# Growth in an OLG Economy with Polluting Non-Renewable Resources

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**Abstract**

This paper analyses the effects of flow pollution implied by the use of necessary non-renewable resources, fossil fuel for example, on overlapping generations (OLG) economies. Notably, it shows that, on the balanced growth path, flow pollution reduces the (negative) resources contribution to growth and increases resources conservation, capital accumulation, and growth. Flow pollution thus increases the ability of an economy to sustain a non-decreasing consumption path. Some of the results are due to (or magnified by) the OLG structure of the economy. In addition, the paper highlights the need for public intervention and shows that the optimal allocation may be decentralized using a tax on resources use and transfers.

*Keywords:* Non-renewable Resources; Growth; Pollution; Overlapping Generations

*JEL Codes:* Q32; Q38; Q53

## 1. Introduction

Since the two first industrial revolutions, economic development has been permitted by the consumption of large quantities of oil, coal, gas, and non-renewable

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resources in general. Because those resources are finite, the feasibility of long-run growth is often questioned. In the 1970s the Club of Rome alerted general audience on the physical "limits to growth" (Meadows *et al.* , 1972). The pessimistic Malthusian point of view adopted in this report has been challenged by neoclassical economists: thanks to technological progress, increasing returns, and substitution possibilities between non-renewable resources and man-made capital, long-run economic development might be feasible (Dasgupta & Heal, 1974, 1979; Solow, 1974; Stiglitz, 1974). More recently, the general audience's attention is being focused on pollution issues. These issues are related since a wide part of emissions is linked to resources use or extraction. A better understanding of the interactions between pollution, resources use and economic development is needed.

In this paper, I propose an overlapping generations model in which emissions are the byproducts of non-renewable resources use or extraction. The non-renewable resources are held by young households and are necessary for production, but their use is associated with a flow of emissions that affects the current factor productivity. Each generation is selfish and doesn't take into account the loss in natural capital its economic activities impose on future generations.

The model highlights the central roles played by flow emissions on sustainability. Because the economy is made of selfish overlapping generations, the labor share of income has to be high enough to ensure the feasibility of a non-declining consumption path (Agnani *et al.* , 2005). If the labor share is not high enough, savings (and thus capital accumulation) cannot be sufficient to compensate for resources depletion. The flow emissions associated with resources extraction and use allow to soften this constraint. Because of the OLG demographic structure, there is room for policy intervention. The role of the policy maker is to decentralize the optimal allocation, which is determined in a way that reflects inter-generational fairness.

The first contribution of this paper is to highlight that the detrimental effects that pollution may have on current factor productivity enhances the ability of the economy to sustain a non-decreasing consumption path. The more the pollution affects factor productivity, the larger is the long run rate of growth. The combination of three effects allows to explain this result. Firstly, because resources are non-renewable, they have a negative growth contribution in the long-run, due to the necessity to compensate for resources depletion. Because resources are the source of pollution, the more pollution affects current factor productivity, the less is this negative contribution, and the larger is the long-run rate of growth. To put it differently, the net resources contribution to growth, which is negative (resources extraction must be decreasing in the long-run), is reduced by the adverse effect of pollution on productivity. Secondly, because the detrimental effect of pollution causes higher growth, the marginal productivity of resources increases faster, and so do resources prices. Natural resources constitute an asset in which young households can invest to finance their old age. If resources prices increase faster, the incentives in investing in resources is stronger, enhancing resources conservation. Finally, because households can invest in resources or physical capital, the returns provided by both assets should be the same to prevent arbitrage possibilities. Thus, resources prices have to increase at the interest rate. When the detrimental effect of pollution increases, the marginal productivity of capital increases, which gives incentives to households to save in physical capital. To summarize, when pollution impacts on productivity are stronger, the net resources contribution to growth is reduced, extraction and consumption are postpone, and capital accumulation increases.

The second contribution of this paper is to highlight the role that policy intervention can play. The existence of a finite asset (the resource) banishes the possibility of capital over-accumulation, which is usual in OLG models. Thus, dynamic inefficiency is ruled out. The model also include an environmental externality, but the specific flow pollution which is considered here does not call for public intervention. Because it is a flow, its effects are not persistent: the generation that pollutes is affected by its own emissions. In addition, it is the extraction and use of resources by firms that affects the productivity of firms. Because we are considering a representative firm, it is affected by its own emissions. Thus, the allocation reached by the market is Pareto-Efficient. Finally, the role of the policy maker is to decentralize the (utilitarian) optimal allocation, which is determined in a way that reflects inter-generational fairness. I characterize the tax that allows the optimal allocation to be decentralized.

