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# Mathematical Modeling and Supercomputer Technologies

20th International Conference, MMST 2020  
Nizhny Novgorod, Russia, November 23–27, 2020  
Revised Selected Papers

 Springer



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Dmitry Balandin · Konstantin Barkalov ·  
Victor Gergel · Iosif Meyerov (Eds.)

# Mathematical Modeling and Supercomputer Technologies


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# Preface

The 20th International Conference and School for Young Scientists “Mathematical Modeling and Supercomputer Technologies” (MMST 2020) was held during November 23–27, 2020, in Nizhni Novgorod, Russia. The conference and school were organized by the Mathematical Center “Mathematics of Future Technologies” and the Research and Educational Center for Supercomputer Technologies of the Lobachevsky State University of Nizhni Novgorod. The conference was supported by reputable sponsors (Intel, Huawei, and RSC). It was organized in partnership with the International Congress “Russian Supercomputing Days”.

The topics of the conference and school covered a wide range of problems related to mathematical modeling of complex processes and numerical methods of research, as well as new methods of supercomputing aimed at solving state-of-the-art problems in various fields of science, industry, business, and education.

This edition of the MMST conference was dedicated to the 100th birthday of Professor Yuri Neimark. Professor Neimark’s contribution to science and higher education is truly immense. He has left considerable heritage in the qualitative theory of differential equations, the theory of stability, the theory of adaptive and robust control, pattern recognition, nonholonomic and gyroscopic systems mechanics, optimization, and mathematical modeling. Yuri Neimark founded the USSR’s first Faculty of Computational Mathematics and Cybernetics and the Research Institute of Applied Mathematics and Cybernetics. He is the author of about 600 published works, including at least 12 for inventions and 10 monographs.

The scientific program of the conference featured the following plenary lectures given by outstanding scientists:

- Alexander Boukhanovsky (Russia): Generative design of value-based systems.
- Vladimir Voevodin (Russia): AlgoWiki, the structure of algorithms and the impossible becomes possible.
- Alexander Gorban (UK): The new centaur: man and artificial intelligence.
- Andrei Gritsun (Russia): Predicting Earth climate change using the INM RAS Earth system model.
- Aleksey Eliseev (Russia): The IAP RAS Earth system model: state of the art and review of key findings.
- Aleksey Koronovskii (Russia): Multistability in an intermittent generalized chaotic synchronization regime.
- Evgeniy Mareev (Russia): Current problems in geophysical electrodynamics.
- Alexander Moskovsky (Russia): Unique and effective RSC solutions for complex problems.
- Sergey Pavlov (Russia): ARM Ecosystem Development for AI, Cloud and High Performance Computing.
- Arkady Pikovsky (Germany): Low-dimensional reduction for ensembles of noisy oscillators.



# On the Problem of Choosing the Optimal Parameters for the Wind Farm in the Arctic Town of Tiksi

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**Abstract.** The paper considers the problem of choosing the optimal parameters for the operation of 3 horizontal wind turbines of the wind farm in the town Tiksi, in the Sakha Republic. The open-source WRF-ARW and FLORIS packages are used to calculate the physical parameters in the wind farm. During the calculation, the values of wind velocity, temperature, pressure fields, and the value of the generated power of wind power plants were obtained. The optimization problem is formulated, the objective function is defined, and 2 variable parameters are selected, namely, yaw angle and angle of attack. The calculations were carried out on the computing cluster of ISP RAS.

**Keywords:** Wind turbine · Wind farm · Calculation · Domain · Grid · Wind velocity · Objective function · Wind turbine parameters · Yaw angle · Power

## 1 Introduction

The design and selection of optimal parameters for horizontal wind turbines in the wind farm is an urgent task, especially in the case of the construction of new wind farms in the Russian Federation. One of the interesting areas for research may be the problem of choosing the best operating modes of the wind farm, taking into account the influence of local weather conditions, as well as the relative location of the wind turbine in a real area. The variable parameters include the angular rotation velocity of the wind turbine, the yaw angle, the angle of rotation of the blade with respect to the vector of the incoming flow, and others.

