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A B S T R A C T

We study price personalization in a two period duopoly with vertically differentiated prod- ucts. In the second period, a firm not only knows the purchase history of all customers, as in standard Behavior Based Price Discrimination models, but it also collects detailed in- formation on its old customers, using it to engage in price personalization. The analysis reveals that there exists a natural market for each firm, defined as the set of customers that cannot be poached by the rival in the second period. The equilibrium is unique, ex- cept when firms are *ex-ante* almost identical. In equilibrium, only the firm with the largest natural market poaches customers from the rival. This firm has highest profits but not nec- essarily the largest market share. Aggregate profits are lower than under uniform pricing. All consumers gain, total welfare is higher herein than under uniform pricing if firms' nat- ural markets are sufficiently asymmetric. The low quality firm chooses the minimal quality level and a quality differential arises, though the exact choice for the high quality depends upon the cost specification.

1. Introduction

In recent years, firms have remarkably enhanced their ability to engage in sophisticated price discrimination strategies. On the one hand, they are able to collect unprecedented amounts of information on their customers and use such information to produce very accurate customer profiling. On the other hand, firms have access to technologies that enable them to target prices individually according to customers' preferences. We now have plethoric evidence[3] on the use of such sophisticated pricing schemes, including personalized discounts, targeted coupons, personalized post-sale services or included warranties. In this paper, we investigate the competitive and welfare effects driven by firms' ability to target personalized price offers[4] to their returning customers within a vertical differentiation (VD) duopoly with behavior-based price discrim-

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[1] This research has been financed by the Norte 2020 and by the Portuguese public funds through FCT (Fundação para a Ciência e a Tecnologia) in the framework of project NORTE-01-0145-FEDER-028540 and UIDB/04105/2020.

[2] We thank an Editor and two anonymous referees for their helpful suggestions.

[3] See for instance Blattberg and Deighton (1991), Shaffer and Zhang (1995),Ezrachi and Stucke (2016). Mohammed (2017) and Wallheimer (2018) also refer to the increasingly sophisticated pricing strategies currently used in many markets.

[4] Note that perfect price discrimination with a continuum of buyers is a tractable reference point for micro-segmentable markets.

ination (BBPD). In our set-up, one firm has a quality advantage over the other and a higher per unit production cost. We investigate how the interplay between this asymmetry and price personalization can affect the poaching strategies and the competitive process in general; whether both firms succeed in poaching or only one firm, and if so which of the two? Are profits related to quality and cost differentials, and how? Is there an advantaged seller? What will be the effects on prices and welfare? Finally, is there an effect on the choice of quality levels? We will show that the answers to these questions crucially hinge on the relative sizes of what we call the "natural markets" of the firms[5], which depend here upon the quality adjusted cost differential. This concept will play a key role in our model: on the one hand, it delimits the socially optimal allocation of varieties to consumers; on the other hand, it delimits, for each firm, the set of customers that cannot be profitably poached by the rival in the second period, significantly affecting the answer to the previous questions.

The possibility to engage in price discrimination by exploiting the information on the different evaluation of quality by heterogeneous consumers is at the heart of the theory of monopolist price discrimination. In this sense, the extension to duopoly under vertical differentiation is natural and it has already been the object of study in a number of works (e.g. Liu and Serfes, 2005 and 2004 deal with segmentation precision in a vertically differentiated static duopoly). Choudhary et al. (2005) look at price personalization within a static setting of vertical differentiation and they report several real-world examples of price discrimination practices in vertically differentiated industries. For instance, they refer to the use of ROI (Return On Investment) as an indicator of willingness to pay to personalize prices by software vendors (see also Jeffery et al., 2017). The authors also refer other real-world cases pertaining to the healthcare sector, the chemicals industry, and online retailers[6]; other instances include the airline and the hotel industry. Another good example of price discrimination in a vertical differentiation context is represented by wireless carriers, like Verizon, T-Mobile or Sprint. Indeed, these providers exhibit differences in regional coverage, an index of service quality (70% of the US territory for Verizon, 62% for T-Mobile and only 30% for Sprint)[7] In addition, their pricing practices often involve offering discounts to new customers.

Among the many forms of price discrimination allowed by new generation technologies, the literature has devoted particular attention to intertemporal price discrimination practices based on customers' purchase history, better known as BBPD (Behavior Based Price Discrimination). After the seminal works by Fudenberg and Tirole (2000) and Chen (1997), the many contributions investigating the outcomes and the mechanisms underlying BBPD mostly deal with a horizontal differentiation (HD) set-up (see Fudenberg and Villas-Boas, 2007 for a survey). BBPD is shown to usually lead to lower profits for firms; further, it causes welfare losses due to inefficient shopping by those consumers who switch from one to the other seller in the second period (excess "transportation costs" in the Hotelling linear city).

Despite the evidence that a large number of firms are engaging in sophisticated pricing strategies within vertically differentiated set-ups, to the best of our knowledge the extant literature on BBPD has mostly focused on horizontal differentiation (with some exceptions, like for example Jing, 2016 and Rhee and Thomadsen, 2016). Moreover, in most studies on BBPD firms condition the second period prices on the first period shopping patterns of customers, engaging in third-degree price discrimination between new and old customers. However, many firms not only recognize who are their old customers but they also gather finer data on their preferences, using it to engage in market hypersegmentation and price discrimination.[8]

In this vein, the recent paper by Choe et al. (2018) analyzes the outcomes of second period personalized pricing to old customers within a symmetric HD model.[9] They find that firms are harmed by this possibility, which actually intensifies the negative profit effects identified by Fudenberg and Tirole (2000). In their setting, for initially fixed symmetric locations, contrary to standard[10] BPPD models, (i) there are two mirror asymmetric equilibria (one firm prices more aggressively in the first period, ripping off higher intertemporal profits than the rival), (ii) only one-way poaching occurs, instead of two-way poaching.

We use the same information hypotheses as in Choe et al. (2018) but we look at the case of a vertically differentiated duopoly, in which the high quality good has a higher marginal cost than the low quality one. We subsequently show that, under our full market coverage assumption[11], and with exogenous qualities, the present model is isomorphic to a generalization of the horizontal differentiation model *à la* Choe et al. (2018), where their original model is extended to a (possibly) asymmetric setting with exogenous differences in reservation values (and/or differences in marginal costs/ firms' initially exogenous locations). Both models lead indeed to the same equilibrium prices and profits as functions of two parameters:

[5] The "natural market" of a firm is formed by the set of "natural customers" who would buy the good from this firm were the two goods priced at their respective marginal costs.

[6] The trend towards personalization is also seen as self-evident in the literature dealing with fairness and privacy issues (e.g. Richards et al., 2016).

[7] We are grateful to an anonymous reviewer for drawing our attention to this example.

[8] Following the seminal work by Thisse and Vives (1988), other authors have investigated firms' strategies under the possibility of price personalization (e.g. Shaffer and Zhang, 2002; Esteves, 2010; Esteves, 2014; Anderson et al., 2015; Anderson et al., 2019), in most cases in a static set-up. Shy and Stenbacka (2016) and Colombo (2018) are particularly interesting contributions to this discussion, since they analyze the problem within a two-period framework (respectively, using a switching costs model and a Hotelling framework).

[9] Note that Choe et al. (2018) also consider endogenous location choices, showing the existence of two asymmetric location choices so that, in that case, firms natural market shares do differ.

[10] A noticeable exception is Carroni, 2016, who looks at a Hotelling model with cost asymmetries, although in his work firms are unable to set personalized prices.

[11] The full market coverage assumption is crucial to get this result. Under partially uncovered markets, the VD model is no longer isomorphic to the HD model, as shown in Wauthy (2010) in a uniform pricing set-up.

natural market share and price sensitivity of the (static) demand[12] However, the two models differ in their interpretations, implications and in the initial stages (endogenous choice of qualities vs. endogenous location choices, for instance). The welfare analysis is also different in the two settings (see Schmidt, 2009). Moreover, there is no *a priori* reason to suppose in a vertical differentiation model that the natural market shares are equal. Differently, symmetry is a very natural and indeed the most common assumption in HD models (e.g. as in Fudenberg and Tirole, 2000 or Choe et al., 2018), possibly due to the symmetric outcomes of endogenous "location" choices, which are obtained when cost symmetry is also imposed.

We find that the second period equilibria analysis of the pricing game is similar to Choe et al., 2018, and the one-way poaching property is shown to be robust. However, differently from them, herein, the poaching firm (in the second period) is always the one with the largest natural market[13]. In addition, as a consequence of the ex ante asymmetry between the firms' natural market shares, our first-period equilibrium is unique in almost the whole parameter space. Mirror asymmetric equilibria exist only when the natural markets shares are close to equality.

In the competition process, we find that the firm with the largest natural market gives up a part of it to the rival in the first period. This is so because firms (especially the one with the smallest natural market) get informational advantages from serving a large fraction of the market in the first period. This incentive translates into lower first period prices in comparison with both benchmarks of uniform pricing and of BBPD under vertical differentiation (studied in Jing, 2016). The ranking of market shares in terms of size also follows a rich pattern and is different in the two periods, except for when one firm has "extreme" market dominance (namely a natural market larger than 7/10 of the total). As a general rule, the firm that starts with the smallest natural market (the "underdog") carves a market segment beyond its natural market in the first period. In the second period the underdog looses only a part of the conquered natural market of the rival (while the underdog's poaching price is "ineffective" in that it will not attract new customers). The firm with the largest natural market, "the top dog", in spite of being the one giving up part of its "natural customers" in the first period, ends up enjoying higher intertemporal profits than the underdog, except for a relatively small domain of parameters, for which the natural markets are almost equal at the start.

Concerning the overall profit effects, we find that firms are hurt by the possibility to engage in BBPD with personalized pricing (both with respect to the uniform pricing benchmark and with respect to the vertical differentiation setting with standard BBPD studied by Jing, 2016).[14] In horizontal differentiation models consumers gain from BBPD; this is only due to the price decrease, absent any benefit from reallocation of varieties to consumers (to the contrary, when switching to the more distant firm in the second period, some consumers induce a loss in total travel costs with respect to the static benchmark). In our model, consumers gain (though not all of them) from lower prices in both periods and from being induced to switch to the socially efficient product choice. Overall, it is here confirmed that all consumers benefit from BBPD with personalized prices. The highest gains accrue to customers that are more contestable, who end up paying lower prices in both periods (other consumers—those with more extreme preferences—only pay lower prices in the first period, in comparison to the other two benchmarks).

In comparison with the uniform pricing benchmark, provided natural markets are asymmetric enough, the surplus loss generated from qualities misallocation is reduced: price personalization induces a market allocation closer to firms' natural markets, reducing the amount of inefficient shopping from the contestable customers. This is never the case in Choe et al. (2018). Despite this welfare improvement, some degree of misallocation remains. This is due to the efforts of the underdog to attract customers from the rival's natural market in the first period: this distortion is corrected in the second period, when the top dog poaches some customers from the rival, but it is not fully eliminated.

Finally, we briefly analyze endogenous quality choices. Under reasonable (but not general) assumptions on the dependence of marginal costs on qualities, a quality increase by the low quality firm will, *ceteris paribus,* lead to a decrease in its natural market, with adversal effects on its profits. An increase in the quality difference also relaxes price competition. Thus, it is easy to predict that this firm will choose as low a quality as possible, as in the standard vertical differentiation model. The high quality firm decreases its natural market if it increases quality and hence the incentive to relax price competition runs opposite that of creating a hefty natural market. This firm will then go for a quality lead, but the precise quality level depends heavily on the specification of costs.

The plan of the paper is as follows. Section 2 lays down the assumptions and defines the model set-up. Section 3 introduces the notion of natural market and uses backwards induction to compute the equilibrium pricing decisions. Section 4 characterizes the economic properties of the equilibria regarding pricing patterns, consumers' surplus, profits, and social welfare. Section 5 briefly analyzes the issue of endogenous qualities. Section 6 shows the isomorphism between

[12] Of course, the two parameters correspond to different fundamentals in the two models: (i) the price sensitivity of demand relates to the inverse of the quality differential in our model and to the inverse of the transportation cost parameter in the HD model, (ii) the natural market share in our model corresponds to the ratio of the marginal cost differential to the quality differential while in the HD model, it depends on the difference between the reservation values.

[13] In Section 2, we will show that the identity of the firm with the largest natural market will depend on whether the parameter μ - which delimits firms' natural markets in our model - is above or below 1/2.

[14] In the HD model, profits decrease with the use of BBPD (Fudenberg and Tirole, 2000) with respect to the static benchmark, and they further decline when personalized pricing can be used (Choe et al., 2018). In the VD case, the first negative effect of standard BBPD is only partially confirmed as Jing, 2016 and Rhee and Thomadsen, 2016 show that firms may actually be better off with standard BBPD for some (small) parameter regions.

our VD model and an asymmetric extension of Choe et al. (2018) HD model. Finally, Section 7 summarizes the main findings and discusses possible extensions. The proofs are left to the Appendix.