The paper is related to the seminal papers of Stiglitz (1974), Solow (1974), and Dasgupta & Heal (1974, 1979) that have highlighted the importance of (exogenous) technical progress, increasing returns and substitution possibilities between man-made and natural capital in order to surpass the problem of resources depletion. Those authors use the infinitely-lived agents (ILA) framework but this choice is not without consequences. In ILA models, agents are implicitly assumed to be intergenerationally altruistic. More precisely, the ILA framework assumes a dynastic altruism where parents are able to maximize the welfare of their children. Such an altruism is not supported by empirical results (Altonji *et al.* , 1992) and makes a sustainable management of resources more probable: since current generations take care of future ones, they are more likely to preserve the resources stock or to invest the rent in order to promote a decent standard of living for future generations following the so-called Hartwick's rule (Hartwick, 1977). Agnani *et al.* (2005) address this shortcoming and use the overlapping generation (OLG) framework, in which each generation is supposed to be perfectly selfish, to study the sustainability of growth with natural resources. They show that the labor share has to be large enough to allow for a positive balanced growth rate. I add to their paper considering the fact that natural resources are polluting. In the present paper, the focus is on the specific effects induced by a flow pollution seen as a byproduct of resources use or extraction. In addition, I show how policy intervention could be used to fit with inter-generational fairness objectives, and I analyze the dynamical properties of the model.

This paper is also related to the literature on the growth/pollution nexus in overlapping economies. Pollution is often modeled as a stock increasing with total output

(Jouvet *et al.* , 2010), consumption (John & Pecchenino, 1994), or capital stock (Gradus & Smulders, 1993). In opposition, few papers have introduced pollution as resulting from resources extraction and/or resources use in the production process. A notable exception is Babu *et al.* (1997) but they focus on Pareto-efficiency and are not interested in feasibility of long-run growth.[2] While the literature usually concentrates on stock pollutants, I introduce pollution as a flow affecting current factor productivity. This assumption shouldn't be shown as restrictive: there exist numerous pollutants with short lifetime (sulphur, black carbon, fine air particulate, nitrogen dioxide, tropospheric ozone...) which may be considered as flow pollutants, especially in the OLG framework where a period accounts for 25-30 years. Those pollutants are known to have effects on health (and thus on worker productivity), land productivity, and may affect negatively plants (through acid rain for example). Nevertheless, the impacts of flow emissions on the economy has known little attention from economists. In addition, in the very long run and abstracting from emission ceilings, the results given in the present paper might be robust to the consideration of stock pollutants: emissions are caused by natural resources which tends to be exhausted so that new emissions tend toward zero which allows the natural absorption to overcompensate new emissions : a stock pollutant will behave like a flow in the very long-run. A general equilibrium model where flow pollution is seen as the result of resources extraction may be found in Schou (2000, 2002). However, this author uses an ILA framework. The consideration of an OLG economy allow us to obtain new effects on savings in capital and resources. More importantly, it allows to show that there is room for policy intervention, which appears unnecessary in an ILA economy (Schou, 2000, 2002).

The rest of the paper is organized as follows. Section 2 presents the decentralized economy, section 3 develops the Ramsey economy, section 4 is devoted to the

[2] I focus here on general equilibrium literature. There also exists an extensive literature that focuses on the exhaustible-resources/pollution problem (see for example Withagen, 1994).

decentralization of the Ramsey optimal allocation while section 5 concludes.

## 2. The decentralized economy

### *2.1. The model*

I use the two-period OLG model with one representative good of Agnani *et al.* (2005). Agents are alive for two periods. For the sake of simplicity, no demographical growth is assumed and the size of the working force is normalized to one.[3]

#### *2.1.1. The resource*

The economy is initially endowed with a quantity $m_{-1}$ of necessary non-renewable resources held by the first generation of aged agents. At each date $t$, elderly agents sell their resources share to the young generation and a quantity $x_t$ of the resources is used in the production process and generates an environmental externality. The resources stock in $t$ is thus denoted by $m_t = m_{t-1} - x_t$ and it belongs to the generation $t$. The rate of exhaustion of the natural asset is:

$$q_t = \frac{x_t}{m_{t-1}} \tag{1}$$

The dynamics of the per worker resources stock is thus:[4]

$$m_t = (1 - q_t)m_{t-1} \tag{2}$$

It leads, associated with the non-renewability of the resources, to the exhaustibility condition:

$$1 \geq \sum_{t=0}^{+\infty} q_t \prod_{j=1}^{t} (1 - q_{j-1}) \tag{3}$$

[3]Lowercases represent per worker variables.