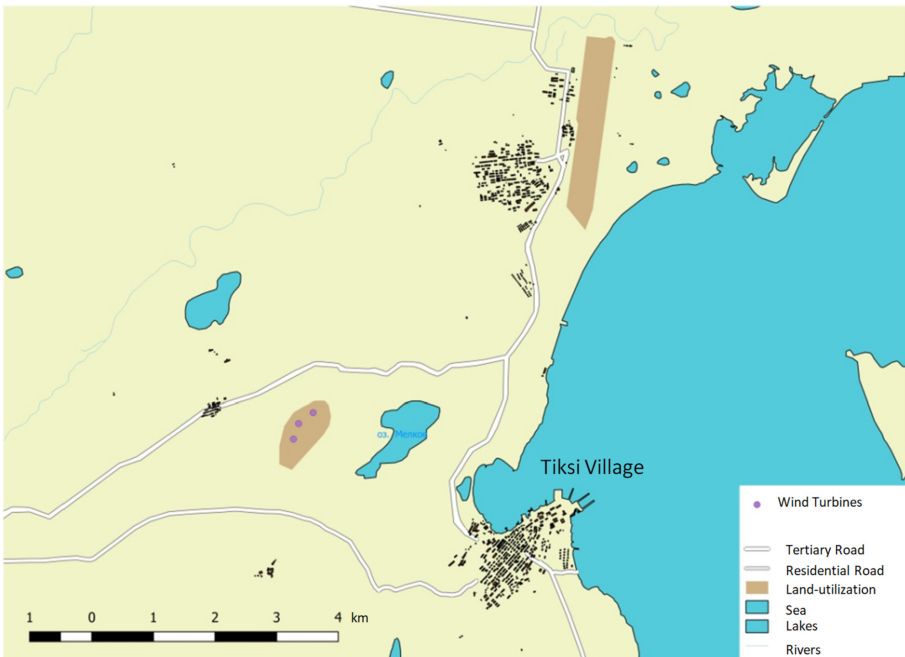


## 2 Wind Farm in the Town Tiksi

The wind farm built by RusHydro together with its Japanese partners in the Arctic town Tiksi, which is home to more than 4.6 thousand people, has demonstrated high efficiency and reliability in the harsh Arctic climate. In the winter of 2018–2020, the air temperature in Tiksi dropped to minus 42°, and the wind velocity reached 30 m/s.

In November 2018, in the Arctic urban-type settlement of Tiksi, which is an important transport hub for the Northern Sea Route, 3 wind turbines from Komaihaltec company from Japan with a capacity of 0.3 MW each were put into operation with the participation of Sakhaenergo and RusHydro. Now the wind farm works together with the existing diesel power plant of the town.

The wind turbines (WT) of the Tiksi wind farm are located at an altitude of about 120 m above sea level, 4 km from the town Tiksi and 2.7 km from the Laptev Sea, Fig. 1.



**Fig. 1.** The map of the town in Tiksi. Open Street Map, 11.09.2019.

Turbines coordinates are: 71°39'24.8"N 128°46'17.6"E (WT1), 71°39'20."N 128°45'55.7"E (WT2), 71°39'12.02"N 128°45'43.57"E (WT3), see Fig. 2.



**Fig. 2.** Satellite image of the wind turbines location. Google Earth, 25.09.2020.

## 2.1 Mathematical Model in WRF-ARW Code

The calculation code WRF-ARW (WRF – Weather Research and Forecasting, ARW – Advanced Research WRF) is actively used to predict the capacity of the wind farm. The WRF-ARW model is a model of a regional weather forecast, for the construction of which it needs boundary and initial conditions [1, 2]. Data sources in this case can be global models, observational data, or forecast reanalysis data. As a parent model for WRF-ARW, data from calculations based on the Global Forecast System (GFS) model is usually used. Figure 3 shows an algorithm for the interaction of different calculation models for different levels of calculation of physical quantities.

The WRF-ARW model allows one to obtain a large range of physical quantities that describe weather conditions. As a rule, the main parameters are the distribution of wind velocity, temperature, and pressure. Secondary parameters may include air humidity, precipitation distribution, precipitation type, and much more. In WRF-ARW, there are various parameterization models, including the model for parameterizing the operation of a wind turbine – the Actuator Disk Model [3].