2. The model

Two firms compete in two periods. Firm H is the high-quality firm: it produces a product of exogenous quality q_H at a constant marginal cost c_H. Firm L is the low-quality firm: it produces a product of exogenous quality q_L at a marginal cost c_L. We assume $\Delta = q_H - q_L > 0$. The high-quality firm has a greater marginal cost than the low-quality one, with $c_H > c_L$. We denote $\mu = \frac{c_H - c_L}{q_H - q_L}$, the incremental unit cost from producing the higher quality product, i.e. the quality-adjusted cost differential. As shown later, this parameter will play a very important role in this model. Consumers always buy one unit of one good or the other, i.e. the market is always covered. A type θ-customer derives a gross utility $U + \theta q_X$ from consuming one unit of the quality X-good in a given period. U is large enough to ensure that the market is always covered. θ is uniformly distributed over $[0, 1]$. All agents discount second period payoffs by the same factor $\delta \in [0, 1]$.

Assumption 1. $\mu \in (0, 1)$.

Social surplus maximization would require that a $\theta-$type consumer buys the good X ($X = H, L$) which provides him/her the greater surplus $U + \theta q_X - c(q_X)$, with $c(q_X) = c_L$ if $q_X = q_L$ and $c(q_X) = c_H$ if $q_X = q_H$. From this point of view, types $\theta > \mu$ (resp. $\theta < \mu$) should consume the high quality good (resp. the low quality good). Assumption 1 ensures that it would never be socially optimal to have all agents consuming the same type of good.

The set of customers who should optimally consume the good offered by firm X (with $X = H, L$) delimits what we call the "natural market" of this firm. In the next Section, we find that the delimitation of the firms' natural market plays a key role in determining the firms' ability to poach (or not) customers from the rival. In particular, a first-period Firm X's customer belonging to its natural market cannot be poached by the rival firm in the second period. The measure of the set of "natural customers" of firm X is denoted λ_X^n, where the superscript n stands for "natural market". In the present model the natural market shares are $\lambda_L^n = \mu$ and $\lambda_H^n = 1 - \mu$, respectively.

The timing of the game is the following. In the first period, the firms non cooperatively set uniform prices p_{1H} and p_{1L}, or, equivalently, they set the uniform margins $m_{1H} = p_{1H} - c_H$ and $m_{1L} = p_{1L} - c_L$ in order to maximize their discounted accumulated profits. Given p_{1H} and p_{1L}, it shall be shown that the consumer population is partitioned into two connected intervals, so that there exists a type θ equal to $z(p_{1H}, p_{1L})$ such that[15] the $\theta-$types smaller than z will buy the low quality good, whereas the others will buy the high-quality good (in the first period). It will be useful to denote by λ_X^1, the first period market share of Firm X, with $\lambda_L^1 = z$ and $\lambda_H^1 = 1 - z$.

In the second period, each firm $X = L, H$ will make use of the customer data it collected in the first period. In line with Choe et al., 2018, we assume that (i) firms are able to distinguish between their own and the rival's old customers, as in standard BBPD models; (ii) in the case of old customers, each firm is able to identify their exact type θ and price discriminate accordingly. In the second-period, each firm $X = H, L$ takes its first-period market shares as given, when taking second-period price decisions. The latter are taken according to the following sequence: first, firms simultaneously set uniform prices P_{2X} targeted to new (unknown) consumers; these prices are always observable by the firms. Differently, consumers that have bought from firm X in the first period do not have access to the offer which X targets to new consumers. This is either because—like in the example of telecom service providers—they are identified as "old" customers and prevented by the supplier, or because they are unable to observe the offer made to new customers—as with online shopping. Afterwards, firms decide the personalized prices $p_{2X}(\theta)$ charged to their old consumers, resulting in profit margins $m_{2X}(\theta) = p_{2X}(\theta) - c_X$ and $m_{2X} = P_{2X} - c_X$. The sequential pricing assumption is common in the literature (see for instance Thisse and Vives, 1988; Choe et al., 2018; Chen et al., 2020). Moreover, from a managerial perspective, it is generally considered that uniform prices are chosen less frequently and at a higher level of management.[16] An implication of this *no-leakage* assumption is that second period price competition in the turf of a given firm is independent of the other turf so that the two market segments can be treated separately. In the next section, we look for the Subgame Perfect Nash Equilibria of the sequential game described above.

3. Equilibrium analysis

3.1. Second-period equilibrium

In the second period, given the first period market shares $\left(\lambda_L^1, \lambda_H^1\right)$, $\lambda_L^1 = z$, $\lambda_H^1 = 1 - z$, firm X ($= H, L$) first proposes a uniform price P_{2X} to its rival's old customers and then chooses a personalized price $p_{2X}(\theta)$ to each of its own type $\theta-$old customers.

[15] As in standard models of BBPD with forward-looking customers, when taking their decision on which good to buy in the first-period, customers account for their overall utility (anticipating second-period equilibrium outcomes when they compare the two-period overall utility of buying good H or good L in the first period).

[16] We are grateful to an anonymous reviewer for drawing our attention to this point.

The sets of consumers who choose Firm H and Firm L in the first period are, as shown by a standard revealed preference argument, connected intervals, so that we can define a unique value z such that all $\theta \in [0, z]$ (resp all $\theta \in [z, 1]$) buy the low-quality (resp. high quality) good in period one. We first determine under which conditions a Firm X's first period customer may or may not be poached by the rival (no-poaching conditions); then we study the second period equilibria conditional on the first period market shares (z and $1-z$).

3.1.1. *The no-poaching conditions*

A given type-θ of Firm H's old customer cannot be poached in the second period by Firm L iff

$$\theta q_H - c_H \geq \theta q_L - c_L \Leftrightarrow \theta \geq \mu. \tag{1}$$

Such a customer will choose H whenever $\theta q_H - p_{2H}(\theta) \geq \theta q_L - P_{2L}$. Firm L never benefits from pricing below marginal cost so that one must have $P_{2L} \geq c_L$. Accordingly, whenever condition (1) holds, because of Firm H's ability to engage in personalized pricing, for given P_{2L}, firm H can undercut any price $P_{2L} \geq c_L$ by setting a personalized price $p_{2H}(\theta) \geq c_H$ and make a positive profit.

By a similar argument, a Firm L's type $\theta-$old customer cannot be poached by Firm H iff

$$\theta q_H - c_H \leq \theta q_L - c_L \Leftrightarrow \theta \leq \mu. \tag{2}$$

If a given natural customer of Firm X has already bought good X $(= L, H)$, its rival is unable to poach this customer in the second period. In each market segment only one of the firms is able to engage in price personalization (in BBPD models, data collection about customers requires some previous interaction between them and the firms). Hence, the firm which is trying to retain its "natural customers" can always offer them a better bargain than the rival.[17]

The no-poaching conditions just derived are going to facilitate the analysis of the second period equilibrium. Indeed, (i) the equilibrium values of $p_{2L}(\theta)$ and P_{2H} for $\theta \in [0, z]$ are not affected by the choices of price schedules in the segment $[z, 1]$; (ii) the equilibrium values of $p_{2H}(\theta)$ and P_{2L} for $\theta \in [z, 1]$ are not affected by the choices of price schedules in the segment $[0, z]$. Therefore, two cases must be distinguished according to the values of z and μ: (i) $z \geq \mu$ and (ii) $z \leq \mu$. These two cases are analyzed separately in the following subsections. Before we proceed with the equilibrium analysis, it is important to note that sequential pricing in the second period may lead to multiple equilibria when considering a firm i which chooses a uniform price in its rival j's non contestable market. Since j is always able to successfully protect its turf, Firm i's uniform price is indeterminate. To break this indeterminacy, we borrow from Lemma 2 in Chen et al. (2020), a refinement argument which, as shown in Result 1 (in Appendix) allows us to select the unique equilibrium where i 's uniform price equals its marginal cost.[18]

3.1.2. *Second-period equilibrium when $z \geq \mu$.*

When the low quality firm inherits from the first period an actual market share higher than its natural one, i.e. $\lambda_L^1 = z \geq \lambda_L^n = \mu$, only Firm H is able to poach customers (namely those located in $[\mu, z]$). Starting by the last stage of our sequential game, we first look at the personalized pricing decisions of H, given Firm L's second-period uniform pricing and first-period market shares, $\left(\lambda_L^1, \lambda_H^1\right)$.

Henceforth we assume that an indifferent consumer buys from Firm H; therefore H's best reply on each θ, denoted $b_H(P_{2L}, \theta)$, in its own turf is the "conventional one": $b_H(P_{2L}, \theta) = P_{2L} + \theta\Delta$, which makes θ just indifferent between the two products.[19] Lemma 1 sums up the main results regarding second-period price competition when $z \geq \mu$.

Lemma 1. *When $z \geq \mu$, the second period equilibrium price decisions are such that:*

(i) *In Firm H' s turf, corresponding to the $[z, 1]$ interval, the equilibrium price schedules are*

$$p_{2H}(\theta) = \theta\Delta + c_L \, \textit{and} \, P_{2L} = c_L. \tag{3}$$

(ii) *In Firm L's turf, corresponding to the interval $[0, z]$, equilibrium price schedules are*

$$P_{2H} = c_L + (\Delta/2)(\mu + z) = \frac{c_L + c_H + \Delta z}{2} \text{ and } p_{2L}(\theta) = Max\{P_{2H} - \theta\Delta, c_L\}. \tag{4}$$

The second period indifferent customer locates at $\overline{\theta}^* = (z + \mu)/2$ and price schedules of H and L are therefore continuous over $[0, z]$, with

$$p_{2L}(\theta) = c_L + \frac{\mu + z}{2}\Delta - \theta\Delta \text{ for } \theta \in [0, (z + \mu)/2] \tag{5}$$

$$= c_L \text{ for } \theta \in [(z + \mu)/2, z]. \tag{6}$$

[17] Recall that the natural customers of a firm are those preferring the good of this firm in the limit case of marginal cost pricing.

[18] This equilibrium is the limit of a perturbed game in which both firms compete with uniform prices on a small subinterval of j's market.

[19] We use the conventional best replies since using mixed strategies equilibria would yield the same payoffs (see Blume, 2003; Kartik, 2011; De Nijs, 2012).

Proof. See the Appendix. □

Then, from equation (4) in Lemma 1 the Firm H's second period uniform profit margin on poached customers, $M_{2H} = \frac{\Delta}{2}(z-\mu)$ is a linear increasing function of the quality gap Δ.[20]It is also increasing with the gap between z and μ: when it goes up, the set of customers that firm H may possibly poach enlarges and this firm becomes less aggressive, as it may get a higher mark-up on a possibly larger set of customers. Comparing $M_{2H} = \frac{\Delta}{2}(z-\mu)$ to $m_{2H}(\theta) = \Delta(\theta-\mu) \geq \Delta(z-\mu)$, it turns out that, for any $\theta \in [z, 1]$, the profit margin on loyal customers is more than twice the profit margin on new customers.

The second period equilibrium market shares are: $(z+\mu)/2$ for firm L and $1-(z+\mu)/2$ for firm H; the second period market share of Firm X $(=H, L)$ is simply the average of its natural market share and its first period market share, so that Firm H has a second period market share lower than its natural market and the reverse is true for Firm L. The quantity of customers poached by firm H is $(z-\mu)/2$. The second period equilibrium profits[21] can then be expressed simply as functions of their respective natural and first period market shares:

$$\pi_H^+ = \frac{\Delta}{2}\left[\lambda_H^1(2\lambda_H^n - \lambda_H^1) + \frac{(\lambda_H^n - \lambda_H^1)^2}{2}\right], \text{ and } \pi_L^+ = \frac{\Delta}{2}\left(\frac{\lambda_L^1 + \lambda_L^n}{2}\right)^2. \tag{7}$$

The superscript ''+'' is used in (7) to denote that the profit expressions refer to the case $z \geq \mu$.

3.1.3. Second-Period equilibrium when $z \leq \mu$

We consider now the case when the high quality firm has a first period market share larger than its natural market. This is a mirror of the case $z \geq \mu$ and the arguments are parallel. As shown in Lemma 2, results are also similar, the only difference being the identity of the poaching firm. Thus, all observations made after Lemma 1 remain valid.

Lemma 2. *When $z \leq \mu$, the second period equilibrium price decisions are such that:*

(i) *For θ in $[0, z]$ the unique Nash equilibrium schedules are:*

$$P_{2H} = c_H \text{ and } p_{2L}(\theta) = c_H - \theta\Delta = c_L + \Delta(\mu - \theta). \tag{8}$$

(ii) *Over $[z, 1]$, the unique equilibrium price schedules are:*

$$P_{2L} = c_H - \Delta(\mu+z)/2 = (c_H + c_L - z\Delta)/2, \tag{9}$$

$$p_{2H}(\theta) = \max\left\{\left(c_H - \Delta\frac{(\mu+z)}{2} + \theta\Delta\right),\ c_H\right\}$$

One can verify that $P_{2L} = c_H - \Delta\overline{\theta}$, where again the second period indifferent consumer is located at $\overline{\theta}^* = (\mu+z)/2$ and that $p_{2H}(\theta)$ is continuous.

