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# Understanding the power of Max-SAT resolution through UP-resilience

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## Maximum Satisfiability (Max-SAT)

### Max-SAT Problem

**Input:** a formula  $\Phi$  in Conjunctive Normal Form (CNF)

**Output:** the maximum (resp. minimum) number of satisfied (resp. falsified) clauses in  $\Phi$  over all possible variable assignments

### Branch & Bound (BnB) for Max-SAT

Binary search algorithm which maintains and constantly updates two values :

- **Upper Bound (UB):** value of the best known solution
- **Lower Bound (LB):** estimation of the best accessible solution

**Cut:** if  $LB \geq UB$  then backtrack

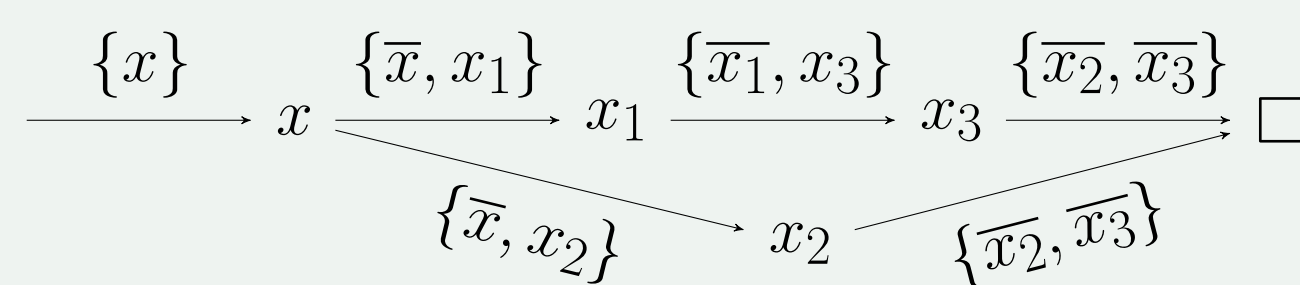
### Lower Bound Estimation

At each node with current assignment  $I$ , we compute a new estimation of  $LB$ .

$$LB = FC(\Phi|_I) + IS(\Phi|_I)$$

- $FC(\Phi|_I)$ : number of falsified clauses in  $\Phi|_I$
- $IS(\Phi|_I)$ : number of **disjoint Inconsistent Subsets (IS)** detected in  $\Phi|_I$  by **Simulated Unit Propagation (SUP)**

$\psi = \{\{x\}, \{\bar{x}, x_1\}, \{\bar{x}, x_2\}, \{\bar{x}_1, x_3\}, \{\bar{x}_2, \bar{x}_3\}\}$  is an IS detected through SUP represented in the form of an **implication graph** :



To ensure that detected ISs are disjoint, they are temporarily deleted or transformed by **Max-SAT resolution** in which case they can be maintained in the current subtree.