The WRF-ARW model is based on non-hydrostatic equations for a compressible fluid written in Cartesian coordinates horizontally and using the orographic coordinate  $\eta$  vertically. In older versions of the package,  $\eta$  was determined using the hydrostatic pressure  $p_h$ :

$$\eta = \frac{p_d - p_t}{p_s - p_t}, \quad (1)$$

where  $p_d$  is the hydrostatic component of dry air pressure,  $p_s$  and  $p_h$  refer to values of  $p_d$  along the surface and top boundaries respectively. In WRF-ARW

version 4, the vertical coordinate was generalized to reduce the influence of the surface on the coordinate grid with increasing height:

$$p_d = B(\eta)(p_s - p_h) + [\eta - B(\eta)](p_0 - p_h) + p_h,$$

where  $p_0$  is a reference sea-level pressure. Here  $B(\eta)$  defines the relative weighting between the terrain-following sigma coordinate and a pure pressure coordinate, such that  $\eta$  corresponds to the sigma coordinate (1) for  $B(\eta) = \eta$  and reverts to a hydrostatic pressure coordinate for  $B(\eta) = 0$ . For a smooth transition from the sigma coordinate near the surface to the pressure coordinate at the upper levels,  $B(\eta)$  is defined by a third order polynomial.

The governing equations system is represented as follows. The momentum equations are written as

$$\partial_t U + m_x[\partial_x(Uu) + \partial_y(Vu)] + \partial_\eta(\Omega u) \tag{2}$$

$$+ (m_x/m_y)(\alpha/\alpha_d)[\mu_d(\partial_x\phi' + \alpha_d\partial_x p' + \alpha'_d\partial_x\bar{p}) + \partial_x\phi(\partial_\eta p' - \mu'_d)] = F_U,$$

$$\partial_t V + m_y[\partial_x(Uv) + \partial_y(Vv)] + (m_y/m_x)\partial_\eta(\Omega v) \tag{3}$$

$$+ (m_y/m_x)(\alpha/\alpha_d)[\mu_d(\partial_y\phi' + \alpha_d\partial_y p' + \alpha'_d\partial_y\bar{p}) + \partial_y\phi(\partial_\eta p' - \mu'_d)] = F_V,$$

$$\partial_t W + m_x[\partial_x(Uw) + \partial_y(Vw)] + \partial_\eta(\Omega w) \tag{4}$$

$$- m_y^{-1}g(\alpha/\alpha_d)[\partial_\eta p' - \bar{\mu}_d(q_v + q_c + q_r)] + m_y^{-1}\mu'_d g = F_W,$$

and the mass conservation equation and geopotential equation is given by

$$\partial_t \mu'_d + m_x m_y [\partial_x U + \partial_y V] + m_y \partial_\eta \Omega = 0, \tag{5}$$

$$\partial_t \phi' + \mu_d^{-1} [m_x m_y (U \partial_x \phi + V \partial_y \phi) + m_y \Omega \partial_\eta \phi - m_y g W] = 0. \tag{6}$$

The conservation equations for the potential temperature  $\Theta_m$  and the scalar moisture  $Q_m$  equations:

$$\partial_t \Theta_m + m_x m_y [\partial_x (U \theta_m) + \partial_y (V \theta_m)] + m_y \partial_\eta (\Omega \theta_m) = F_{\Theta_m}, \tag{7}$$

$$\partial_t Q_m + m_x m_y [\partial_x (U q_m) + \partial_y (V q_m)] + m_y \partial_\eta (\Omega q_m) = F_{Q_m}, \tag{8}$$

and the diagnostic equation for dry hydrostatic pressure:

$$\partial_\eta \phi' = -\bar{\mu}_d \alpha'_d - \alpha_d \mu'_d, \tag{9}$$

with the diagnostic relation for the full pressure (dry air plus water vapor):

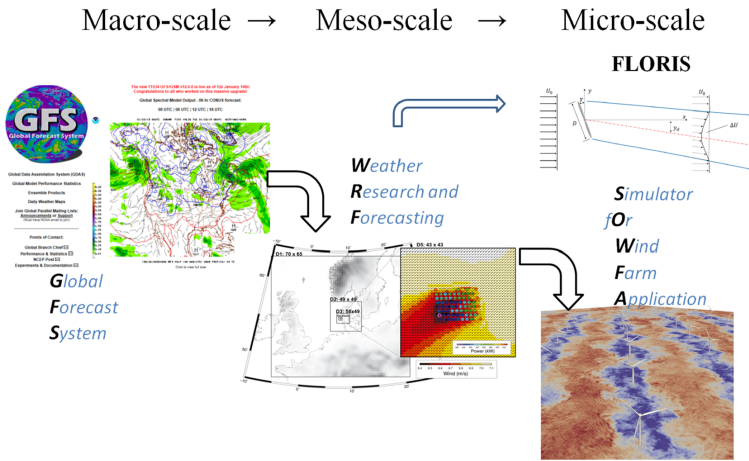
$$p = p_0 \left( \frac{R_d \theta_m}{p_0 \alpha_d} \right)^\gamma. \tag{10}$$