[4]It may also be interpreted as resources market clearing condition as in Agnani *et al.* (2005).

*2.1.2. The consumers*

Agents are alive for two periods and maximize the following log-linear utility function:[5]

$$u(c_t; d_{t+1}) = \ln(c_t) + \frac{1}{1+\rho}\ln(d_{t+1}) \tag{4}$$

where $c$ represents the consumption while young, $d$ the consumption while old, and $\rho$ is the individual rate of time preferences.

In its first period of life, the agent works and earns a wage $w_t$ which may be consumed $c_t$, saved as physical capital $s_t$, or used to buy property rights on the resources stock $m_t$ at a price $p_t$ in terms of the representative good. His/her first period budget constraint is:

$$w_t = c_t + s_t + p_t m_t \tag{5}$$

While old, the agent gets his/her savings increased at the interest rate, and he/she sells his/her resources rights at a price $p_{t+1}$. His/her second period budget constraint is:

$$d_{t+1} = (1 + r_{t+1})s_t + p_{t+1}m_t \tag{6}$$

Combining (5) and (6), the following inter-temporal budget constraint can be obtained (IBC hereafter):

$$w_t = c_t + \frac{d_{t+1}}{1+r_{t+1}} - \frac{p_{t+1}m_t}{1+r_{t+1}} + p_t m_t \tag{7}$$

The maximization of utility with respect to $m_t$, $c_t$, $d_{t+1}$ subject to the IBC leads to the following first order conditions:

$$\frac{d_{t+1}}{c_t} = \frac{1+r_{t+1}}{1+\rho} \tag{8}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1} \tag{9}$$

[5] The consideration of a more general CES utility function doesn't affect the guidelines given in this paper.

(8) is the standard Euler equation which established that the marginal rate of substitution between consumption while young and consumption while old has to be equal to their relative price. (9) is the well known Hotelling's rule, i.e. a non-arbitrary condition between the two assets in this economy, capital and resources, which established that the resources price increases at the interest rate.

*2.1.3. The firms*

Firms produce the representative good $Y_t$ using a Cobb-Douglas technology. They use capital $K_t$, labor $N_t$, and resources $X_t$, with constant returns to scale for a given level of technology $A_t$, which grows at an exogenous rate $a$ such that:

$$A_{t+1} = (1 + a)A_t \tag{10}$$

The extraction and use of the resources in the production process generate a flow of pollution $e_t$ such that:

$$E_t = \phi X_t \tag{11}$$

Pollution resulting from the use of the resources in the production process generates a productivity loss. The consideration of flow pollution is not so reductive: there exists a wide variety of pollutant with short lifetime that may cause productivity losses. For example sulfur dioxide resulting from the burning of fossil fuel is a major cause of acid rain. Tropospheric ozone resulting from fossil fuel burning[6] generates health issues that may affect directly the productivity of workers (Zivin & Neidell, 2012).[7] $\theta$ captures the detrimental impact of pollution on the level of production. Nielsen *et al.* (1995) and Schou (2000) model the detrimental effect from pollution on productivity in the same way. The production function is thus $Y_t = A_t K_t^{\alpha} L_t^{\beta} X_t^{\nu} E_t^{-\theta}$ which gives

[6]More precisely, ozone is produced through the interaction of nitrogen dioxide with dioxygen and sun.