Proof. Since the proof is the mirror of the proof of Lemma 1 it has been omitted. □

For the sake of completeness, remark that $P_{2L} - c_L = (\Delta/2)(\mu - z)$ is positive if $z < \mu$. Again, firms' second period equilibrium profits can be expressed as functions of λ_X^1, λ_X^n, $X = L, H$[22]:

$$\pi_H^- = \frac{\Delta}{2}\left(\frac{\lambda_H^1 + \lambda_H^n}{2}\right)^2 \text{ and } \pi_L^- = \frac{\Delta}{2}\left[\lambda_L^1(2\lambda_L^n - \lambda_L^1) + \frac{(\lambda_L^n - \lambda_L^1)^2}{2}\right]$$

These expressions are the exact mirror of those we obtained in the case $z \geq \mu$ (see (7)). The superscript '' - '' is intended to specify that the expressions above refer to the case $z \leq \mu$.

To sum up, the firms' equilibrium profits in the second period are respectively given by

$$\pi_H(z) = \begin{cases} \pi_H^+(z) = \frac{\Delta}{2}(1-z)(1+z-2\mu) + \Delta(\frac{z-\mu}{2})^2 \text{ if } z \geq \mu, \\ \pi_H^-(z) = (\Delta/8)(z+\mu-2)^2 \text{ if } z \leq \mu. \end{cases} \tag{10}$$

$$\pi_L(z) = \begin{cases} \pi_L^+(z) = (\Delta/8)(z+\mu)^2 \text{ if } z \geq \mu, \\ \pi_L^-(z) = \Delta z\left(\frac{2\mu-z}{2}\right) + \Delta\left(\frac{\mu-z}{2}\right)^2 \text{ if } z \leq \mu. \end{cases} \tag{11}$$

Notice that the profit functions are continuous with respect to z but not continuously differentiable at $z = \mu$. Consider first $\pi_H(z)$. For $z \leq \mu$, one obtains $\frac{\partial \pi_H^-(z)}{\partial z} = (\Delta/4)(z+\mu-2) < 0$. For $z \geq \mu$, $\frac{\partial \pi_H^+(z)}{\partial z} = (\Delta/2)(\mu - z) \leq 0$. At $z = \mu$,

[20] Notice that (i) an increase in the quality gap allows firm H to increase the poaching price to its new customers and (ii) the personalized price schedules of L ($p_{2L}(\theta)$ in (5)) and of H ($p_{2H}(\theta)$ in (3)) also increase if the quality gap Δ increases.

[21] This is simply because over the interval $[z, 1]$ firm H's profit margin is $(\theta-\mu)\Delta$ on each customer, yielding profits $\int_z^1(\theta-\mu)\Delta d\theta$. In period two L's profits in this segment are zero. Over $[z, 1]$ second period profits are $\pi_H^2 = (1/2)\Delta(1-z)(1+z-2\mu)$ and $\pi_L^2 = 0$. Over $[0, z]$, one has $\pi_H^2 = \Delta(\frac{z-\mu}{2})^2$ and $\pi_L^2 = (1/8)\Delta(z+\mu)^2$.

[22] The equilibrium payoffs when $z \leq \mu$ are as follows. On $[z, 1]$, where firm L poaches $\left[z, \frac{\mu+z}{2}\right]$ one has $\pi_H = (\Delta/8)(z+\mu-2)^2$ and $\pi_L = (\Delta/4)(\mu-z)^2$. On $[0, z]$, since all customers are served by firm L, which enjoys a margin $\Delta(\mu-\theta)$ on each θ. Thus, one has $\pi_H = 0$ and $\pi_L = \int_0^z \Delta(\mu-\theta)d\theta = \Delta z(\mu - (z/2))$.

the left-derivative equals $\frac{\Delta}{2}(\mu-1)<0$ while the right-derivative equals 0. Consider then $\pi_L(z)$. For $z\le\mu$, one obtains $\frac{\partial\pi_L(z)}{\partial z}=\frac{\Delta}{2}(\mu-z)\ge 0$. For $z\ge\mu$, $\frac{\partial\pi_L(z)}{\partial z}=\frac{\Delta}{4}(z+\mu)>0$. At $z=\mu$, the left-derivative equals 0 while the right-derivative equals $\frac{\Delta}{2}\mu>0$. So, in both cases, around $z=\mu$, the sensitivity of a firm's second period profits with respect to its first period market share is smaller when it is poaching its rival's customers than when the reverse occurs: with poaching, the variation in the profits made out of new customers compensates (locally) the opposite variation in the profits made out of old returning customers.

3.2. First period equilibria

3.2.1. First period demand functions

Consider first the case $z\ge\mu$, where only Firm L's customers can be poached. The marginal customer in the first period (i.e., the *z−type* one) is then indifferent between staying with Firm H for the two periods and choosing Firm L in the first period and switch to Firm H only in the second one (buying the high-quality good at discount as he/she benefits from the poaching price). Recalling that one can write $P_{2H}=(1/2)(\Delta z+c_H+c_L)$ given (3) and (4), it must then be that:

$$zq_H-p_{1H}+\delta(zq_H-z\Delta-c_L)=zq_L-p_{1L}+\delta(zq_H-(\Delta z+c_H+c_L)/2).$$

Now consider the case when $z\le\mu$, where only Firm H's customers can be poached. The marginal customer z is now indifferent between choosing Firm H in the first period and then switching to Firm L or staying with Firm L for the two periods. Given (8) and (9),

$$zq_H-p_{1H}+\delta(zq_L-((c_H+c_L-z\Delta)/2))=zq_L-p_{1L}+\delta(zq_L-c_H+z\Delta).$$

The solution for z, denoted as $z(p_{1H},p_{1L})$ is the same in both cases and given by:

$$z(p_{1H},p_{1L})=\frac{(1/\Delta)(p_{1H}-p_{1L})-\mu(\delta/2)}{1-(\delta/2)}. \tag{12}$$

This can be conveniently rewritten as

$$z(m_{1H},m_{1L})=\mu+\frac{m_{1H}-m_{1L}}{\Delta(1-(\delta/2))}, \tag{13}$$

so that, according to our definition of (L's) natural market, $z(0,0)=\mu$: when firms L and X price at marginal cost, their respective market shares are given by μ and $1-\mu$. The sensitivity of the first period market shares with respect to the first period price differential $(p_{1H}-p_{1L})$ is increasing with the discount factor δ. The following Remark highlights the specific features of z when $\delta=1$.[23]

Remark 1. *Notice that when $\delta=1$, if we take zgiven by (12), we get that, in the case $z\ge\mu$, all the consumers to the right of the marginal customer do not strictly prefer H over L but are indifferent between the two, just as the marginal customer does (This is clear when writing the difference in utilities from buying at the firms as a separate function of the consumer's type θ and the expected value of z: the terms in θ cancel out when $\delta=1$).*

The result in Remark 1 no longer arises for $\delta<1$, where all customers at the right of z strictly prefer to buy H instead of L. In addition, even for $\delta=1$, we get that $z(p_{1H},p_{1L})$ as defined by (12) remains the equilibrium marginal customer (in the sense that, in equilibrium, all customers to the right of z do buy good H). Would indeed the marginal customer be some $z>z(p_{1H},p_{1L})\ge 0$, then the poaching price P_{2H} would be such that at least all types θ-customers in $[z(p_{1H},p_{1L}),z]$ would strictly prefer H over L, a contradiction. It is also worth noting that when letting $\Delta=2t$ and looking at a symmetric case $(\mu=1/2)$, then $z(p_{1H},p_{1L})$ corresponds exactly to Choe et al. (2018) first period demand function (Section 3.2. page 12).[24]

3.2.2. Equilibrium prices

To simplify the exposition, herein we set $\delta=1$, while the equilibrium results with $\delta\in[0,1]$ are presented in the Appendix. The firms' two-period discounted profits are equal to:

$$\Pi_H(m_{1H},m_{1L})=\begin{cases}(1-z)m_{1H}+\pi_H^+(z)\ \text{if} z\ge\mu,\\(1-z)m_{1H}+\pi_H^-(z)\ \text{if} z\le\mu.\end{cases} \tag{14}$$

and

$$\Pi_L(m_{1H},m_{1L})=\begin{cases}z\,m_{1L}+\pi_L^+(z)\ \text{if} z\ge\mu,\\z\,m_{1L}+\pi_H^-(z)\ \text{if} z\le\mu.\end{cases} \tag{15}$$

where z is defined by (13). These profit functions are continuous with respect to the first period margins m_{1H} and m_{1L}. However, owing to the discontinuity of the derivatives of the second period profits with respect to z, they are not continuously differentiable. This non-surprisingly results into discontinuous best-reply functions. Indeed, starting with firm H:

[23] We thank an anonymous referee for pointing this result. The remark is written for $z\ge\mu$. A symmetric remark can be made for the case $z\le\mu$.

[24] Unit costs are zero in their model.

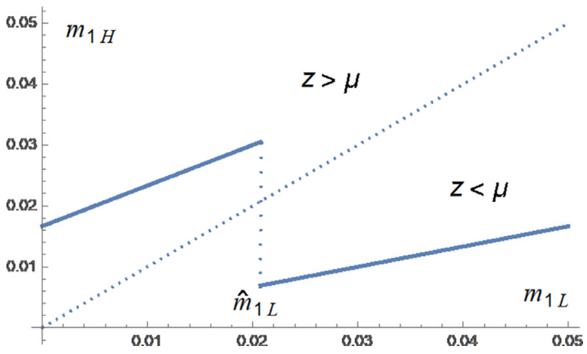

Fig. 1. H's best reply function.

For a given value of m_{1L} the profit to firm H is given by the first line in (14) as far as $m_{1H} \le m_{1L} \Longleftrightarrow z \ge \mu$; while for $m_{1H} \ge m_{1L} \Longleftrightarrow z \le \mu$ it is given by the second line. Both functions in (14) are concave with respect to m_{1H} and get a unique maximum. Hence, a best reply is either defined by the maximizer of the first line or of the second one, according to a comparison of the maximum values of the two functions. The result of this comparison depends upon whether m_{1L} is above or below a given threshold. This is the intuition behind the following Claims.

Claim 1. *Let,* $\widehat{m}_{1L} = (\Delta/2)(\sqrt{2}-1)(1-\mu)$, *the best-reply function of firm H is*

$$BR_H(m_{1L}) = \begin{cases} \frac{1}{3}m_{1L} \;(\Rightarrow\; z \le \mu) \text{ if } m_{1L} \ge \widehat{m}_{1L} \\ \frac{1}{6}\Delta(1-\mu) + \frac{2}{3}m_{1L} \;(\Rightarrow\; z \ge \mu) \text{ if } m_{1L} \le \widehat{m}_{1L}; \end{cases},$$

Claim 2. *Let* $\widehat{m}_{1H} = (\Delta/2)(\sqrt{2}-1)\mu$, *the best-reply function of firm L is*

$$BR_L(m_{1H}) = \begin{cases} \frac{1}{6}\Delta\mu + \frac{2}{3}m_{1H} \;(\Rightarrow\; z \le \mu) \text{ if } m_{1H} \le \widehat{m}_{1H} \\ \frac{1}{3}m_{1H} \;(\Rightarrow\; z \ge \mu) \text{ if } m_{1H} \ge \widehat{m}_{1H} \end{cases},$$

Given that $1-\mu$ and μ are nothing else than $H's$ natural market share and $L's$ natural market share, the two firms' best-reply functions exactly mirror each other. Furthermore, for any rival's unit margin, a firm logically chooses a smaller margin (and a lower price) when it prefers to expand its first period market share beyond its natural market.

Without loss of generality, we can then consider only Firm H's best reply function. It is discontinuous at $m_{1L} = \widehat{m}_{1L}$; more precisely m_{1H} jumps down when m_{1L} crosses this critical value. What is the intuition for this downward jump of m_{1H}? For $m_{1L} < \widehat{m}_{1L}$, we have $z > \mu$, so that Firm H derives second period profits from new customers it poaches from L. These profits decrease when m_{1H} decreases. For $m_{1L} \ge \widehat{m}_{1L}$, $z \le \mu$, so that H does not attract any new customers. Accordingly it has greater incentives to set a low first period price in order to enlarge its second period customer base. In Figure 1, we draw H's best-reply function for the case $\delta = 1, \mu = 0.5, \ \Delta = 0.2$. Notice that for these values of the parameters, we get $\widehat{m}_{1L} \simeq 0.020711$.