The following notations is used in the system (2)–(10):

$$U = \mu_d u / m_y, \quad V = \mu_d v / m_x, \quad W = \mu_d w / m_y, \quad \Omega = \mu_d \omega / m_y,$$

$$\Theta_m = \mu_d \theta_m, \quad Q_m = \mu_d q_m,$$

where  $\mathbf{v} = (u, v, w)$  – velocity vector,  $\omega = \dot{\eta}$  – vertical velocity in terms of orographic coordinates,  $\mu_d$  defines mass of the dry air column,  $\phi$  – geopotential,  $m_x, m_y$  – map scale factors.  $\theta_m = \theta(1 + (R_v/R_d)q_v) \approx \theta(1 + 1.61q_v)$  is the moist potential temperature and  $q_m = q_v, q_c, q_r, q_i, \dots$  represents the mixing ratios of moisture variables (water vapor, cloud water, rain water, ...).  $\alpha_d$  is the inverse density of the dry air ( $1/\rho_d$ ) and  $\alpha$  is the inverse density taking into account the full parcel density  $\alpha = \alpha_d(1 + q_v + q_c + q_r + q_i + \dots)^{-1}$ . The right-hand-side terms  $F_U, F_V, F_W, F_{\theta_m}$  and  $F_{Q_m}$  represent forcing terms arising from physical models, turbulent mixing, spherical projections, and the earth’s rotation,  $g$  – acceleration due to the gravity. Hydrostatically-balanced reference state variables (denoted by overbars) are a function of height only and satisfy the governing equations for an atmosphere at rest.

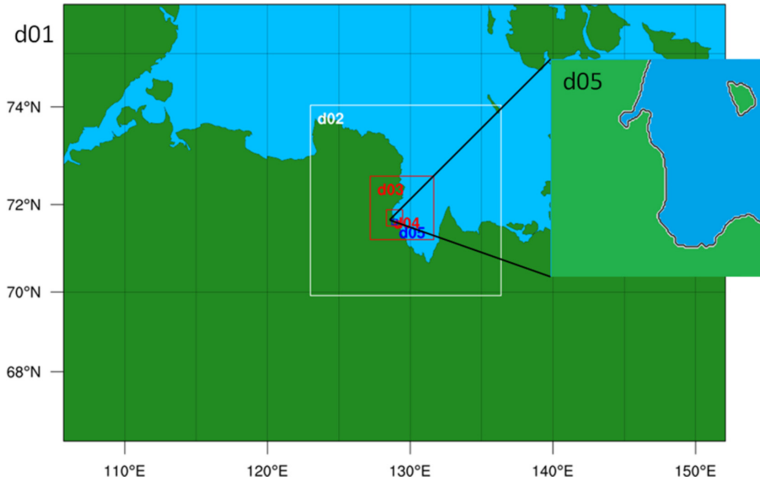


**Fig. 3.** How to model the operation of a wind farm.

WRF-ARW code supports the configuration of the shared and distributed memory of server. The code could be compiled with OpenMP and OpenMPI libraries for parallel calculations. The goal of parallel option is to speed-up the computing time on supercomputer.

**2.2 Numerical Domain, Grid, and Results of Calculation**

The layout of the numerical domain for the area of the town Tiksi with the image of nested domains is shown in Fig. 4. The calculations in the WRF-ARW package were performed for the following calendar dates: 29.09.2019, 30.09.2019, 01.10.2019.



**Fig. 4.** Layout of the main area with the image of nested domains.

To compare the results, the Tiksi weather station was selected:  $71^{\circ}34'48''$  N  $128^{\circ}54'E$ , altitude 7 m above sea level. Data on the temperature at the level of 2 m above the surface, the pressure, and wind velocity at the surface (at the level of the weather station) were displayed at the point closest to the location of the weather station in the d05 region. The graph of the velocity comparison is shown in Fig. 6. The model configuration and simulation results are given in [4].

In general, the results of the model correspond to real data, in any case, they repeat the profile of changes in indicators. Also, the wind farm energy generation was modeled using the built-in model, which was analyzed in [3]. The characteristics of the wind turbine, namely: the power curve and the thrust coefficient as a function of velocity, were taken from the parameters of similar wind turbines from open sources on the Internet (Fig. 5). The result of the simulation is the dynamics of the power of the Tiksi wind farm in a given period, Fig. 7. One can see that the results correspond to the average wind value in the specified area.