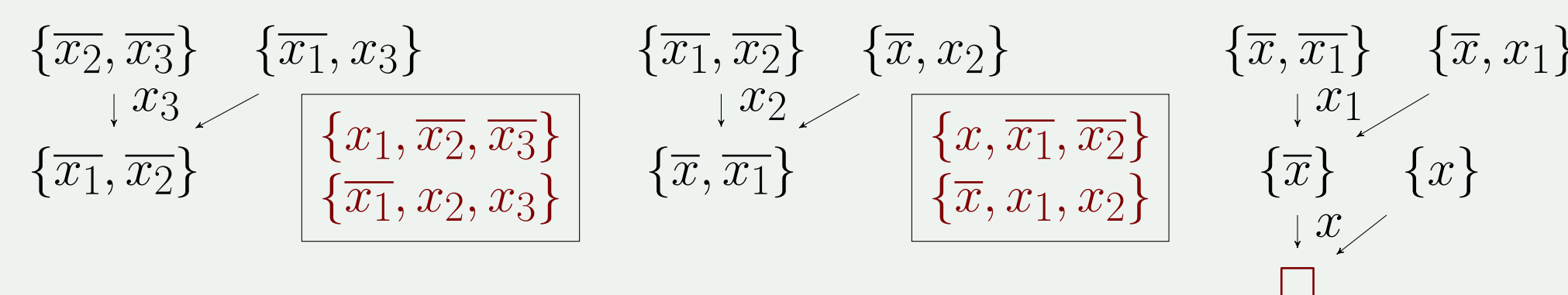
### Max-SAT resolution Transformation

$$\frac{c = \{x, y_1, \dots, y_s\} \quad c' = \{\bar{x}, z_1, \dots, z_t\}}{cr = \{y_1, \dots, y_s, z_1, \dots, z_t\}, cc_1, \dots, cc_t, cc_{t+1}, \dots, cc_{t+s}} \text{ Max-SAT resolution}$$

Compensation clauses

$$\begin{array}{l|l} cc_1 = \{x, y_1, \dots, y_s, \bar{z}_1\} & cc_{t+1} = \{\bar{x}, z_1, \dots, z_t, \bar{y}_1\} \\ cc_2 = \{x, y_1, \dots, y_s, z_1, \bar{z}_2\} & cc_{t+2} = \{\bar{x}, z_1, \dots, z_t, y_1, \bar{y}_2\} \\ \dots & \dots \\ cc_t = \{x, y_1, \dots, y_s, z_1, \dots, z_{t-1}, \bar{z}_t\} & cc_{t+s} = \{\bar{x}, z_1, \dots, z_t, y_1, \dots, y_{s-1}, \bar{y}_s\} \end{array}$$

Transformation of the IS  $\psi = \{\{x\}, \{\bar{x}, x_1\}, \{\bar{x}, x_2\}, \{\bar{x}_1, x_3\}, \{\bar{x}_2, \bar{x}_3\}\}$  with respect to the **Reverse Propagation Order (RPO)**  $\langle x_3, x_2, x_1, x \rangle$ :



## UP-resilience

### Definition

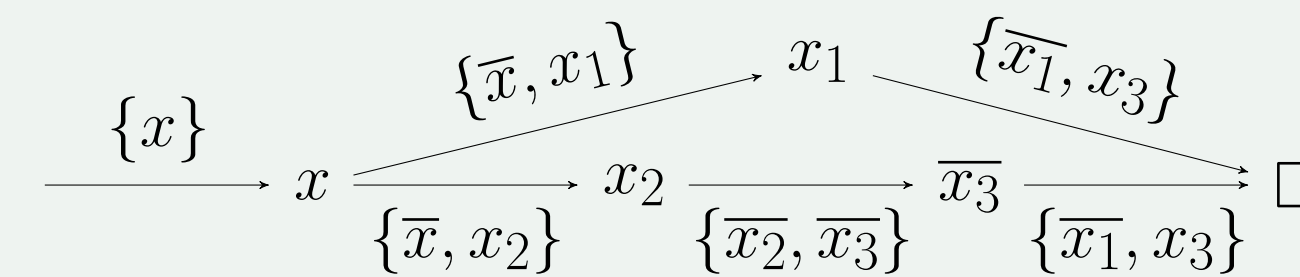
The transformation of an IS  $\psi$  is UP-resilient for a literal  $l$  in  $\psi$  iff :

$$\forall N \in \text{pneigh}_{\psi}(l) : \square \in N \text{ or } l \text{ can be propagated in } \Theta(\psi)|_N$$

- $\text{pneigh}_{\psi}(l)$ : **possible neighborhood** of  $l$ , i.e. the set of its different neighborhoods in all the implication graphs enabling the detection of the IS  $\psi$
- $\Theta(\psi)|_N$ : result of the Max-SAT resolution transformation of the IS  $\psi$

The transformation of  $\psi$  is UP-resilient iff is UP-resilient for all the literals in  $\psi$ .