From the last two Claims it follows that $z > \mu$ implies both: $m_{1L} < \widehat{m}_{1L}$ and $m_{1H} > \widehat{m}_{1H}$; while $z < \mu$ implies the reversion of both inequalities. Hence there are only two possible equilibria. Either the first period equilibrium decisions $\left(m^*_{1H}, m^*_{1L}\right)$ are such that $z\left(m^*_{1H}, m^*_{1L}\right) > \mu$, i.e. $m^*_{1L} < \widehat{m}_{1L}$ and $m^*_{1H} > \widehat{m}_{1H}$ (as in equilibrium A), or they are such that $z\left(m^*_{1H}, m^*_{1L}\right) < \mu$, i.e. $m^*_{1L} > \widehat{m}_{1L}$ and $m^*_{1H} < \widehat{m}_{1H}$ (as in equilibrium B).

Proposition 1.

(A) *Let* $\mu^A = 0.50852$ *then iff* $\mu \le \mu^A$, *there exists a unique pure strategy Nash Equilibrium* $\left(m^{A*}_{1H}, m^{A*}_{1L}\right)$ *such that* $z = z^{A*} > \mu$, *with*

$$\begin{aligned} m^{A*}_{1L} &= \Delta(1-\mu)/14 = \frac{1}{14}(q_H - q_L - c_H + c_L), \\ m^{A*}_{1H} &= 3\Delta(1-\mu)/14 = \frac{3}{14}(q_H - q_L - c_H + c_L), \\ z^{A*} &= (5\mu+2)/7 \text{ and } \bar{\theta}^{*A} = (6\mu+1)/7, \end{aligned} \tag{16}$$

where firm H poaches some of firm L's old customers (in the second period).

(B) *Let* $\mu^B = 0.491\,48$; *then, iff* $\mu \ge \mu^B$, *there exists a unique pure strategy Nash-equilibrium* $\left(m^{B*}_{1H}, m^{B*}_{1L}\right)$ *such that in equilibrium* $z = z^{B*} < \mu$, *with*

$$\begin{aligned} m^{B*}_{1L} &= 3\Delta\mu/14 = \frac{3}{14}(c_H - c_L), \\ m^{B*}_{1H} &= \Delta\mu/14 = \frac{1}{14}(c_H - c_L), \end{aligned} \tag{17}$$

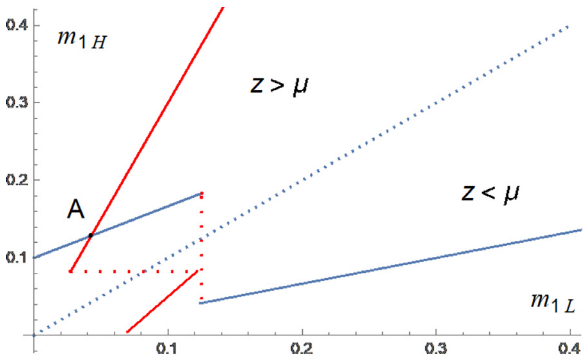

Fig. 2. Type-A Equilibrium

$$z^{B*} = 5\mu/7 \text{ and } \bar{\theta}^{*B} = 6\mu/7,$$

where now the low-quality firm L poaches in the second period some of firm H′s old customers.

Proof. See Appendix. □

The results in Proposition 1 illustrate that the specificity of the vertical differentiation model appears clearly and the symmetry between firms breaks down when the unit margins are expressed as functions of the cost and quality differentials**.** It is worth emphasizing that though the final expressions for margins in equilibrium B do not contain the quality differential Δ (since $\Delta\mu = c_H - c_L$), this is only true in the limit case for the discount factor $\delta = 1$ that we are considering here only to simplify the exposition.[25]

In the tiny range of $\mu-$values in [0.49148,0.50852] the two equilibria coexist. Otherwise a unique equilibrium exists. In the range $\mu < 0.49148$, the type A equilibrium obtains, with firm H poaching some customers. In the equilibrium of type B it is firm L that poaches (this equilibrium is unique for $\mu > 0.50852$). Fig. 2 illustrates the case where a type A equilibrium exists (and is unique). It is drawn for $\mu = 0.4$, $\delta = 1$, $\Delta = 1$. H's best-reply function is drawn in blue, L's in red.

Suppose that $\mu = 0.6$, $\delta = 1$ and $\Delta = 1$. Then, the representation of Equilibrium B mimics, *mutatis mutandis,* the representation of Equilibrium A in Fig. 2.[26]

Proposition 1 illustrates the symmetry between outcomes in Equilibrium A and Equilibrium B. Comparing first period equilibrium mark-ups, we get that the behavior of equilibrium unit margins as functions of the firm's natural market share (μ for Firm L, $1-\mu$ for Firm H) and the quality gap Δ (which is the inverse of the sensitivity of demand with respect to the price differential) are completely symmetric in the two equilibria. Indeed, when we look at a given firm $X = L, H$, its equilibrium mark-up is equal to: (i) $\frac{3\Delta}{14}\lambda_X^n$ when firm X poaches some consumers in the second period; (ii) $\frac{\Delta}{14}(1-\lambda_X^n)$, otherwise. When the natural market of the high (resp. low) quality firm is large, in the sense that $\mu \le \mu^A$ (resp. $\mu \ge \mu^B$), firm H (resp. firm L) gives up on serving all its natural customers in the first period, in order to get higher mark-ups. Remark that the share of contestable customers (and poached ones) increases with the asymmetry of natural markets, namely it decreases with μ in equilibrium A and with $1-\mu$ in equilibrium B.

4. Characterization

4.1. Dominance and price strategies

We proceed to discuss the main features and economic implications of the equilibrium analysis. To start with, it is interesting to evaluate the market shares in the two periods for the ranges of μ in the intervals $A_\mu \equiv [0, \mu^B]$ and $B_\mu \equiv [\mu^A, 1]$ where the equilibrium is unique.

Remark 2. The firm with the largest natural market, the "top dog", obtains the largest market share in both periods only if its natural market is at least equal to 70% of the total.

For $\mu \in A_\mu$ in equilibrium A, the period 1 market share of L is $z^{A*} = (5\mu+2)/7$ and therefore $z^{A*} > 1/2 \Longleftrightarrow \mu > 0.3$. Period 2 market share of L is $\bar{\theta}^A = (6\mu+1)/7$, hence $\bar{\theta}^A > 1/2$ if $\mu \in (0.416, \mu^B]$. Let $a_1 = 0.3$ and $a_2 = 0.416$ define the

[25] The general expressions for all prices can be found in the proof of Proposition 1 in the Appendix. For $\delta < 1$, when we go back from the notion of natural market to the primitives of the vertical differentiation model, we have that $p_L^{B*} = c_L + \Delta\frac{8(1+\mu)-\delta(8+5(2-\delta)\mu)}{24-10\delta}$ and $p_H^{B*} = c_H + \Delta\frac{4(4-\delta)(1-\delta)-\mu(8+\delta(\delta-10))}{24-10\delta}$. Thus, it follows that firms' mark-ups in equilibrium B are not independent of Δ, for $\delta < 1$.The general expression for z^{A*} is $z^{A*} = \frac{4\mu-2\delta+\mu\delta+4}{12-5\delta}$ while $z^{B*} = \frac{4-4\delta+4\mu+\delta\mu}{12-5\delta}$, which are both unambiguously increasing in μ.

[26] To visualize the representation of Equilibrium B, it is enough to make a small number of adaptations to Fig. 2: switch the axes and let the blue (red) lines now be L's (H's) best-reply function. Let also the area above (resp. below) the bisector correspond now to $z < \mu$ (resp. $z > \mu$).

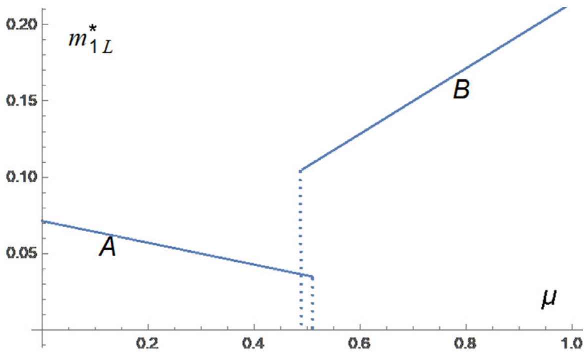

Fig. 3. Firm L's margin as a function of μ

intervals as $A_1 = [0, a_1]$, $A_2 = [a_1, a_2]$, $A_3 = [a_2, \mu_B]$ and $B_1 = [\mu_A, 1 - a_2]$, $B_2 = [1 - a_2, 1 - a_1]$, $B_3 = [1 - a_1, 1]$. For μ in A_1, firm H dominates in both periods, for μ in A_2 firm L dominates in the first period only and H in the second period, for μ in A_3 firm L dominates in both periods. A similar analysis applies *mutatis mutandis* for equilibrium B, so that for μ in B_1 firm H dominates in both periods, for μ in B_3 firm L dominates in both periods, and for μ in B_2 firm H dominates in the first while firm L dominates in the second period. Hence the underdog firm can obtain a dominant market share in both periods, though only when its natural market share is at least roughly 2/3 of the rival's share. In the range for μ in $[\mu_B, \mu_A]$ both equilibria coexist and no clear-cut prediction is possible.

Remark 3.

(i) The underdog prices more aggressively in the first period than the top dog: the profit margin of the former is one third the profit margin of the latter.
(ii) More asymmetry in the natural markets relaxes first period price competition.
(iii) An increase in the quality differential also relaxes first period price competition.

By charging a low first period price, a given firm increases the size of its own turf, getting information superiority over the rival (in the second period) within a larger set of customers. The incentive to set a low first period price is stronger for the underdog firm, since it starts with a small natural market and therefore its base of customers for personalized pricing is naturally smaller. Concerning part (ii) in the Remark above, it is worth noting that if a firm starts with an extremely small natural market, the rival is more easily willing to give up on a fraction of natural customers in the first period (anticipating that it will be able to recover at least some of them later). This effect counterbalances the incentives to price low so as to increase one's own turf. When the natural market of the underdog grows large enough, the second period poaching market share becomes small (as both natural markets are close to 1/2) and therefore firms attach more value to acquire/defend the own turf, which leads to tougher competition in the first period.

In both equilibria, the firm which poaches in the second period (which is almost always the "top dog") exploits its natural advantage by setting a first period unit margin thrice as large as the unit margin of the "underdog" (the poached firm). Although the top dog starts by loosing some customers to its rival, it has a larger mark-up on each first period customer.

The first period unit margin of the poaching firm is increasing in its natural market share while that of the poached firm is decreasing in its natural market share. This reflects the differences in the strategies of the two firms: the top dog translates all the more its natural market advantage into a greater margin; the underdog has to price all the more aggressively to attract first period customers (in order to extend its turf behind its natural market). Fig. 3 illustrates this result for Firm L—The same Figure obtains for Firm H simply replacing m^*_{1L} by m^*_{1H} and μ by $1 - \mu$. We get that for both firms, equilibrium margins (prices) decrease as the sizes of the natural market shares converge towards equality.

4.2. Comparisons

In order to shed some light on the economic effects of using accurate information on the preferences of returning customers to engage in price personalization within a BBPD framework, it is useful to compare our results with two benchmarks: (i) the static uniform price competition benchmark; and (ii) the standard BBPD setting with vertical differentiation (studied in Jing, 2016).

Let us start by looking at the difference between first period prices in our setting and the *static uniform prices*, which can be interpreted as a measure of firms' aggressiveness in the first period. Let p^u_X denote the price of firm X in the static game. Then one retrieves $p^u_L = (\Delta + c_H + 2c_L)/3$, and $p^u_H = (2\Delta + 2c_H + c_L)/3$, so that:

$$m^u_X = (\Delta/3)(1 + \lambda^n_X), \text{ for } X = H, L.$$

When X is the poaching firm (H in equilibrium A, L in equilibrium B) the difference $m^u_X - m^*_{1X} = \frac{\Delta}{42}\left(14 + 5\lambda^n_X\right)$ is strictly positive and increasing in firm X' s natural market share. When X is the poached firm, we have $m^u_X - m^*_{1X} = \frac{\Delta}{42}(11 + 17\lambda^n_X)$, which is again strictly positive and increasing in the firm's natural market share though at a higher rate. One can then say that BBPD with personalized prices intensifies first period price competition**.** The poached firm becomes relatively more aggressive than the rival when its natural market increases, confirming that first period price competition is more intense when firms are *ex-ante* more symmetric.