The results of the calculation showed that the efficiency of the wind farm does not exceed 20% for the selected days. In this regard, there is a need to choose the optimal parameters for the operation of wind turbines. This parameter can be the angle of rotation of the nacelle (the yaw angle).

The calculations were performed using 12-cores computer node. Several cases with different configuration of physical parameters were launched simultaneously. The average computation time for one case was about 24 h.

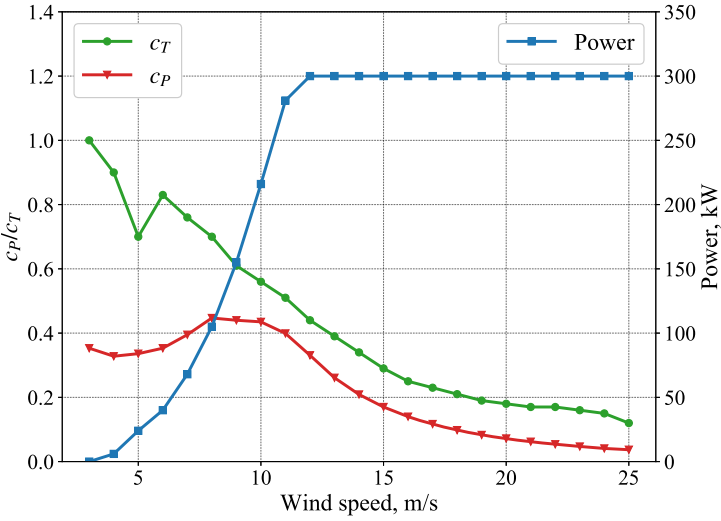


Fig. 5. The power curve and  $c_p$ ,  $c_t$  coefficients for a single wind turbine.

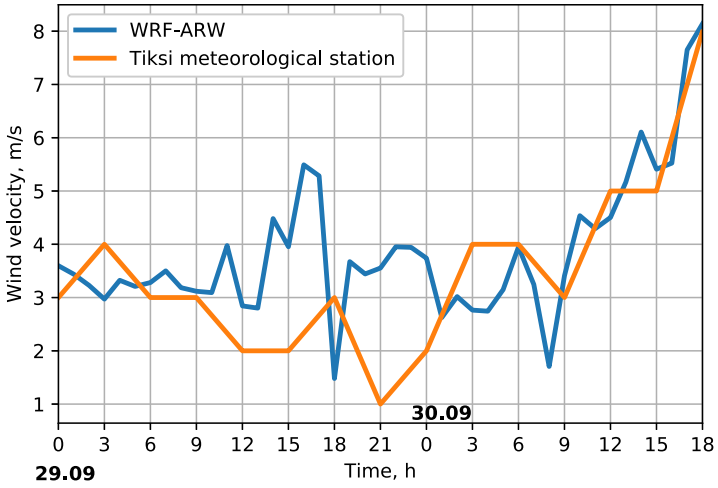
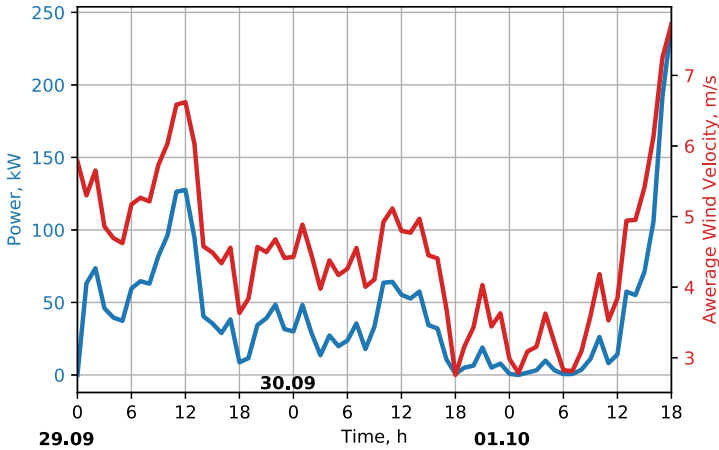


Fig. 6. Comparison of wind velocity at the location of the Tiksi weather station for model and real data.



**Fig. 7.** Simulation change in the total power generation capacity of the Tiksi wind farm on 29.09–01.10.

The simulations were done using the resources of the UniHUB HPC cluster of ISP RAS with 32 computer nodes.

### 2.3 Setting Task of Optimizing the Generated Capacity of the Wind Farm

It is known that the front wind turbines have a significant impact on the operation of the rear wind turbines. There is a concept of velocity deficit. The vortex trail behind the wind turbine may deviate.