[7]An effect of pollution on health also affects agents utility and would be better modeled if also introduced in the utility function. For the sake of simplicity, I do not consider the impact of pollution on utility which could be analyzed in future research.

in per worker variables:

$$y_t = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \tag{12}$$

with $\alpha + \beta + \nu = 1$. Thus, the model assumes constant returns to scale from the firm point of view. Indeed, the environmental externality is not taken into account by individual firms but they consider the aggregated level of pollution as given while they decide their production plans.[8]

Capital is remunerated at the interest rate $r_t$ and depreciates at a rate $\delta$ over the period. Firms pay a wage $w_t$ to their workers and buy the natural input at its price $p_t$. The profit of the representative firm is thus:

$$\Pi_t = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} - (r_t + \delta) k_t - w_t - p_t x_t \tag{13}$$

The focus is put on the case $\theta < \nu$ and $\nu$ and $\theta$ are both positive. That is, for an identical amount of resources and emissions, I assume that the positive impact of resources on income outweighs its negative one. A similar assumption may be found in Schou (2000), where pollution is also seen as a flow. The standard profits maximization leads to the following first order conditions:

$$r_t = \alpha A_t k_t^{\alpha-1} x_t^{\nu} e_t^{-\theta} - \delta \tag{14}$$

$$p_t = \nu A_t k_t^{\alpha} x_t^{\nu-1} e_t^{-\theta} \tag{15}$$

$$w_t = \beta A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \tag{16}$$

Each factor is thus remunerated at its marginal productivity.

From equation (14)-(16), it appears that a decrease in the level of emissions increases the marginal productivity of production factors. Such a decrease is conditioned to a reduction of resources use. A reduction of resources use cause an increase in the marginal productivity of resources and a decrease in the marginal productivity of other factors.

[8] This is consistent for global pollution. Results and insights given in this paper are not qualitatively affected if one consider local pollution.

*2.2. Intertemporal Equilibrium and Balanced Growth Path*

The economy produces a representative good which may be consumed or saved as physical capital. Following Diamond (1965), the good market clearing condition is:[9]

$$s_t = k_{t+1} \tag{17}$$

**Lemma 1.** An intertemporal competitive equilibrium is a solution of the following system for $k_0, m_{-1}, m_0$ given

$$k_{t+1} = \left[\frac{\beta}{2+\rho} - \frac{\nu m_t}{m_{t-1} - m_t}\right](1+a)^t k_t^\alpha (m_{t-1} - m_t)^{\nu-\theta}\phi^{-\theta} \tag{18}$$

$$\frac{(1+a)k_{t+1}^\alpha (m_t - m_{t+1})^{\nu-\theta-1}}{k_t^\alpha (m_{t-1} - m_t)^{\nu-\theta-1}} = 1 + \alpha(1+a)^{t+1} k_{t+1}^{\alpha-1}(m_t - m_{t+1})^{\nu-\theta}\phi^{-\theta} - \delta \tag{19}$$

Equation (18) comes from the combination of the market clearing condition with the intertemporal budget constraint while equation (19) comes from the Hotelling rule.

This paper mainly focuses on balanced growth path because they constitute the only cases where long-run positive growth is possible, as noted in Agnani *et al.* (2005). Moreover, it is in accordance with stylized facts of growth literature.

**Definition 1.** An intertemporal equilibrium where all variables grow at a constant rate is defined as a balanced growth path (BGP hereafter).

Let $\mu_h$ be the BGP notation of the ratio $h_{t+1}/h_t$. According to definition 1, $\mu_m$ should be constant. Thus, (2) implies a constant rate of extraction along the BGP, i.e. $q_t = q_{t+1} = q$. The system represented in Lemma 1 allows to characterize the BGP of this economy.

**Proposition 1.** The balanced growth path of the economy exists, is unique, and is

[9]Proof is reported in Appendix A.

characterized by the following growth factors:

$$\mu_y = \mu_k = \mu_c = \mu_d = \mu_s = \mu_w = \mu = (1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}$$

$$\mu_x = \mu_m = \mu_e = 1-q$$

$$\mu_p = \frac{\mu}{\mu_x}$$

$$\mu_A = 1+a$$

$$\mu_r = 1$$

and $q$ satisfying the following non-linear equation:

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}\alpha(2+\rho)q}{\beta q - \nu(1-q)(2+\rho)} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}}{1-q} - (1-\delta) \tag{20}$$

*Proof.* Proof is reported in Appendix B. □

From proposition 1, it appears that a higher extraction rate is associated with a lower growth while looking at (1) and (12), an increase in $q$ implies a higher income. It is thus necessary to distinguish between short and long run impacts of a higher extraction rate. In the short run, an increase in $q$, *ceteris paribus*, implies an increase of one input in the production process. Current production thus increases. Nevertheless, it will be harder to maintain this level of production, because less natural resources are available to produce. In the long run, the higher the extraction rate, the more the economy needs capital to compensates for resources depletion. The pressure on natural resources thus limits future growth possibilities.