It is also interesting to compare our equilibrium first period prices with the corresponding counterparts in the standard VD model with BBPD. To allow the direct comparison between our results in the simplified model, we set $\delta = 1$ in the results of Jing (2016), obtaining, after the necessary notation adaptation[27], the following first period equilibrium mark-ups:

$$m^J_{1X} = (\Delta/24)(13 + 6\lambda^n_X), \text{ for } X = H, L,$$

where J stands for Jing (2016). Keeping in mind that the main difference between the two settings lies in the ability to personalize prices to old customers (which is only considered in our set-up), for the poaching firm we get: $m^J_{1X} - m^*_{1X} = \frac{1}{168}\Delta\left(6\lambda^n_X + 91\right)$ which is strictly positive and increasing in the natural market of X. As for the firm which looses customers to the rival, the difference is $m^J_{1X} - m^*_{1X} = \frac{1}{168}\Delta(54\mu + 79)$ which is again positive and increasing in its natural market.

Remark 4. The possibility to exercise third-degree price discrimination among new and old customers relaxes first period price competition in comparison with the static uniform pricing scenario. However, if we allow firms to send personalized pricing offers to each returning customer, first period price competition becomes more intense, with:

$$m^J_{1X} > m^u_X > m^*_{1X}.$$

This result depart from results obtained within standard BBPD models, in which the possibility to discriminate prices between old and new customers ends up relaxing first period price competition. Below we compare our second period equilibrium prices for switching customers and for loyal ones and the corresponding counterparts in the benchmarks of uniform pricing and VD with standard BBPD.

Remark 5. In the second period, in both equilibria, A and B, (i) the poaching prices are lower than the static uniform prices; (ii) there is always a subset of consumers for whom the personalized prices are higher than the static uniform prices (in particular those buying from H and with the highest types in equilibrium A, and those buying from L and with the lowest types in equilibrium B) and a subset for whom the reverse is true.

Proof. See the Appendix. □

These results are very intuitive: only customers whose taste for quality is either extremely low or extremely high may possibly pay a higher price herein than under no discrimination; the customers with a high θ, for instance, are naturally within the top quality firm as they are resistant to substitute a low price for a high quality.

When we compare our second period equilibrium prices with standard VD with BBPD (studied in Jing, 2016), we find that the possibility to personalize prices tends to result in lower prices, except for those customers with very extreme preferences. In those cases, the high-quality (resp. low-quality) firm is able to identify their very high (low) willingness to pay for quality, using this information to rip off additional rents. However, for most of customers (namely all those with intermediate $\theta-$ values) equilibrium prices are lower herein than in Jing (2016). In the next subsection, we look in more detail at the consumer welfare and profit effects.

4.3. Consumer surplus and firms' profits

Although some customers may pay higher second period prices, we find that overall all consumers benefit when firms become able to send type-dependent prices to old customers. The largest gains are obtained by those consumers that have less extreme taste parameter θ.

Proposition 2. *All consumers gain under price personalization with respect to static pricing.*

Corollary 1. *Since in neither equilibrium one has $z = \mu$ the allocation of qualities to consumers is socially inefficient.*

In the neighborhood of the threshold that defines the firms' natural markets (μ), there is a subset of customers who end up buying inefficiently in the first period to get a better bargain in the second period: among these customers, some will start buying efficiently in the second period, whereas a subset of customers will stick to the initial provider (who is able to charge him/ her a personalized attractive price).

Recalling that maximization of the social surplus requires that all consumers' types $\theta \geq \mu$ consume the high quality good in both periods while types $\theta \leq \mu$ consume the low quality good, the equilibria characterized in Proposition 1 implies that, in both periods, too many consumers are buying the low quality good in equilibrium A (and the reverse result in equilibrium B).

[27] More precisely we denote by Δ the quality gap (which is equal to $s_H - s_L$ in Jing, 2016). In addition, we denote the adjusted cost differential $\frac{c_H - c_L}{q_H - q_L}$ by μ, whereas Jing (2016) denotes this ratio by d.

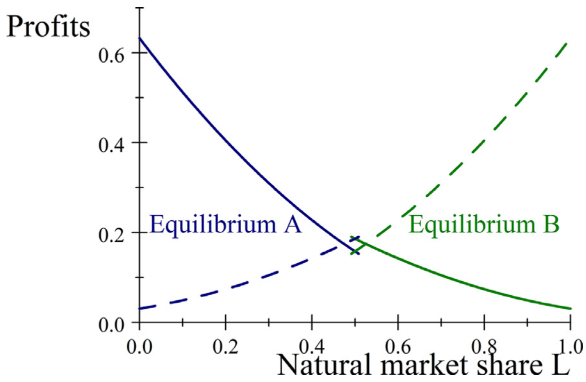

Fig. 4. Equilibrium profits as function of μ.

The quality adjusted price differential $(p_H^{A*} - p_L^{A*})/\Delta$ between firm H and firm L in type A equilibrium is equal to $(1 + 6\mu)/7$. The quality adjusted price difference in type B equilibrium is $\left(p_H^{B*} - p_L^{B*}\right)/\Delta = (6/7)\mu$. For a given quality gap Δ, an increase in μ leads to an increase in the quality adjusted (and not adjusted) price differential.

Total (intertemporal) profits in equilibrium A are[28]

$$\Pi_L^{A*} = \Delta\left(15\mu + 31\mu^2 + 3\right)/(98) \text{ and } \Pi_H^{A*} = (31/49)\Delta(1-\mu)^2 \tag{18}$$

whereas in equilibrium B[29], they are

$$\Pi_L^{B*} = (31/49)\Delta\mu^2 \text{ and } \Pi_H^{B*} = (31/98)\Delta\mu^2 - (11/14)\Delta\mu + (\Delta/2). \tag{19}$$

More generally, and also more simply, the equilibrium profits of the poaching firm Y and of the poached one Z can be expressed as functions of their respective natural market shares:

$$\Pi_Z^* = \Delta\left(15\lambda_Z^n + 31\lambda_Z^{n\,2} + 3\right)/(98), \text{ and } \Pi_Y^* = \Delta(31/49)\lambda_Y^{n\,2},$$

where obviously $\lambda_Z^n + \lambda_Y^n = 1$. In the following figures, we represent the (quality-adjusted) profit of the low quality firm (dashed lines) and the (quality-adjusted) profit of the high quality firm (solid lines). In the figure, the blue (green) lines represent equilibrium A (equilibrium B).

Fig. 4 shows that profits are increasing in firms' own natural market shares. In the region where two equilibria exist, the figure shows that a firm always obtain greater profits at the equilibrium where it is the poached firm than at the equilibrium where it is the poaching one (namely Firm H prefers equilibrium B while Firm L prefers equilibrium A). Fig. 4 also shows that, except for tiny regions close to the symmetric market shares, the firm with the largest natural market tends to enjoy a higher overall profit than the rival.[30]

Remark 6. (i) The profit of each firm increases with the size of its natural market, with $\partial\Pi_H^{j*}/\partial\mu < 0$ and $\partial\Pi_L^{j*}/\partial\mu > 0$, for $j = A, B$. (ii) The profits of the poaching firm (Firm H in equilibrium A, Firm L in equilibrium B) are greater than those of the poached one whenever its natural market share is greater than $\simeq 0.52528$. Hence, almost everywhere the firm with the largest natural market has higher profits than its rival.[31]

It is also interesting to investigate how the equilibrium profits change with the firms' quality levels. As reported in the remark below (whose proof is omitted as results can be obtained through simple comparative statics), the formal symmetry between firms with regard to natural market shares and quality differential breaks down when one looks at the comparative statics with respect to the fundamental underlying parameters.

Remark 7. *Keeping costs constant:*

(i) in either equilibrium, A or B, Firm H's profits are increasing in q_H *and decreasing in Firm L's quality level* q_L;
(ii) Firm L's profits in equilibrium A are increasing in q_H*and decreasing in* q_L *if* $\mu < \sqrt{3/31} \simeq 0.311$*and decreasing in* q_H *and increasing in* q_L*if* $\mu \in (\sqrt{3/31}, 0.50852]$;

[28] The period t equilibrium profit for each firm is denoted as $\pi_{X,t}^{A*}$, for $X = A, B$ and one finds $\pi_{H,2}^{A*} = \frac{47}{98}\frac{(\Delta-(c_H-c_L))^2}{\Delta}$ and $\pi_{L,2}^{A*} = \frac{1}{98\Delta}(\Delta + 6c_H - 6c_L)^2$, while $\pi_{H,1}^{A*} = \frac{15}{98}\frac{(\Delta-(c_H-c_L))^2}{\Delta}$ and $\pi_{L,1}^{A*} = \frac{1}{98}(\Delta - c_H + c_L)\frac{2\Delta+5c_H-5c_L}{\Delta}$.

[29] The second period equilibrium profits for each firm are:$\pi_{H,2}^{B*} = \frac{1}{98}\Delta(6\mu-7)^2$ and $\pi_{L,2}^{B*} = \frac{47}{98}\Delta\mu^2$.The first period equilibrium profits are $\pi_{H,1}^{B*} = \frac{1}{98}\Delta\mu(7-5\mu)$ and $\pi_{L,1}^{B*} = \frac{15}{98}\Delta\mu^2$.

[30] For $\mu \in [0.474, 0.508]$ in type A equilibrium; and for $\mu \in [0.494, 0.525]$ in type B equilibrium (rounded numbers).

[31] At equilibrium A, H's profits are greater than L's for $\mu < 0.47472$ and smaller if $\mu \in [0.47472, 0.50852]$. At equilibrium B, L's profits are greater than H's if $\mu > 0.52528$ and smaller only if $\mu \in [0.49148, 0.52528]$.

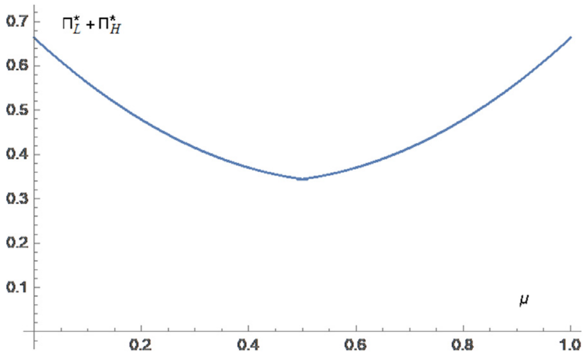

Fig. 5. Aggregate profits as function of μ.

(iii) Firm L's profits in equilibrium B are always decreasing in q_H*and increasing in* q_L.

We have already shown that at each equilibrium, the profits of a given firm are increasing in its natural market share. What about aggregate profits?

Lemma 3. *Total industry profits are increasing in the asymmetry of natural markets, with:*

$$\Pi_L^{A*} + \Pi_H^{A*} = (\Delta/98)\left(\left(65 - 109\mu + 93\mu^2\right)\right), \ \ \mu \in [0, 0.508518],$$

$$\Pi_L^{B*} + \Pi_H^{B*} = (\Delta/98)\left(49 - 77\mu + 93\mu^2\right), \ \ \mu \in [0.49148, 1].$$

Both functions are convex in μ. Despite the very small segment $[0.49148, 0.508518]$ where the two equilibria coexist, it is clear that aggregate profits are the greater the more unequal are the firms' natural market shares (see Fig. 5, which depicts industry profits as a function of μ, for $\Delta = 1$).

Finally it is interesting to compare equilibrium profits in our model with the two benchmark identified above. The two-period repetition of the static uniform pricing equilibrium gives to each firm equilibrium profits $\Pi_X^u = (2\Delta/9)(1 + \lambda_X^n)^2$, $X = H, L$, which are always greater than equilibrium profits in our model.[32] The equilibrium profits under BBPD with second period market segmentation are computed in equations (23) and (24) of Jing (2016), pages 15–16:

$$\Pi_X^J = \frac{\Delta\left(708\lambda_X^n + 540\lambda_X^{n2} + 599\right)}{2304}, \ \ X = H, L.$$

These are greater than the profits of the poached firm Y in this model[33] The same turns out to be true for the poaching firm Z.[34]

Remark 8. The possibility to engage in first-degree price discrimination among old customers ends up hurting firms: In comparison both with the case of uniform pricing and the case of standard third-degree BBPD, the total intertemporal profits unambiguously decrease when firms are able to send personalized price offers to old customers.

Recall that the comparison between profits under uniform pricing and profits under third-degree price discrimination leads to more ambiguous results. According to Jing (2016) (Proposition 1, pages 13–14), third-degree discrimination reduces both firms' profits when their natural market shares are not too different and increases the profits of the firm with the smallest natural market share, when the latter is small enough (smaller than ≈ 0.2824), while reducing the profits of its rival.

4.4. Total welfare

From Fudenberg and Tirole (2000), we know that BBPD may lead to some welfare distortions (within symmetric HD models). There, the first period allocation is optimal (with $z_1 = 1/2$ for symmetric locations), but poaching in the *second period* implies consumers traveling in excess, which increases transportation costs. This is pure waste and it reduces welfare as shown in this seminal work. When firms have the possibility to engage in price personalization, the first period allocation of varieties becomes distorted with respect to the first best (i.e. $z_1 \neq 1/2$) as already shown in Choe et al. (2018), for HD

[32] Indeed, they are always greater than the equilibrium profits of the poached firm Y since $\Pi_Y^u - \Pi_Y^* = \frac{1}{441}\Delta\left(196\lambda_Y^n - 181\left(\lambda_Y^n\right)^2 + 98\right) > 0$. They are also greater than the equilibrium profits of the poaching firm Z since $\Pi_Z^u - \Pi_Z^* = \frac{1}{882}\Delta\left(257\lambda_Z^n - 83\lambda_Z^{n2} + 169\right) > 0$.