Data from a weather station located 10 km from the wind farm shows that the wind direction changes in the range of angles from  $\theta_1 = 0^\circ$  to  $\theta_2 = 90^\circ$ . Thus, there is the problem of choosing the optimal angle of rotation of the engine nacelle of the wind turbine (yaw rate), the angle of rotation of the blade to obtain maximum power capacity of wind turbines.

Work in the field of optimization calculations for wind farms and the design of wind turbines was practically not carried out earlier in the Russian Federation. Among the international teams that deal with optimization issues in the design of new wind farms, there are research groups from JHU, USA (Prof. C. Meneveau); KU Leuven, Belgium (Dr. Johan Meyers); EPFL, Lausanne, Switzerland (Prof. Fernando Porte-Agel); NREL (Dr. P. A. Fleming); Stanford University (Dr. J. Park); Aalborg University, Aalborg, Denmark (Prof. Zheng Chen) [5–14]. These teams considered the following tasks:

1. optimization of the location of wind turbines in the wind farm [5–7];
2. selection of optimal parameters (yaw angle of the wind wheel axis, angular rotation velocity, control parameters, etc.) [10–14];

3. optimization of the geometric shape of the wind turbine blade in order to reduce the noise level on the ground and increase the durability of the structure made of composite material.

To solve such problems, an objective function was formed, which took into account various criteria and restrictions on the parameters to be changed. Among the optimization methods used, one can distinguish:

- genetic algorithms;
- sequential quadratic programming;
- cross-entropy method;
- an approach based on a combination of the conjugate gradient method, the Polak-Ribière method for determining weight coefficients, and the Brent search algorithm [8, 9];
- machine learning methods based on “Bayesian ascent” [10, 11];
- the particle swarm method [12, 13].

The yaw angle’s optimization problem for the wind wheel axis was studied in papers [15–17].

The authors of the work [9] solved the optimization problem using an approach with a cooperative game. All agents (participants) strive to maximize the overall goal. Interaction between agents is taken into account.

The vector maximization problem is solved:

$$\mathbf{x}^* = \arg \max_{\mathbf{x}} \sum_{i=1}^N \varphi_i f_i(\mathbf{x}), \quad \varphi_i = 1, i = \overline{1, N}; \tag{11}$$

$$\max_{\mathbf{x}} f(\mathbf{x}) \triangleq \sum_{i=1}^N P_i(\boldsymbol{\alpha}, \mathbf{o}, U, \theta^W), \quad \text{subject to } \mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^u, \tag{12}$$

where  $\mathbf{x}^l, \mathbf{x}^u$  – lower and upper bounds,  $\varphi_i$  represents the weighting coefficient on the objective function  $f_i(x)$ .

Because it is difficult to accurately determine the analytical function for the cardinality, the authors used Bayesian optimization (BO).

The problem is solved:

$$\mathbf{x}^* = \arg \max_{\mathbf{x}} f(\mathbf{x}) \tag{13}$$

Observing for the behavior of the function:

$$y = f(\mathbf{x}) + \epsilon. \tag{14}$$

The BO uses 2 phases: training and optimization.

In this paper, the objective function  $f(\mathbf{x})$  associated with the total power generation was formulated.

It is known that the power for a single wind turbine is equal to:

$$P = \frac{1}{2} \rho A U^3 C_p(\alpha, 0). \tag{15}$$



An arbitrary set of parameters can be considered in an optimization problem:

$$\mathbf{x} = (x_1, \dots, x_i, \dots, x_N). \quad (16)$$

The target function for the total capacity of the wind farm is defined as:

$$f_i(\mathbf{x}) = P_i(\boldsymbol{\alpha}, \mathbf{o}, U, \theta^W) \quad (17)$$

The goal of any optimization is to achieve the maximum values of performance indicators while meeting the specified limits. At the stage of setting the problem, the researcher determines a set of parameters that need to be maximized, minimized, or limited. The indicators are optimized by varying the input parameters. The researcher sets the composition of the variable variables, the ranges of their changes, and, possibly, the relationship with other input parameters.

Usually, when conducting optimization studies, it is necessary to solve not one, but several optimization problems that differ:

- the composition of variable variables (from minimal changes in the project to a complete redesign);
- values of constraints (analysis of the possibility of relaxing individual requirements for the project);
- the number and composition of optimization criteria (from a complete set of alternative projects to a single option).

