers struggling, for example, with questions of national assessments and skills) and the long duration of research had an impact on the progress of the project, as did the injunction to work collectively remotely in connection with the health management of the Covid-19 crisis.

This modelling of the collaborative process, which is based on a theoretical framework derived from the ATD, thus makes it possible to identify the ingredients of the process, to characterise the roles/responsibilities/interactions of the actors and to infer elements of understanding that give meaning to the work carried out. However, formalisation via the proposed research model should be tested in other collaborative research contexts to extend the validity of the model produced. One of the challenges would be to identify the ecological conditions and constraints that characterise each empirical context of collaborative research in order to consider to which extent the constitution of a research system is productive, or enables the evolution of relationships to which types of objects, etc. Of course, the proposed model is meant to evolve. It could be extended in order to better understand how individual actions come together in collective activities, how relationships/interactions between researchers and

<sup>5</sup> In middle school, the special general and vocational education units take in students with serious and persistent academic difficulties that cannot be remedied by the prevention, assistance and support measures of the traditional system.

actors develop, how the collaborative dynamic is maintained, or how disagreements and differences that may arise in the collaboration are resolved. In addition to the aspects related to the institutionalisation of collaboration, our model could therefore find points of convergence with the meta-didactic transposition model proposed by Aldon et al. (2013) which would speak, for example, of 'shared praxeology' to designate the response obtained without, it seems to us, considering the system producing the response as a single institution.

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