[33] Since $\frac{\Pi_Y^{BJ} - \Pi_Y^*}{\Delta} = 0.307\lambda_Y^n - 0.398\left(\lambda_Y^n\right)^2 + 0.26$.

[34] Since $\frac{\Pi_Z^{BJ} - \Pi_Z^*}{\Delta} = 0.154\lambda_Z^n - 0.082\lambda_Z^{n2} + 0.2294 > 0$.

settings. The second period one-way poaching, by allowing the poaching firm to regain part of its natural market, reduces but does not eliminate this waste.

We find that overall welfare may be above the standard uniform pricing welfare level provided that firms' natural markets are *ex-ante* sufficiently asymmetric. Letting z_t represent the marginal consumer in period t, the period-t total surplus under a z_t allocation is given by:

$$W_t(z_t) = \int_0^{z_t} (\theta q_L - c_L)d\theta + \int_{z_t}^1 (\theta q_H - c_H)d\theta,$$

so that total intertemporal surplus if the discount factor is $\delta = 1$, is $\sum_{i=1}^{2} (W_t(z_t))$. Obviously, the maximizer of $W_t(z_t)$ is $z_t = \mu$. It is then possible to state the following result.

Proposition 3. *Let $\mu^W \equiv 0.243\,78$. For asymmetric enough natural markets, with $\mu < \mu^W$ or $\mu > 1 - \mu^W$, total welfare is larger under BBPD with price personalization than under uniform static pricing.*

Proof. The proof is in the Appendix. □

Total welfare is higher here than under uniform pricing, provided that natural markets are sufficiently asymmetric. Prices in the first and second period determine the allocation of consumers to firms. Otherwise they act as pure transfer mechanism and should not affect welfare under full market coverage. First period allocation of consumers to firms are inefficient in both equilibria. However the share of first period misallocated consumers ($z^{A*} - \mu$ in equilibrium A and $\mu - z^{B*}$ in equilibrium B) changes with the asymmetry in natural markets (namely it decreases with μ for equilibrium A and it increases with μ for equilibrium B). Second period poaching allows reallocation of consumers to the "appropriate" firm and reduces the welfare loss from inefficient shopping.

5. Endogenous qualities

Let us now consider each firm X independently choosing initially, irreversibly and observably, its quality level q_X, at a stage 0, before price competition takes place. Of course, changes in the quality level q_x impact the marginal production cost level c_X. Moreover there may exist, in addition, a fixed cost $F(q_X)$ of quality. This cost is independent of the exact production level and it represents the R&D costs needed to develop a product of quality q_X. The outcome of the competition in quality levels will heavily depend on the properties of the cost function $c(q_X)$.

Let us consider the intertemporal profits, gross of the fixed costs of quality, given by the equations (18) and (19) for the two types of equilibria. They are in all cases increasing in the quality differential Δ and in the firm X's natural market (μ for Firm L, $1 - \mu$ for Firm H).

We shall consider from now on the following Assumption.

Assumption 2. $c(q) \geq 0,\ c'(q) > 0,\ c''(q) \leq 0.$

Assumption 2 means that marginal production costs are positive and non-decreasing in quality. Moreover the function is concave: an increase in the quality level entails a *less than proportional increase* in the corresponding marginal cost of production. A commonly used rationale (see e.g. Shaked and Sutton 1983) for this assumption is that increasing production quality mainly requires an increase in the fixed cost—e.g. R&D expenditures.

Assumption 3. $F(q) \geq 0,\ F(0) = 0,\ F'(q) > 0$ and $\exists \bar{q} > 0 : F(q) \underset{q \to \bar{q}}{\rightarrow} +\infty.$

Assumption 3 imposes an upper threshold ($\bar{q}$) on the firms' economically viable quality choices.

Lemma 4. *Under Assumption 2, Firm L's natural market share $\mu(q_H, q_L)$ is decreasing in both, q_H and q_L (equivalently Firm H's natural market share is increasing in both, q_H and q_L).*

Proof. See the Appendix. □

The above Lemma shows that the high quality firm H increases its natural market share by increasing the quality level of its product, while the reverse holds for the low quality firm L. This result follows from the assumed concavity of $c(q)$: given this concavity, any increase in q_H has a smaller (positive) impact on the cost differential than on the quality differential while any increase in q_L reduces more the cost differential than the quality differential.

Note that the profits of a firm X gross of its fixed costs depend on its own quality level through two channels: (i) they are a linear increasing function of the quality differential and (ii) a linear-quadratic increasing function of its natural market share. Both channels impact gross profits positively. Overall, Firm L's gross profits are decreasing in its quality level q_L while Firm H's gross profits are increasing in its quality level q_H. Given Assumption 3, Proposition 4 follows directly.

Proposition 4. *Under Assumptions 2 and 3, Firm L always chooses the minimal quality level $q_L = 0$.*

Since L's fixed cost are increasing in q_L and its gross profits are decreasing in q_L, L's profits net of quality costs are decreasing in q_L as well. Thus, the optimal choice of q_L is always 0 (i.e. the minimal quality). What about the equilibrium quality level of Firm H? It is in fact very difficult to say something precise in general, given that there is a range of values of μ, and accordingly, a range of values of q_H, for which the equilibrium profits are not uniquely determined. The only general thing that we can say, given Assumption 3, is that any possible equilibrium value of q_H should belong to the interval $(0, \overline{q})$. In order to be more specific, Example 1 illustrates a special case.

Example 1. Let the function $c(q)$ be affine in quality level, namely $c(q_X) = a + bq_X$, where $a \geq 0$ and $b \in [0, 1]$ and $F(q_X) = (q_X)^2$. We immediately deduce that $\mu = b$, i.e.; the firms' natural market shares are unaffected by quality choices. So we have to deal with three separate cases:

(i) if $b \leq 0.49148$, the only possible continuation equilibrium is a type A one; optimizing with respect to q_H we obtain $q_H^{*A} = (31/98)(1-b)^2 > 0$;
(ii) if $b \geq 0.508$, the only possible continuation equilibrium is a type B one; optimizing with respect to q_H we obtain $q_H^{*B} = (49 - 77b + 31b^2)/196 > 0$;
(iii) if $b \in [0.49148, 0.508]$, there are two possible continuation equilibria so that the optimal value of q_H depends on the arbitrary probability that Firm H attributes to each of them. If ρ is the probability of equilibrium A, then $q_H^* = \rho q_H^{*A} + (1-\rho) q_H^{*B} > 0$.

Notice that (i) the equilibrium qualities selected by Firm H in the above example are increasing in its natural market share and (ii) that over the interval $b \in [0.49248, 0.50852]$, $q_H^{*A} < q_H^{*B}$. In the above example H's profits net of fixed costs are equal to $\frac{961}{9604}(1-b)^4$ in equilibrium A and $\frac{1}{38\,416}\left(-77b + 31b^2 + 49\right)$ in equilibrium B. In both cases they are steeply increasing in H's natural market share $1-b$. In this specific example, b corresponds to the increase in production cost due to a quality increase (which ends up coinciding with μ). As expected, when quality becomes more costly (b goes up), the natural market of firm H goes down (and its profit shrinks).

6. Vertical vs horizontal differentiation

In Sections 2 to 5, we have shown that in both periods equilibrium profit margins and profits can be expressed as functions of only two parameters, μ and Δ.[35] It is possible to show that, for exogenous qualities in our vertical differentiation model, one can construct an isomorphic HD model with asymmetric firms (and with quadratic transportation costs) such that the pricing games originated by the two models are strategically equivalent.[36] This implies that some features, like the equilibrium uniqueness, translate also to such asymmetric HD models (not to the Choe et al. (2018) symmetric version). Our VD two-stage game, where product qualities are endogenous, is not equivalent to any HD two-stage game where locations are endogenous.[37]

Consider an HD model where two firms L and H are located at the two extremes of the Hotelling unit line over which consumers are uniformly distributed. A given type θ consumer obtains gross utilities $v_L - t\theta^2$ from buying at Firm L and $v_H - t(1-\theta)^2$ from buying at Firm H. The natural market shares of Firm L and Firm H are respectively and straightforwardly obtained as $\nu = \frac{1}{2} + \frac{v_L - v_H}{2t}$ and $1 - \nu$. The price sensitivity of demand in the static game is $\frac{1}{2t}$. The framework is otherwise identical to our model's: a two-period equilibrium in which the firms compete in uniform prices in the first period , then in the second period, they first set a uniform price to new customers and they decide on the personalized prices targeted to their old customers. The discount factor $\delta = 1$. We are simply extending Choe et al. (2018) to allow for an asymmetry between the two products' reservation prices.[38]

In this HD asymmetric model, we assume unit marginal costs are zero while they are positive in the VD model so that in the former unit profit margin equal prices. Both the HD and the VD models assume that markets are always covered—otherwise the isomorphism breaks down (Wauthy, 2010). With covered markets the net utility differential from buying at the two firms matters, which is the gross utility differential minus the cost differential. When we study the pricing game, it follows that for any model with positive and constant marginal costs, like the VD model, there is indeed an isomorphic HD model with zero marginal costs, in which consumers' utilities from buying at a firm are redefined as the gross utilities minus the marginal costs.

In our VD model the net utility differential is

$$\Lambda_{VD} = u_H(\theta) - u_L(\theta) - c_H + c_L = \Delta(\theta - \mu). \tag{20}$$

In the HD model it is:

$$\Lambda_{HD} = u_H(\theta) - u_L(\theta) = (v_H - v_L) - t(1 - 2\theta). \tag{21}$$

---

[35] Notice that Δ is formally the inverse of the price sensitivity of demand with respect to the price differential in the static game (that is $\frac{\partial z}{\partial p_H} = -\frac{\partial z}{\partial p_L} = \frac{1}{\Delta}$).

[36] We are grateful to the Editor and the Reviewers for drawing our attention to this point.

[37] In general, when considering the strategy space at the first stage, the payoff equivalence breaks down and two games cannot be said to be equivalent in that case (see Thompson, 1952 for a definition of equivalence in games). We thank an anonymous referee for pointing out that one cannot require comparative static analysis to correspond between qualities in our model and locations in HD.

[38] Note that our analysis of HD with ex-ante asymmetry on reservation prices can be easily extended to other sources of asymmetry (e.g. generally asymmetric HD models with different unit costs and/or asymmetric exogenous locations). Only the definition of the primitives behind the natural markets and of the price sensitivity of (static) demand parameters would change.

For the two to be isomorphic we then need $\Delta = 2t$ and $\mu = \nu = \frac{1}{2} + \frac{v_L - v_H}{2t}$. Indeed, substituting $\Delta/2$ for t and $\Delta/2(1-2\mu)$ for $v_H - v_L$ in (21) one obtains the same net utility differential $\Lambda = \Delta(\theta - \mu)$ as in the VD model.

Accordingly, after this transformation, in the second and first period equilibrium, unit profit margins, profits, as well as the first period demand, are exactly the same functions of parameters μ and Δ in our VD model and in this HD benchmark. We obviously obtain the same two equilibria A and B, under the same conditions, as in Proposition 1, with now Δ simply replaced by $2t$. This allows for meaningful comparisons (in the pricing games) between our results and Choe et al. (2018). They found that there exist two mirror asymmetric pure strategy equilibria whereas we obtain a unique price equilibrium except over a very small range of values of (L's) natural market share μ, which includes the case $\mu = 1/2$, which is the case studied in Choe et al. (2018). More asymmetry in natural market shares is shown here to ensure uniqueness of a price equilibrium in both models.

7. Concluding remarks

The paper considers a vertically differentiated duopoly where firms are able to offer personalized prices to returning customers. Our results show that price competition evolves according to patterns that depend upon the magnitude of "structural" factors pertaining to the primitives of the vertical differentiation model, namely: the quality gap and the quality-adjusted cost differential, which delimits firms natural markets in our setting. Poaching into the rival's turf is one-directional, in accordance with the horizontal differentiation analysis in Choe et al. (2018). However, differently from the equilibrium outcomes arising in their symmetric framework, here the equilibrium is unique, with the exception of a tiny range of parameters. In addition, the identity of the poaching firm is uniquely determined: only the firm with the largest natural market is able to poach customers from the rival.

According to our results, having the highest natural market is not sufficient to guarantee the highest equilibrium market share, neither in the first nor in the second period. In particular, in the first period, the firm with the smallest natural market always replies to the rival's price by quoting a price that conquers a fraction of the consumers belonging to the natural market of the rival. It is only in the second period that a natural market is a barrier to poaching. This first period pricing behavior bites into the natural market of the "top dog" to the point that the "underdog" may become dominant in both periods (serving a large fraction of the market) if the initial asymmetry is not too large.