In the course of solving the problem, it is necessary to solve the problem of finding the maximum of the objective function  $f(x)$ :

$$\max_{\mathbf{x}} f(\mathbf{x}) \triangleq \sum_{i=1}^N P_i(\boldsymbol{\alpha}, \mathbf{o}, U, \theta^W), \quad \mathbf{x}^l \leq \mathbf{x} \leq \mathbf{x}^u, \quad (18)$$

where  $\mathbf{x} = (\alpha_1, o_1, \dots, \alpha_N, o_N)$ ,  $N$  – the number of wind turbines.

The generated power of wind turbines depends on 4 parameters: the angle of rotation of the blades (angle of attack), the values of the flow velocity, the angle of direction of the vector flow velocity, yaw rate. The range of changes in 2 selected parameters out of 4 set according to changes in weather data in the area of the location of the wind farm in the town Tiksi, as well as the technical characteristics of a wind turbine with a power of  $P = 0.3$  MW. The range for changing all parameters is known, the yaw angle  $\mathbf{o}$  varies from  $0^\circ$  to  $25^\circ$ , the angle of rotation of the blade  $\alpha$  from  $-10^\circ$  to  $10^\circ$ .

### 3 Analytical Models for Calculation of Wake Parameters

Analytical models in comparison with models based on the solution of partial differential equations are simpler and require less computing resources of the computer. The open package FLORIS (FLOW Redirection and Induction in Steady-state) contains various analytical models for calculating the vortex traces

of wind turbines in the wind farm, [18]. The FLORIS package was developed in the Python programming language at NREL (USA), TU Delft.

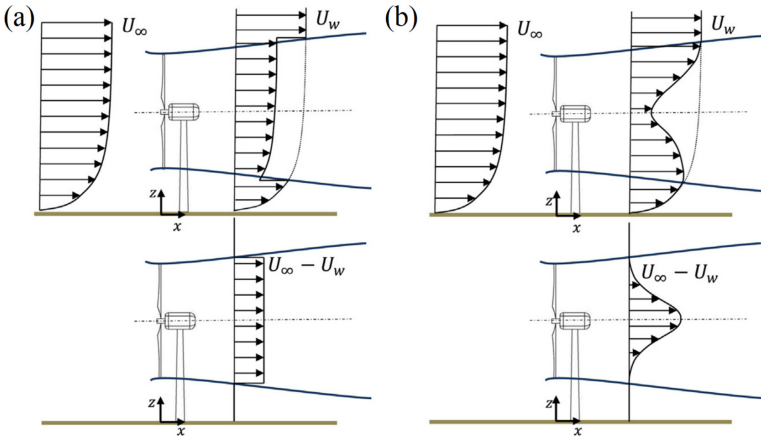
One of the first analytical models for calculating the parameters of the vortex wake was developed by N. Jensen in Denmark in 1983 [19]. In this paper, it was proposed to use the “top-hat shape” model for the cylinder to calculate the velocity deficit in the track (see Fig. 8) and written in the form:

$$\frac{\Delta U}{U_\infty} = (1 - \sqrt{1 - c_T}) / \left( 1 + \frac{2k_{wake}x}{d_0} \right)^2, \tag{19}$$

where  $\frac{\Delta U}{U_\infty}$  is a normalized (dimensionless) velocity deficit.

$$\frac{\Delta U}{U_\infty} = \frac{U_\infty - U_W}{U_\infty}. \tag{20}$$

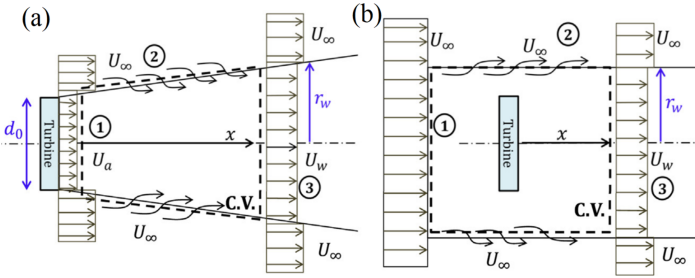
The Eq. (19) has been widely used in the literature and has been implemented in commercial software (WasP, WindPRO, WindSim). However, there were two limitations to this model: 1) the assumption of the distribution of the velocity deficit was unrealistic; 2) only the law of conservation of mass was used to derive Eq. (19).



**Fig. 8.** Velocity profile for the wind turbine according to the a) “top-hat shape” model; b) Gaussian distribution model. Picture is taken from [20].