*2.3. Local dynamics*

Noting that

$$(1+a)^t k_t^{\alpha-1}(m_{t-1}-m_t)^{\nu-\theta}\phi^{-\theta} = \frac{1}{\alpha}\left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-\theta-1} - 1 + \delta\right]$$

and taking the ratio of equation (19) evaluated in both $t$ and $t+1$, the system given in Lemma 1 may be written in growth factors :

$$\begin{cases} \mu_{k,t+1} = \frac{1}{\alpha}\left[\frac{\beta}{2+\rho} - \frac{\nu(1-q_t)}{q_t}\right]\left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-\theta-1} - 1 + \delta\right] \\ (1+a)\mu_{k,t+1}^{\alpha-1}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-\theta} = \frac{(1+a)\mu_{k,t+1}^{\alpha}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-\theta-1} - 1 + \delta}{(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-q_{t-1})}{q_{t-1}}\right]^{\nu-\theta-1} - 1 + \delta} \end{cases}$$

Then, defining $z_{t+1} = q_t$, the system may be written as follows

$$\begin{cases} \mu_{k,t+1} = \frac{1}{\alpha}\left[\frac{\beta}{2+\rho} - \frac{\nu(1-q_t)}{q_t}\right]\left[(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-z_t)}{z_t}\right]^{\nu-\theta-1} - 1 + \delta\right] \\ (1+a)\mu_{k,t+1}^{\alpha-1}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-\theta} = \frac{(1+a)\mu_{k,t+1}^{\alpha}\left[\frac{q_{t+1}(1-q_t)}{q_t}\right]^{\nu-\theta-1} - 1 + \delta}{(1+a)\mu_{k,t}^{\alpha}\left[\frac{q_t(1-z_t)}{z_t}\right]^{\nu-\theta-1} - 1 + \delta} \\ z_{t+1} = q_t \end{cases}$$

Linearizing this system around the BGP, we get

$$\begin{pmatrix} d\mu_{k,t+1} \\ dq_{t+1} \\ dz_{t+1} \end{pmatrix} = \begin{pmatrix} A & B & C \\ \frac{A(G-D)+H}{E-I} & \frac{B(G-D)+J-F}{E-I} & \frac{C(G-D)+K}{E-I} \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} d\mu_{k,t} \\ dq_t \\ dz_t \end{pmatrix}$$

where

$$A = \left(\frac{\beta q - \nu(1-q)(2+\rho)}{(2+\rho)q}\right)\frac{1}{1-q}$$
$$B = \left(\frac{\beta q - \nu(1-q)(2+\rho)}{(2+\rho)q}\right)\frac{\mu(\nu-\theta-1)}{(1-q)q\alpha} + \frac{\nu}{\alpha q^2}\left(\frac{\mu}{1-q} - 1 + \delta\right)$$
$$C = -\frac{\mu(\nu-\theta-1)}{(1-q)^2 q\alpha}\left(\frac{\beta q - \nu(1-q)(2+\rho)}{(2+\rho)q}\right)$$
$$D = \frac{\alpha}{1-q}$$
$$E = \frac{(\nu-\theta-1)\mu}{q(1-q)}$$
$$F = -\frac{(\nu-\theta-1)\mu}{q(1-q)^2}$$
$$G = \frac{\alpha-1}{\mu}\left(\frac{\mu}{1-q} - 1 + \delta\right)$$
$$H = \frac{\alpha}{1-q}$$
$$I = \frac{\nu-\theta}{q}\left(\frac{\mu}{1-q} - 1 + \delta\right)$$
$$J = \frac{(\nu-\theta-1)\mu}{q(1-q)} - \frac{\nu-\theta}{q(1-q)}\left(\frac{\mu}{1-q} - 1 + \delta\right)$$
$$K = -\frac{(\nu-\theta-1)\mu}{q(1-q)^2}$$

The model has been calibrated with the following annual rates : $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$. We assume that one period account for 25 years and we take $\alpha = 0.3$, $\beta = 0.65$, $\nu = 0.05$ and $\theta = 0.5\nu$. It follows that $q \approx 0.39$ and $\mu \approx 2.63$. With these values, it appears that the system exhibits saddle-path stability. Indeed, our system include two predetermined variables and one forward looking variable, and we obtain only one eigenvalue with absolute value larger than one.[10] Sensitivity analysis has been performed and this result appears very robust.