It is also worth noting that in equilibrium, neither the first nor the second period market shares coincide with the natural markets—with greater distortions arising in the *first* period (because in the second period, the top dog is able to poach some of its natural customers). Hence, the allocation of varieties to consumers is always socially inefficient. Despite this inefficiency, we find that there may be welfare gains in comparison with benchmarks like the standard uniform pricing. Those gains arise when the firms' natural markets are very asymmetric**.** Consumers are always better-off herein than under standard uniform pricing or than under standard BBPD. Interestingly, some consumers pay higher second period prices in our model than under standard uniform pricing, a phenomenon that does not occur under standard BBPD. However, they all gain in terms of intertemporal utility due to the low first period prices in our setting (driven by firms' attempts to enlarge their own turf and obtain informational advantages for the second period). The same kind of conclusion arises when we compare our results to those in VD with standard BBPD (Jing, 2016).

Concerning the intertemporal profits, we find that a firm's profit are increasing in the dimension of its natural market and the firm with the largest natural market obtains the highest profit. In a sense, even though the firm with the smallest natural market ends up attracting some of the customers belonging to the rival's natural market, it still remains an "underdog" in terms of profitability.

Hence, our results indicate that strategies enhancing the natural market of a firm, like a reduction in the own cost to quality ratio, increase its competitiveness; our definition of the natural market boundary as the ratio of costs to quality differential implies that the largest natural market can be obtained by a firm either by increasing its quality or by decreasing its marginal cost, or both at the same time.

If quality choice is endogeneized, as we argue in a short discussion, the resulting equilibrium quality difference heavily depends upon the assumptions about the relation of production costs to quality. Here, quality not only affects the substitutability between products and hence the intensity of price competition, it also affects the dimension of firms' natural market. While, all other things constant, the low quality firm has an interest in picking the minimal quality, the high quality firm, under reasonable assumptions about the dependence of costs on quality, faces contrasting incentives.

All our results have been obtained under the assumption that the market is fully covered, as ensured with a consumers' reservation value high enough to guarantee that they always buy. This assumption has two important implications. On the one hand, it implies that our vertical differentiation pricing setting is isomorphic to that in an asymmetric horizontal differentiation duopoly (e.g. when consumers attach different reservation values to the goods, or firms have asymmetric costs/ locations). As shown by Wauthy (2010), the isomorphism in pricing games between HD and VD no longer holds if we allow for uncovered markets.[39] On the other hand, the full market coverage assumption is also critical for our result that firms' equilibrium margins and profits are identical when they play the same role, either the top dog or the underdog, independently of them being the high or the low quality firm. In other words, the model is "functionally symmetric", although it is

[39] Indeed under VD the market shares of the two firms are two connected intervals, while under HD they will be typically disconnected.

not generally symmetric because firms have different natural market shares. Again, this functional symmetry breaks down when one consider uncovered markets. Exploring what may change when we allow for uncovered markets is extremely difficult[40] but is certainly worthwhile. Our intuition, based on some computations and on numerical examples, is that while the high quality firm's best reply is unaffected, the low quality adopts a much more aggressive stance to compensate the consumers lost at the bottom of the scale: it tries to use aggressive first period prices in order to expand its first period market share beyond its natural market and to retain a greater pool of old customers to exploit later via personalized prices.[41] We leave this issue as the subject of further research.

Credit Author Statement

All three authors have equally contributed to the paper.

Appendix

Result 1.

(i) Suppose some $\Theta \geq z \geq \mu$ and consider an ϵ -neighborhood of Θ where both firms compete in uniform prices, $P_{2L}(\epsilon)$ and $P^O_{2H}(\epsilon)$.[42] Then $\lim_{\epsilon \to 0} P_{2L}(\epsilon) = c_L$.
(ii) Suppose some $\Theta \leq z \leq \mu$. Consider an ϵ -neighborhood of Θ where both firms choose uniform prices $P_{2H}(\epsilon)$ and $P^O_{2H}(\epsilon)$. Then $\lim_{\epsilon \to 0} P_{2H}(\epsilon) = c_H$.

Proof.

(i) Let the relevant interval be $\left[\Theta - \frac{\epsilon}{2}, \Theta + \frac{\epsilon}{2}\right]$. The demand to Firm H is $\Theta + \frac{\epsilon}{2} - \frac{P^O_{2H} - P_{2L}}{\Delta}$. The demand to Firm L is $\frac{P^O_{2H} - P_{2L}}{\Delta} - \Theta + \frac{\epsilon}{2}$. From the first order conditions $P_{2L} - c_L = \frac{1}{6}\Delta(2(\mu - \Theta) + 3\epsilon)$ and $P^O_{2H} - c_H = \frac{1}{6}\Delta(-2(\mu - \Theta) + 3\epsilon)$. But $\frac{1}{6}\Delta(2(\mu - \Theta) + 3\epsilon) \leq 0$ when $\epsilon \leq \frac{2}{3}(\Theta - \mu)$ so that $P_{2L} - c_L = 0$ when ϵ is small enough.
(ii) We don't repeat the proof since it is exactly along the same lines.

□

Proof of Lemma 1. Let us first look at the second period equilibrium in Firm *H*'s turf. After observing P_{2L}, Firm H's best reply is to defend successfully its turf by choosing $p_{2H}(\theta) = \theta\Delta + P_{2L}$ for all $\theta \in [z, 1]$. So Firm L would make zero profit on this consumer whatever the uniform price $P_{2L} \geq c_L$ it chooses. Sequential pricing results then in a continuum of equilibria. The refinement described in Result 1 allows us to select the equilibrium where $P_{2L} = c_L$.

Further, on Firm *L*'s turf, we have that for any price P_{2H} the best reply by firm *L* obviously is $p_{2L}(\theta) = Max\{P_{2H} - \theta\Delta, c_L\}$, since if $P_{2H} - \theta\Delta > c_L$ firm L can secure a sale to customer θ at a price above marginal cost. It can be shown that the value of θ for which $P_{2H} - \theta\Delta = c_L$ must lay in the interval $[\mu, z]$. In fact, in order to have an equilibrium, the value of θ for which $P_{2H} - \theta\Delta = c_L$ _ call it $\overline{\theta}$ _ must be lower than z, otherwise firm H has zero demand in equilibrium in $[0, z]$ and zero profits, while it could lower its price P_{2H} (still with $P_{2H} > c_H$) attract some customers in $[\mu, z]$ and make positive profits. Further, it cannot be $\overline{\theta} < \mu$ because then firm H would be poaching customers that cannot be poached according to (2). Hence, forcibly, $\overline{\theta} = \frac{P_{2H} - c_L}{\Delta}$, and it is such that $\mu \leq \overline{\theta} \leq z$ in equilibrium. Firm H chooses P_{2H} in order to maximize the strictly concave "poaching profits" function $(z - (P_{2H} - c_L)/\Delta)(P_{2H} - c_H)$, so that one obtains

$$\begin{aligned} P_{2H} &= c_L + (\mu + z)(\Delta/2) = c_H + \Delta(z - \mu)/2, \\ \overline{\theta}^* &= (z + \mu)/2. \end{aligned} \tag{22}$$

This is also a weak Nash equilibrium[43] □

Proof of Claim 1. From (14) and (12), $\Pi_H(p_H, p_L)$ is concave with respect to p_H both for $z(p_H, p_L) \geq \mu$ and for $z(p_H, p_L) \leq \mu$. When they exist, from the FOC, local interior maxima with respect to p_{1H} obtain respectively at p^1_{1H} (in the first branch of the profit function) and p^2_{1H} (in the second branch of the profit function):

$$p^1_{1H} = \frac{\Delta(2 - \delta)^2 + \delta\Delta\mu(4 - \delta) + 2(2 - \delta)c_H + 4p_{1L}}{2(4 - \delta)} \tag{23}$$

[40] The technical reason is that the equilibrium existence conditions do not depend uniquely upon a unique parameter value (namely μ). Indeed, for uncovered markets, several (three) parameters enter into play and it becomes much more difficult to disentangle the equilibrium types—as well as their specific economic properties.

[41] In all the numerical examples we computed the equilibrium is a type A one.

[42] Notice that $P^O_{2H}(\epsilon)$ is the uniform price of firm *H* to its *old* customers, and needs not be equal to the price P_{2H} offered to its new customers, as in standard BBPD models.

[43] Indeed, consider $\varepsilon > 0$, $\eta > 0$, and $A = [\bar{\theta} + \eta, z - \eta]$. A unilateral deviation by firm L to a schedule $p^d_L(\theta; \eta)$ over $[0, z]$ where, say, $p^d_L(\theta) = c_L + \varepsilon$ if $\theta \in A$, and $p^d_L(\theta) = Max\{P_H - \theta\Delta, c_L\}$, otherwise, leaves equilibrium demand to *L* unchanged. For any ε there exists $\eta(\varepsilon)$ small enough so as to leave H's the best reply to $p^d_L(\theta; \eta)$, equal to the equilibrium strategy $P_H = c_L + \Delta\bar{\theta}$, and it is not convenient for *H* to raise its price to sell to customers in *A* and loosing all the others. Hence this equilibrium is not unique in an irrelevant way since equilibrium payoffs remain constant over all equilibria.

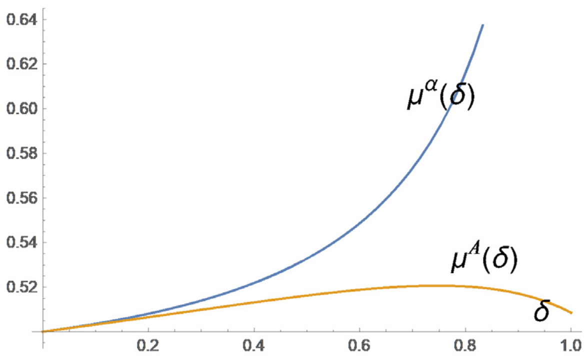

Fig. 6. $\mu^{\alpha}(\delta)$ and $\mu^{A}(\delta)$.

$$p^2_{1H} = \frac{\Delta(4-\delta(6-2\delta-3\mu+2\delta\mu))+2(2-\delta)c_H+(4-3\delta)p_{1L}}{(8-5\delta)}. \tag{24}$$

Equation (23) defines an interior maximum in the area $z(p_H,p_L)\geq\mu$, iff $p_L\leq\overline{p}_{1L}=c_H+\Delta(1-2\mu-\frac{1}{2}\delta(1-\mu))$. Equation (24) defines an interior maximum in the area $z(p_H,p_L)\leq\mu$ iff $p_L\geq\underline{p}_{1L}=c_H+\Delta(1-2\mu-\delta(1-\mu))$. Since $\overline{p}_{1L}-\underline{p}_{1L}=\frac{1}{2}\delta\Delta(1-\mu)>0$, $\Pi_H(p_H,p_L)$ is double-peaked w.r.t. p_H over the interval $\left[\underline{p}_{1L},\overline{p}_{1L}\right]$. Simple computations now show that H obtains greater (resp. smaller) profits at (23) than at (24) iff $p_L\leq\widehat{p}_L$ (resp. $p_L\geq\widehat{p}_L$), where for $\delta=1$, we obtain:[44]

$$\widehat{p}_L=c_H-(\Delta/2)\left(\mu+\sqrt{2}(\mu-1)+1\right). \tag{25}$$

□

Proof of Claim 2. The Proof goes along the same lines of the Proof of Claim 1, with

$$p^1_{1L}=\frac{2(2-\delta)c_L+(4-3\delta)p_{1H}-(3-2\delta)\delta\Delta\mu}{(8-5\delta)}, \tag{26}$$

$$p^2_{1L}=\frac{2(2-\delta)c_L+4p_{1H}-(4-\delta)\delta\Delta\mu}{2(4-\delta)}. \tag{27}$$

Equation (27) defines an interior maximum in the area $z(p_H,p_L)\leq\mu$, iff $p_{1H}\leq c_L+\frac{1}{2}(4-\delta)\mu\Delta$. Equation (26) defines an interior maximum in the area $z(p_H,p_L)\geq\mu$, iff $p_{1H}\geq c_L+(2-\delta)(c_H-c_L)$. Again, $\Pi_L(p_H,p_L)$ is double-peaked w.r.t. p_L over the interval $\left[\underline{p}_{1H},\overline{p}_{1H}\right]$. Simple computations show that L obtains greater (resp. smaller) profits at (27) than at (26) iff $p_{1H}\leq\widehat{p}_H$ (resp. $p_{1H}\geq\widehat{p}_H$), where for $\delta=1$, we have:

$$\widehat{p}_H=c_L+\left(\left(\sqrt{2}+1\right)/2\right)\Delta\mu. \tag{28}$$

□

Proof of Proposition 1. For equilibrium A, the Nash candidates are $p^{A*}_L=c_L+\Delta\frac{(8-10\delta)(1+\mu)+\delta^2(3+\mu)}{24-10\delta}$ and $p^{A*}_H=c_H+\Delta\frac{8(2-\mu)-\delta(18-5\delta+5(\delta-2)\mu))}{24-10\delta}$. Given Claims 1 and 2, in order to show that $\left(p^{A*}_H,p^{A*}_L\right)$ constitute a pure strategy Nash equilibrium of the game (PSNE), it is necessary and sufficient to show that $p^{A*}_H\geq\widehat{p}_H$ and $p^{A*}_L\leq\widehat{p}_L$:

(i) Given (25) and (16), the difference $\widehat{p}_L-p^{A*}_L$ is a linear decreasing function of μ and is positive iff $\mu\leq\mu^{\alpha}(\delta)$, where

$$\mu^{\alpha}(\delta)=\frac{48-28\delta+4\delta^2-(12-5\delta)\sqrt{64-56\delta+10\delta^2}}{32\delta-8\delta^2-(12-5\delta)\sqrt{64-56\delta+10\delta^2}}.$$

(ii) Given (28) and (16), the difference $p^{A*}_H-\widehat{p}_H$ is a linear decreasing function of μ and is positive iff $\mu\leq\mu^{A}(\delta)$, where

$$\mu^{A}(\delta)=\frac{3(2-\delta)(8-5\delta)}{20\delta^2-32\delta+(12-5\delta)\sqrt{64-56\delta+10\delta^2}}.$$

(iii) Finally $\mu^{\alpha}(\delta)\geq\mu^{A}(\delta)$ for all $\delta\in[0,1]$, as pictured in Fig. 6, so that $\mu\leq\mu^{A}(\delta)\Rightarrow\mu\leq\mu^{\alpha}(\delta)$.