Subsequently, Frandsen [21] applied the equations of conservation of mass and amount of motion for the control volume (Fig. 9) around the wind turbine and proposed the following expression:

$$\frac{\Delta U}{U_\infty} = \frac{1}{2} \left( 1 - \sqrt{1 - 2 \frac{A_0}{A_W} c_T} \right). \tag{21}$$



**Fig. 9.** Example of using the control volume: a) quadrilateral; b) rectangle. Picture is taken from [20].

Vortex traces behind bodies in free flow have been actively studied in the theory of shear flows. In these studies, a self-similar Gaussian profile was found to calculate the velocity deficit.

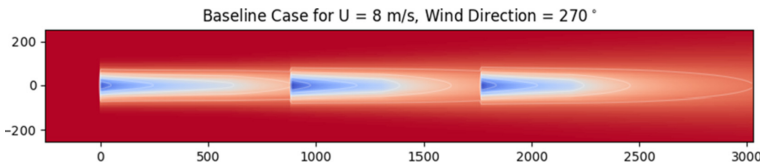
The self-similar Gauss profile was discovered in experiments in wind tunnels, in numerical calculations, and in operating wind farms. An analytical model with a self-similar Gauss profile was obtained in [20]:

$$\frac{\Delta U}{U_\infty} = \left( 1 - \sqrt{1 - \frac{c_T}{8(k^*x/d_0 + \varepsilon)^2}} \right) \times \exp \left( -\frac{1}{2(k^*x/d_0 + \varepsilon)^2} \left\{ \left( \frac{z - z_h}{d_0} \right)^2 + \left( \frac{y}{d_0} \right)^2 \right\} \right). \tag{22}$$

The analytical models (19)–(22), which were proposed by different authors to predict the deficit of the streamwise velocity in the of wind turbine, were implemented in open source software code FLORIS. FLORIS code is written on Python and is available on github.com [18].

In this paper, the calculation was carried out for a model wind farm with 3 wind turbines according to the Jimenez model, taking into account the influence of changes in the yaw angle for the nacelle [7].

An example of the calculation for a wind farm in the FLORIS package is shown in Fig. 10. The size of the numerical domain was selected with 500 m in the OY direction and 3200 m in the OX direction. The velocity was defined as 8 m/s at the inlet of the numerical domain. The wind farm consisted of 3 NREL 5 MW wind turbines. They were located on the same straight line, respectively, with coordinates 0, 7D, 14D.



**Fig. 10.** Initial position of wind turbines in wind farm.

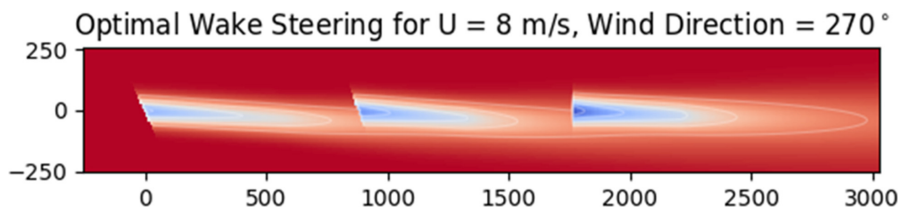


Fig. 11. Velocity value field for 3 wind turbines.

The wind direction was  $270^\circ$ . The initial position of wind turbines is shown in Fig. 10.

The possible range of yaw angle changes for wind turbines is from 0 to  $25^\circ$ . We used the SLSQP (Sequential Least Squares Programming) optimization algorithm. SLSQP is a nonlinear, gradient-based algorithm that can handle inequality constraints. Iterations are generated by solving quadratic sub-problems. The solution to the optimization problem, the best option is to rotate the nacelle for the first wind turbine is at a 25-degree angle, for the 2nd wind turbine the yaw angle is  $19^\circ$ , for the 3rd wind turbine the yaw angle is  $4.3^\circ$  (Fig. 11). The total power gain amounted to 15.1%.

On of the future directions of work could be dealt with using Globalizer Software, developed at the Lobachevsky National Research University for multi-extreme optimization problems, see the paper [22] to solve the optimization problems for wind farms.

## 4 Conclusion

This approach, using meteorological data, wind farm data, open-source software WRF-ARW and FLORIS, allows us to formulate the problem of choosing the optimal parameters for the operation of wind turbines in the wind farm. In the future, it is planned to apply the chosen approach to solve the problem of choosing the optimal parameters for the operation of 3 Japanese wind turbines in the wind farm in the Arctic town Tiksi, taking into account the wind rose and complex terrain.

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