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Understanding the power of Max-SAT resolution through UP-resilience

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Maximum Satisfiability (Max-SAT)

Max-SAT Problem

Input: a formula Φ in Conjunctive Normal Form (CNF) Output: the maximum (resp. minimum) number of satisfied (resp. falsified) clauses in Φ over all possible variable assignments

Branch & Bound (BnB) for Max-SAT

Binary search algorithm which maintains and constantly updates two values :

- Upper Bound (UB): value of the best known solution
- Lower Bound (LB): estimation of the best accessible solution

Cut: if $LB \ge UB$ then backtrack

Lower Bound Estimation

At each node with current assignment I, we compute a new estimation of LB.

 $LB = FC(\Phi|_I) + IS(\Phi|_I)$

- $FC(\Phi|_I)$: number of falsified clauses in $\Phi|_I$
- $IS(\Phi|_I)$: number of **disjoint Inconsistent Subsets (IS)** detected in $\Phi|_I$ by Simulated Unit Propagation (SUP)

 $\psi = \{\{x\}, \{\overline{x}, x_1\}, \{\overline{x}, x_2\}, \{\overline{x_1}, x_3\}, \{\overline{x_2}, \overline{x_3}\}\}\$ is an IS detected through SUP represented in the form of an **implication graph** : $\begin{array}{c} \{x\} \\ \hline \end{array} \begin{array}{c} x \\ \hline \end{array} \end{array}$

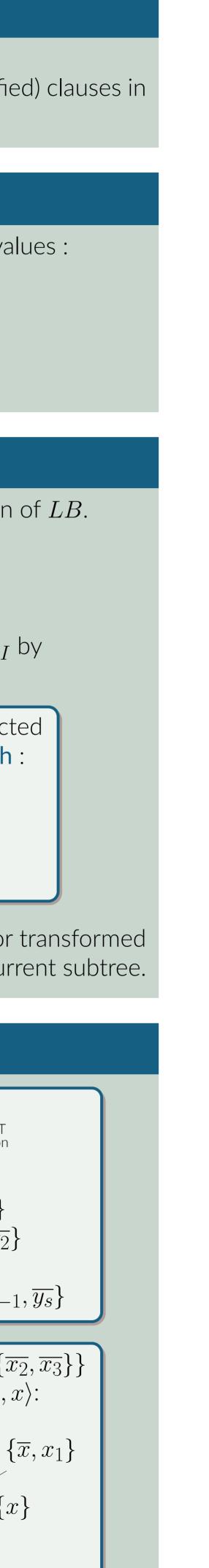
$$\{\overline{x}, x_2\} \xrightarrow{} x_2 \xrightarrow{} \{\overline{x_2}, \overline{x_3}\}$$

To ensure that detected ISs are disjoint, they are temporarily deleted or transformed by Max-SAT resolution in which case they can be maintained in the current subtree.

Max-SAT resolution Transformation

 $\frac{c = \{x, y_1, \dots, y_s\}}{cr = \{y_1, \dots, y_s, z_1, \dots, z_t\}, cc_1, \dots, cc_t, cc_{t+1}, \dots, cc_{t+s}} \operatorname{Max-SAT}_{\text{resolution}}$ Compensation clauses $cc_{t+1} = \{\overline{x}, z_1, \dots, z_t, \overline{y_1}\}$ $cc_1 = \{x, y_1, \dots, y_s, \overline{z_1}\}$ $cc_2 = \{x, y_1, ..., y_s, z_1, \overline{z_2}\}$ $cc_{t+2} = \{\overline{x}, z_1, ..., z_t, y_1, \overline{y_2}\}$ $cc_{t} = \{x, y_{1}, \dots, y_{s}, z_{1}, \dots, z_{t-1}, \overline{z_{t}}\} | cc_{t+s} = \{\overline{x}, z_{1}, \dots, z_{t}, y_{1}, \dots, y_{s-1}, \overline{y_{s}}\}$ Transformation of the IS $\psi = \{\{x\}, \{\overline{x}, x_1\}, \{\overline{x}, x_2\}, \{\overline{x_1}, x_3\}, \{\overline{x_2}, \overline{x_3}\}\}$ with respect to the **Reverse Propagation Order (RPO)** $\langle x_3, x_2, x_1, x \rangle$: $\{\overline{x_2},\overline{x_3}\}$ $\{\overline{x_1},x_3\}$ $\{\overline{x_1}, \overline{x_2}\} \quad \{\overline{x}, x_2\}$ $\{\overline{x}, \overline{x_1}\} \quad \{\overline{x}, x_1\}$ $\downarrow x_2$ $\downarrow x_1$ $\{x_1, \overline{x_2}, \overline{x_3}\}$ $\{x, \overline{x_1}, \overline{x_2}\}$ $\{\overline{x},\overline{x_1}\}$ $\{\overline{x}\}$ $\{\overline{x_1},\overline{x_2}\}$ $\{x\}$ $\{\overline{x}, x_1, x_2\}$ $\downarrow x$