### *2.4. The impact of flow pollution on sustainability*

In this work, I define sustainability as the ability of the economy to sustain a non-declining balanced consumption path. The economy will contract if $1-(1+a)^{\frac{1}{\theta-\nu}} < \hat{q}$ *i.e.* if $\beta < \frac{(1+a)^{\frac{1}{\theta-\nu}}(2+\rho)\nu}{1-(1+a)^{\frac{1}{\theta-\nu}}} \equiv l(\theta)$. This condition is less likely to be satisfied when $\theta$ increases.

**Proposition 2.** The detrimental effect of pollution on production enhances sustainability.

*Proof.* $\frac{\partial l(\theta)}{\partial \theta} < 0$ □

In addition, it can be shown that an increase in $\theta$ causes a higher growth.

**Proposition 3.** When pollution hurts productivity more severely, growth is higher.

*Proof.* See Appendix C. □

Proposition 2 may be seen as a consequence of Proposition 3. Indeed, if growth increases, the ability of the economy to sustain a non-declining consumption path increases too because consumption increases at the economy rate of growth. *Prima facie*, these two propositions may appear puzzling. Nevertheless, they can be simply explained as the combination of three effects. Firstly, for a given extraction rate, an increase in $\theta$ causes a decrease in the importance of resources for growth. That is, when $\theta$ increases, it diminishes the adverse effect of growth imposed by the necessary decreasing resources extraction. To put it differently, $\theta$ diminishes the net resources contribution to growth and thus its implicit negative contribution. Secondly, an increase in $\theta$ diminishes resources extraction. Indeed, we have just seen that an increase in the detrimental effect of pollution causes higher growth, all been equal. The marginal productivity of resources thus increases faster, and so do resources prices. This provide an incentive for households to buy more resources while young to finance their old age, which reduces resources extraction. Thirdly, capital

[10]Simulated eigenvalues of the Jacobian Matrix are 2.33176, 0.30094, 0.

accumulation is enhanced by an increase in $\theta$, because equation (9) established that the returns of capital has to equalize the returns of resources to avoid any possibility of arbitrage between the two assets. Households thus increase their savings in capital. To summarize, when pollution affects productivity more severely, resources are less important for growth, households postpone consumption while young at the benefit of old age consumption by investing in both resources and capital so that resources are used in smaller amounts, and capital accumulation is enhanced.

## 3. The Ramsey Economy

### *3.1. The Model*

The present section is devoted to the presentation of the Ramsey economy. We assume that the economy is managed by a benevolent social planner which has to maximize the welfare of the dynasty of households. Thus, the planner takes care of both present and future generations. Since each generation prefers present than future consumption, future consumption is discounted at a rate $\rho$ (as in the decentralized economy). $\rho$ is thus an intragenerational discount rate that represents preferences of households. In addition, the planner could decide to attach different weights to different generations in its objective function. This possibility is captured by the parameter $\gamma$ which is the social discount rate. Here, this rate reflects solely the weight that the central planner attaches to each generation and can be seen as an intergenerational discount rate. There exist great debates in the literature on the value of such a rate. One may argue that the central planner shouldn't favor closer in time generation. In such a case, the policy maker should choose $\gamma = 0$ (Ramsey, 1928; Pigou, 1932; Solow, 1986). Nevertheless, the uncertainty about future economic conditions argue for positive discount rate. For example, our framework is not robust to the existence of a backstop technology that may appear in the future and which will modify the production function (resources may become unnecessary in the future). Since this is likely to happen in the very long run, a positive social discount rate allows to avoid an overweighting of distant generations' welfare. Moreover, the existence of far distant generations is not guaranteed.