The IS  $\psi = \{\{x\}, \{\bar{x}, x_1\}, \{\bar{x}, x_2\}, \{\bar{x}_1, x_3\}, \{\bar{x}_2, \bar{x}_3\}\}$  also corresponds to another implication graph:



- $\text{pneigh}(x_2) = \{\{x, \square\}, \{x, \bar{x}_3\}\}$
  - $\Theta(\psi)|_{\{x, \bar{x}_3\}} = \{\{x_1, x_2\}, \{\bar{x}_1, x_2\}, \square\}$
- $x_2$  cannot be propagated in  $\Theta(\psi)|_{\{x, \bar{x}_3\}}$  (**fragmentation phenomenon**)  
→ The transformation of  $\psi$  w.r.t RPO is not UP-resilient

### Properties

- If the transformation of an IS  $\psi$  is UP-resilient for a set of literals  $L$  in  $\psi$  then  $\forall N \in \text{pneigh}(L) : \square \in N$  or  $\forall l \in L, l$  can be propagated in  $\Theta(\psi)|_{N \setminus \{l\}}$ .
- The order of application (variable sequence) of Max-SAT resolution has a direct impact on the UP-resilience of the transformations.
- Efficient learning schemes (**Patterns**) are UP-resilient.

→ **quantifies the impact of transformations on the SUP mechanism**  
→ **provides a theoretical understanding of the efficiency of learning schemes**

### UP-resilience and Main Patterns

Since Max-SAT resolution transformations can negatively affect the efficiency of BnB solvers, they are only performed when they correspond to certain **patterns**.

$$\begin{array}{l} \frac{\{l_1, l_2\}, \{\bar{l}_1, \bar{l}_2\}}{\{l_1\}} (P_1) \\ \frac{\{l_1, l_2\}, \{l_1, l_3\}, \{\bar{l}_2, \bar{l}_3\}}{\{l_1\}, \{l_1, l_2, l_3\}, \{\bar{l}_1, \bar{l}_2, \bar{l}_3\}} (P_2) \\ \frac{\{l_1\}, \{\bar{l}_1, l_2\}, \{\bar{l}_2, l_3\}, \dots, \{\bar{l}_k, l_{k+1}\}, \{\bar{l}_{k+1}\}}{\square, \{l_1, \bar{l}_2\}, \{l_2, \bar{l}_3\}, \dots, \{l_k, \bar{l}_{k+1}\}} (P_3) \end{array}$$

→ **Patterns do not augment the size of formula and produce unit clauses.**

If a subset of an IS  $\psi$  matches the premises of **pattern (P1) or (P2) or (P3)**. Then, the Max-SAT resolution transformation described in the pattern is **UP-resilient w.r.t all possible application orders**.

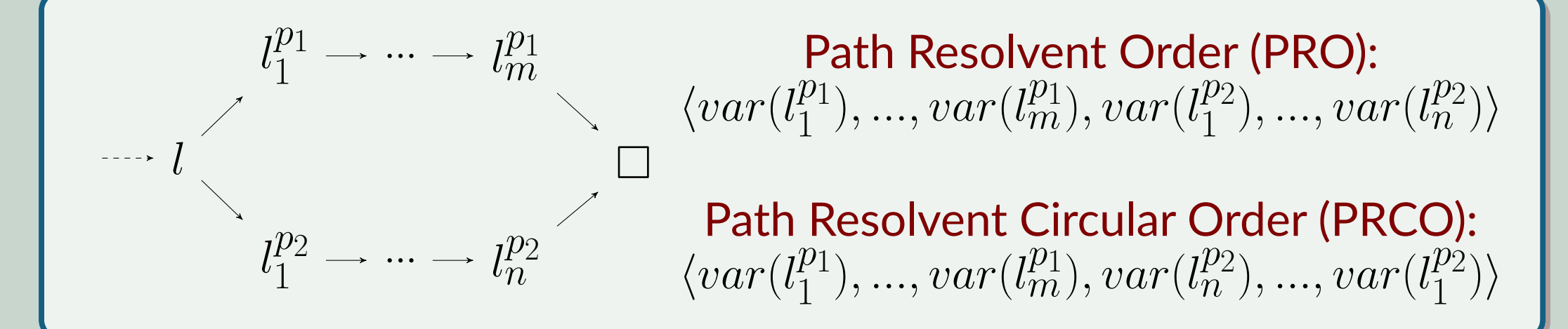
### UP-resilience and Binary UCS Patterns

A binary  $k$ -Unit Clause Subset ( $k^b$ -UCS) is a set of binary clauses  $\{c_1, \dots, c_k\}$  in an IS such that there exists a sequence of Max-SAT resolution steps on  $c_1, \dots, c_k$  that **produces a unit clause resolvent**.