UP-resilience



Definition

The transformation of an IS ψ is UP-resilient for a literal l in ψ iff :

 $\forall \mathbf{N} \in \mathbf{pneigh}_{\psi}(l) : \Box \in \mathbf{N} \text{ or } l \text{ can be propagated in } \Theta(\psi)|_{\mathbf{N}}$

• $pneigh_{\psi}(l)$: possible neighborhood of l, i.e. the set of its different neighborhoods in all the implication graphs enabling the detection of the IS ψ • $\Theta(\psi)|_N$: result of the Max-SAT resolution transformation of the IS ψ

The transformation of ψ is UP-resilient iff is UP-resilient for all the literals in ψ .

The IS $\psi = \{\{x\}, \{\overline{x}, x_1\}, \{\overline{x}, x_2\}, \{\overline{x_1}, x_3\}, \{\overline{x_2}, \overline{x_3}\}\}$ also corresponds to another implication graph:

$$\begin{array}{c} \{x\} \\ \xrightarrow{\{\overline{x}, x_1\}} & x_1 \\ \xrightarrow{\{\overline{x}, x_2\}} & x_2 \\ \xrightarrow{\{\overline{x}, x_2\}} & x_2 \\ \overline{\{\overline{x}, \overline{x}, \overline{x}$$

•
$$pneigh(x_2) = \{\{x, \Box\}, \{x, \overline{x_3}\}\}$$

• $\Theta(\psi)|_{\{x,\overline{x_3}\}} = \{\{x_1,x_2\},\{\overline{x_1},x_2\},\Box\}$

 $\rightarrow x_2$ cannot be propagated in $\Theta(\psi)|_{\{x,\overline{x_3}\}}$ (fragmentation phenomenon)

 \rightarrow The transformation of ψ w.r.t RPO is not UP-resilient

Properties

- If the transformation of an IS ψ is UP-resilient for a set of literals L in ψ then $\forall N \in pneigh(L) : \Box \in N \text{ or } \forall l \in L, \ l \text{ can be propagated in } \Theta(\psi)|_{N \setminus \{l\}}.$
- The order of application (variable sequence) of Max-SAT resolution has a direct impact on the UP-resilience of the transformations.
- Efficient learning schemes (Patterns) are UP-resilient.
- \rightarrow quantifies the impact of transformations on the SUP mechanism
- \rightarrow provides a theoretical understanding of the efficiency of learning schemes

UP-resilience and Main Patterns

Since Max-SAT resolution transformations can negatively affect the efficiency of BnB solvers, they are only performed when they correspond to certain **patterns**.

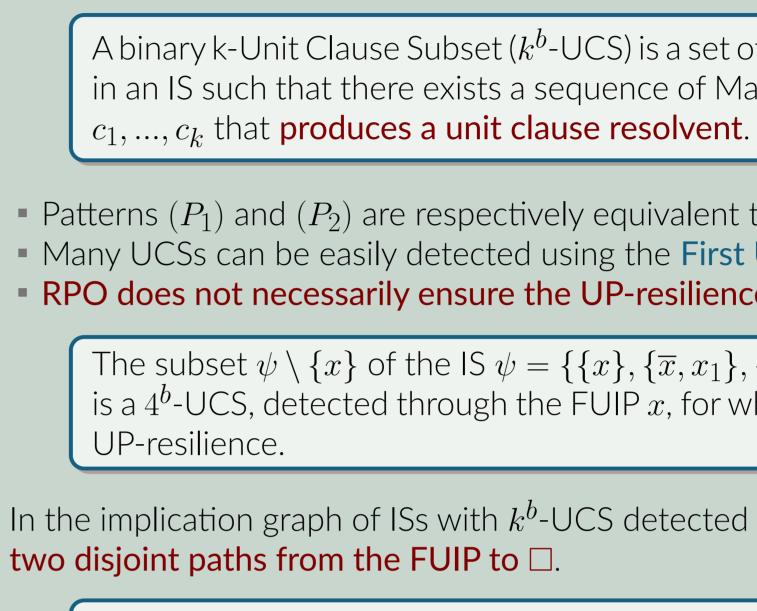
> $\frac{\{l_1, l_2\}, \{l_1, \overline{l_2}\}}{\{l_1\}}(P_1)$ $\frac{\{l_1, l_2\}, \{l_1, l_3\}, \{\overline{l_2}, \overline{l_3}\}}{\{l_1\}, \{l_1, l_2, l_3\}, \{\overline{l_1}, \overline{l_2}, \overline{l_3}\}}(P_2)$ $\frac{\{l_1\}, \{\overline{l_1}, l_2\}, \{\overline{l_2}, l_3\}, \dots, \{\overline{l_k}, l_{k+1}\}, \{\overline{l_{k+1}}\}\}}{\Box, \{l_1, \overline{l_2}\}, \{l_2, \overline{l_3}\}, \dots, \{l_k, \overline{l_{k+1}}\}}(P_3)$