[44] Notice that $\widehat{p}_L\in\left[\underline{p}_{1L},\overline{p}_{1L}\right]$.

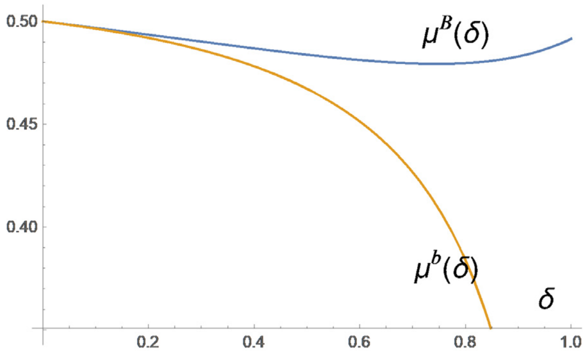

Fig. 7. $\mu^B(\delta)$ and $\mu^b(\delta)$.

As for equilibrium B, the Nash candidates are $p_L^{B*} = c_L + \Delta\frac{8(1+\mu)-\delta(8+5(2-\delta)\mu)}{24-10\delta}$ and $p_H^{B*} = c_H + \Delta\frac{4(4-\delta)(1-\delta)-\mu(8+\delta(\delta-10))}{24-10\delta}$. Given Claims 1 and 2, in order to show that $\left(p_H^{B*}, p_L^{B*}\right)$ constitute a PSNE, it is necessary and sufficient to show that $p_H^{B*} \le \widehat{p}_H$ and $p_L^{B*} \ge \widehat{p}_L$:

(i) Given (25) and (17), the difference $p_L^{B*} - \widehat{p}_L$ is a linear decreasing function of μ and is positive iff $\mu \ge \mu^B(\delta)$, where

$$\mu^B(\delta) = \frac{48 - 22\delta - 5\delta^2 - (12-5\delta)\sqrt{64 - 56\delta + 10\delta^2}}{32\delta - 20\delta^2 - (12-5\delta)\sqrt{64-56\delta+10\delta^2}}.$$

(ii) Given (28) and (17), the difference $\widehat{p}_H - p_H^{B*}$ is a linear decreasing function of μ and is positive iff $\mu \ge \mu^b(\delta)$ where

$$\mu^b(\delta) = \frac{12(4-\delta)(1-\delta)}{8\delta^2 - 32\delta + (12-5\delta)\sqrt{64-56\delta+10\delta^2}}.$$

(iii) Finally, $\mu^B(\delta) \ge \mu^b(\delta)$ for all $\delta \in [0,1]$, as pictured in Fig. 7, so that $\mu \ge \mu^B(\delta) \Rightarrow \mu \ge \mu^b(\delta)$.

To complete the description of the two equilibria. The equilibrium profits in equilibrium A are

$$\begin{aligned} \Pi_H^{A*} &= \Delta\frac{\left(6\mu^2 - 14\mu + 4\right)\delta^3 + \left(64\mu - 31\mu^2 - 9\right)\delta^2 + \left(40\mu^2 - 48\mu - 28\right)\delta + 16\left(\mu^2 - 4\mu + 4\right)}{(12-5\delta)^2}, \\ \Pi_L^{A*} &= \Delta\frac{\left(5\mu^2 + 5\mu - 5\right)\delta^3 + \left(2\mu - 38\mu^2 + 28\right)\delta^2 + \left(32\mu^2 - 56\mu - 52\right)\delta + \left(32\mu^2 + 64\mu + 32\right)}{2(12-5\delta)^2}. \end{aligned} \tag{29}$$

Similarly, the equilibrium profits in equilibrium B are

$$\begin{aligned} \Pi_H^{B*} &= \Delta\frac{32(2-\mu)^2 + 5\delta^3(1+(\mu-3)\mu) + \delta^2(74\mu - 38\mu^2 - 8) + 4\delta(8\mu^2 - 19 - 2\mu)}{2(12-5\delta)^2}, \\ \Pi_L^{B*} &= \Delta\frac{\left(6\mu^2 + 2\mu - 4\right)\delta^3 + \left(-2\mu - 31\mu^2 + 24\right)\delta^2 + \left(40\mu^2 - 32\mu - 36\right)\delta + \left(16\mu^2 + 32\mu + 16\right)}{(12-5\delta)^2}. \end{aligned} \tag{30}$$

□

Proof of Remark 6.. The values of P_{2L} and P_{2H} are directly comparable and the proof is omitted. In equilibrium A the personalized prices of firm H are $p_{2H}^A(\theta) = \theta\Delta + c_L$ and are strictly higher than $p_H^u = (2\Delta + 2c_H + c_L)/3$ for all types with $\frac{2}{3}(1+\mu) < \theta \le 1$. In equilibrium B (where $\mu < 1/2$) one has $p_{2L}(\theta) = c_H - \theta\Delta$ and $p_{2L}(\theta) > p_L^u = (\Delta + c_H + 2c_L)/3$ for $0 \le \theta < \frac{1}{3}(2\mu - 1)$. There are always consumers paying a lower price than the uniform price in the second period. In particular, it can be seen that $p_H^u > \min_{1\ge\theta\ge z^A}\left\{p_{2H}^A(\theta)\right\} = \frac{5\mu+2}{7}\Delta + c_L$ and $p_L^u > \min_{0\le\theta\le z^A}\left\{p_{2L}^A(\theta)\right\} = c_L$ for equilibrium A; and $p_H^u > \min_{1\ge\theta\ge z^B}\left\{p_{2H}^B(\theta)\right\} = c_H$ and $p_L^u > \min_{0\le\theta\le z^B}\left\{p_{2L}^B(\theta)\right\} = c_H - \frac{5}{7}\Delta\mu$ for equilibrium B. □

Proof of Proposition 2.

(a) Equilibrium A ($z > \mu$). In the second period, in H's turf, we find that the consumers with $\theta > z$ buying H pay $p_{2H}(\theta) = c_L + \Delta\theta$ and $c_L + \Delta\theta - p_H^u = \Delta(\theta - (2+2\mu)/3) > 0$ for $\theta > (2\Delta + 2c_H - 2c_L)/3\Delta = \theta'$. The value of θ' being less than 1 for $\mu < 1/2$ implies that some consumers pay a higher price than the uniform price in the second period. However the intertemporal utility of consumer $\theta = 1$, which pays the highest second-period price, can be easily shown to be higher under personalized BBPD than under uniform pricing[45]. In L's turf consider the consumer paying the highest second

[45] Indeed the consumer with $\theta = 1$ pays in total $p_H^{A*} + c_L + \Delta$, with $p_H^{A*} = m_{1H}^{A*} + c_H$ which can be easily seen to be less than $2p_H^U$, with $p_H^u = m_H^u$.

period price, namely $\theta = 0$ who pays $p_L^{A*}(0)$. We find that $p_{2L}(0) + p_L^{A*} - 2p_L^u = -\Delta(19 - 5\mu)/42 < 0$, implying that all consumers will also gain in turf L.

(b) Equilibrium B ($z < \mu$). In H's turf, θ buying for H pays in total at most $p_H^{B*} + p_{2H}(\theta)$ or $(c_H + (\Delta\mu/14)) + (c_H + \Delta\theta - (6\Delta\mu)/7)$, which increases in θ. Simple algebra allows us to show $p_H^{B*} + p_{2H}(1) < 2p_H^u$. Hence all buyers from H gain under personalized BBPD. Buyers from L pay a total equal to $p_L^{B*} + p_{2L}(\theta) = (c_L + (3/14)\Delta\mu) + (c_H - \theta\Delta) - 2((\Delta/3)(1+\mu) + c_L)$ and the difference $p_L^{B*} + p_{2L}(\theta) - 2p_L^u$ can be seen to be negative for any $\theta \geq 0$. However some consumers who buy L will end-up paying a second-period price higher than the uniform price (this is true for $c_H - \theta\Delta - ((\Delta/3)(1+\mu) + c_L) > 0$, which holds for $\theta < (2\mu - 1)/3$).

□

Proof of Proposition 3. Recall that total surplus is $\sum_{i=1}^{2}(W_t(z_t))$. Under static pricing $z_1 = z_2 \equiv z^U = (1+\mu)/3$. Hence $W^U = 2W_1(z^U)$. Therefore it can be found that

$$W^U = \left(\frac{8}{9} - \frac{2}{9}\mu - \frac{1}{9}\mu^2\right)q_H + \left(\frac{1}{9}\mu^2 + \frac{2}{9}\mu + \frac{1}{9}\right)q_L + \left(\frac{2}{3}\mu - \frac{4}{3}\right)c_H - \frac{2}{3}(\mu+1)c_L \tag{31}$$

Under the allocation in Equilibrium A, $z_1^A = (5\mu+2)/7$ and $z_2^A = (6\mu+1)/7$. Straightforward computations yield:

$$W^A = \left(\frac{93}{98} - \frac{16}{49}\mu - \frac{61}{98}\mu^2\right)q_H + \left(\frac{61}{98}\mu^2 + \frac{16}{49}\mu + \frac{5}{98}\right)q_L + \frac{11}{7}(\mu-1)c_H - \left(\frac{11}{7}\mu + \frac{3}{7}\right)c_L,$$

After some algebra, we obtain:

$$W^A - W^U = \Delta\left(347\mu^2 - 302\mu + 53\right)/882$$

This expression, in the range $\mu < 1/2$ is positive if

$$\mu < 0.243\,78 \equiv \mu^W$$

In the case of Equilibrium B, we have that $z_1^B = z^{B*} = 5\mu/7$ and $z_2^B = \bar{\theta}^{*B} = 6\mu/7$. Then,

$$W^B = \left(1 - \frac{61}{98}\mu^2\right)q_H + \left(\frac{61}{98}\mu^2\right)q_L + \left(\frac{11}{7}\mu - 2\right)c_H - \frac{11}{7}\mu c_L, \text{ with}$$

$$W^B - W^U = (347/882)\Delta\mu^2 - (4/9)\Delta\mu + \Delta/9,$$

which is positive if $\mu > 1 - \mu^W = 0.756\,22$. □

Proof of Lemma 3.. Remind that $\mu(q_H, q_L) = \frac{c(q_H) - c(q_L)}{q_H - q_L}$. Accordingly, since $c''(q) \leq 0$,

$$\begin{aligned}\frac{\partial\mu(q_H, q_L)}{\partial q_H} &= \Delta^{-2}\left[c'(q_H)(q_H - q_L) - (c(q_H) - c(q_L))\right] \\ &= \Delta^{-2}\int_{q_L}^{q_H}\left(c'(q_H) - c'(q)\right)dq \leq 0, \text{ and}\end{aligned}$$

$$\begin{aligned}\frac{\partial\mu(q_H, q_L)}{\partial q_L} &= \Delta^{-2}\left[-c'(q_L)(q_H - q_L) + (c(q_H) - c(q_L))\right] \\ &= \Delta^{-2}\int_{q_L}^{q_H}\left(c'(q) - c'(q_L)\right)dq \leq 0.\end{aligned}$$

□

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