- Patterns  $(P_1)$  and  $(P_2)$  are respectively equivalent to  $2^b$ -UCS and  $3^b$ -UCS patterns.
- Many UCSs can be easily detected using the **First Unit Implication Point (FUIP)**.
- **RPO does not necessarily ensure the UP-resilience of  $k^b$ -UCSs for  $k \geq 4$ .**

The subset  $\psi \setminus \{x\}$  of the IS  $\psi = \{\{x\}, \{\bar{x}, x_1\}, \{\bar{x}, x_2\}, \{\bar{x}_1, x_3\}, \{\bar{x}_2, \bar{x}_3\}\}$  is a  $4^b$ -UCS, detected through the FUIP  $x$ , for which RPO does not ensure UP-resilience.

In the implication graph of ISs with  $k^b$ -UCS detected by the FUIP, **there exists exactly two disjoint paths from the FUIP to  $\square$** .

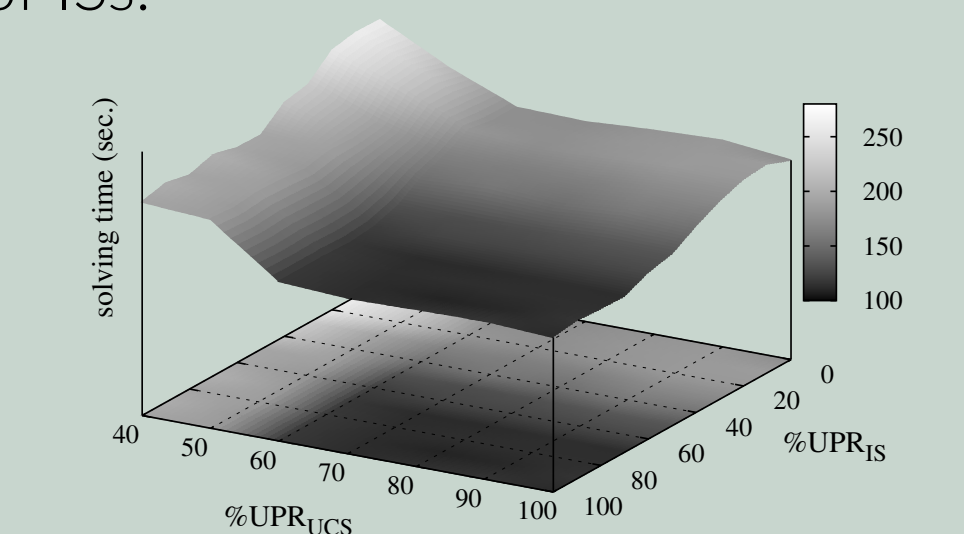


→  $k^b$ -UCSs are UP-resilient with respect to PRO and PRCO  
→ **New approach to extend patterns by UP-resilience**

### Empirical Study on the Relevance of UP-resilience

- **IRS:** learns transformations whose all intermediary resolvents contain less than four literals (MiniMaxSAT learning scheme).
- **PAT:** learns the transformations when the ISs match the three main patterns.
- **PAT+:** learns the transformations when the IS matches the three main patterns or certain  $k$ -UCS patterns ( $k \in \{4, 5\}$ ).
- **UPRes:** learns only UP-resilient transformations of ISs.

Learning scheme	S (T)	D	% L	% UPR
IRS	1033 (210.69)	327864	73.9	79
PAT	1400 (72.51)	95222	21.5	100
PAT+	1402 (65.94)	72683	24.5	98
UPRes	1407 (58.27)	71279	27.6	100



**1776 unweighted and weighted (partial) instances** in total tested with the solver **ahmaxsat**, S = number of solved instances, T = average solving time, D = average number of decision, % L = percentage of learned transformations, % UPR = percentage of learned UP-resilient transformations.

### References

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