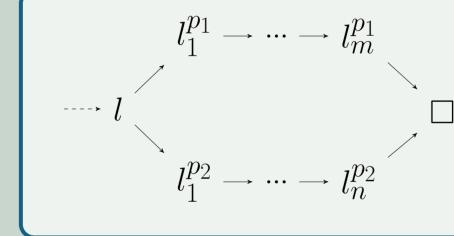
 \rightarrow Patterns do not augment the size of formula and produce unit clauses.

If a subset of an IS ψ matches the premises of **pattern (P1) or (P2)** or (P3). Then, the Max-SAT resolution transformation described in the pattern is **UP-resilient w.r.t all possible application orders**.

$$\underbrace{\{\overline{x_1}, x_3\}}_{\overline{x_3}} \xrightarrow{[\overline{x_1}, x_3]}$$

UP-resilience and Binary UCS Patterns





 $\rightarrow k^b$ -UCSs are UP-resilient with respect to PRO and PRCO \rightarrow New approach to extend patterns by UP-resilience

Empirical Study on the Relevance of UP-resilience

- four literals (MiniMaxSAT learning scheme).
- certain k-UCS patterns ($k \in \{4, 5\}$).
- UPRes: learns only UP-resilient transformations of ISs.

Learning scheme	S (T)	D	% L	% UPR
IRS	1033 (210.69)	327864	73.9	79
PAT	1400 (72.51)	95222	21.5	100
PAT+	1402 (65.94)	72683	24.5	98
UPRes	1407 (58.27)	71279	27.6	100

1776 unweighted and weighted (partial) instances in total tested with the solver **ahmaxsat**, S = number of solved instances, T = average solving time, D = average number of decision, % L = percentage of learned transformations, % UPR = percentage of learned UP-resilient transformations.

- International Conference on Artificial Intelligence, page 268–274, 2015.
- Artificial Intelligence, 289:103397, 2020.



A binary k-Unit Clause Subset (k^b -UCS) is a set of binary clauses $\{c_1, ..., c_k\}$ in an IS such that there exists a sequence of Max-SAT resolution steps on

• Patterns (P_1) and (P_2) are respectively equivalent to 2^b -UCS and 3^b -UCS patterns. • Many UCSs can be easily detected using the First Unit Implication Point (FUIP). • RPO does not necessarily ensure the UP-resilience of k^b -UCSs for $k \ge 4$.

The subset $\psi \setminus \{x\}$ of the IS $\psi = \{\{x\}, \{\overline{x}, x_1\}, \{\overline{x}, x_2\}, \{\overline{x_1}, x_3\}, \{\overline{x_2}, \overline{x_3}\}\}\$ is a 4^b-UCS, detected through the FUIP x, for which RPO does not ensure

In the implication graph of ISs with k^b -UCS detected by the FUIP, there exits exactly

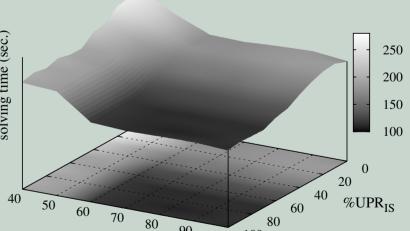
Path Resolvent Order (PRO):

 $\langle var(l_1^{p_1}), ..., var(l_m^{p_1}), var(l_1^{p_2}), ..., var(l_n^{p_2}) \rangle$

Path Resolvent Circular Order (PRCO): $\langle var(l_1^{p_1}), ..., var(l_m^{p_1}), var(l_n^{p_2}), ..., var(l_1^{p_2}) \rangle$

• IRS: learns transformations whose all intermediary resolvents contain less than

• **PAT:** learns the transformations when the ISs match the three main patterns. PAT+: learns the transformations when the IS matches the three main patterns or



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