The planner has to solve the following Ramsey problem:

$$\max_{\{c_t;d_t;M_t;k_t;e_t\}_{t=0}^{\infty}} = \frac{1}{1+\rho}\ln(d_0) + \sum_{t=0}^{\infty} \frac{1}{(1+\gamma)^{t+1}} \left[\ln(c_t) + \frac{1}{1+\rho}\ln(d_{t+1})\right] \tag{21}$$

subject to :

$$y_t = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \tag{22}$$

$$y_t = c_t + d_t + k_{t+1} - (1-\delta)k_t \tag{23}$$

$$A_{t+1} = (1+a)A_t \tag{24}$$

$$e_t = \phi x_t \tag{25}$$

$$m_t = (1-q_t)m_{t-1} \tag{26}$$

$$x_t = q_t m_{t-1} \tag{27}$$

$$m_{-1} = \sum_{t=0}^{\infty} q_t m_{t-1} \tag{28}$$

$$k_0,\, m_{-1},\, e_{-1}, A_0 > 0 \tag{29}$$

where $\gamma$ is the social discount rate. (22) represents the production function. (23) established that the economy consumes or invests exactly its net production in each period. (24) represents the exogenous technological progress. (25) is the emissions implied by the resources use while (26) and (27) represents the dynamics of the resources. (28) is a total exaustibility condition for the resources while (29) represents initial endowments.

The first order conditions of the previous problems may be reduced to:

$$\frac{1+\gamma}{1+\rho} = \frac{d_t}{c_t} \tag{30}$$

$$(1+\rho)\frac{d_{t+1}}{c_t} = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{\nu} e_{t+1}^{-\theta} + 1 - \delta \tag{31}$$

$$\frac{A_{t+1} k_{t+1}^{\alpha} x_{t+1}^{\nu} e_{t+1}^{-\theta} (\nu x_{t+1}^{-1} + \phi\theta e_{t+1}^{-1})}{A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} (\nu x_t^{-1} + \phi\theta e_t^{-1})} = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{\nu} e_{t+1}^{-\theta} + 1 - \delta \tag{32}$$

$$\lim_{t\to\infty} \left(\frac{1}{1+\gamma}\right)^t \frac{k_{t+1}}{c_t} = 0 \tag{33}$$

(30) is an intergenerational optimality condition establishing that the marginal rate of substitution between consumption of young and old has to be equal to one. (31) is an intragenerational optimality condition which states that the marginal rate of substitution between

consumption while young and consumption while old has to be equal to the marginal product of physical capital net of depreciation. (32) characterizes the optimal inter-temporal resources allocation which indicates that the depletion of the resources stock implies an implicit return equal to the physical capital return. This condition implies that the economy should satisfy the Hotelling rule. (33) is the transversality condition associated with the planner's problem.

Combining (22)-(33) one can define the balanced growth path of this Ramsey economy.

**Proposition 4.** The optimal balanced growth path is defined by the following growth rate:

$$\tilde{\mu}_y = \tilde{\mu}_k = \tilde{\mu}_c = \tilde{\mu}_d = \tilde{\mu} = (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{\nu-\theta}{1-\alpha}}$$
$$\tilde{\mu}_x = \tilde{\mu}_m = \tilde{\mu}_e = 1-\tilde{q}$$
$$\tilde{\mu}_A = 1+a$$

and $\tilde{q} = \frac{\gamma}{1-\gamma}$.

*Proof.* Proof is reported in Appendix D. □

The optimal extraction rate only depends on the social rate of time preference. To put it differently, the Ramsey economy follows a path that depends on the central planner trade-off between generations. A higher social preference for the present implies a higher depletion rate of the resources stock, and a lower growth. On the contrary, a society which strongly cares about future generations is more conservative and achieve a larger rate of growth in the long run, because it preserves its resources stock. To decentralize the optimal balanced growth path, a government should find an instrument able to put the extraction rate at the optimal level $\gamma/(1+\gamma)$, keeping the rate of growth of other variables at their optimal level. In section 4, it will be shown that the optimal balanced growth path may be decentralized using a tax.

Before looking at the decentralization of the Ramsey optimal allocation, one should analyze how changes in $\theta$ affects the optimal balanced growth path.

### *3.2. Comparative statics*

This section analysis how movements in $\theta$ impact the optimal rate of growth.

**Proposition 5.** The optimal long run rate of growth increases with the detrimental effect of pollution.

*Proof.*

$$\frac{\partial\tilde{\mu}}{\partial\theta} = -\frac{(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{\nu-\theta}{1-\alpha}}\log(1-\tilde{q})}{1-\alpha} > 0 \tag{34}$$

□

As in the decentralized economy, $\theta$ diminishes the net resources contribution to production. When time goes by, the resources, which are not reproducible, are used in ever small amount. Their necessity in the production process thus imposes a negative drag on growth because an increasing share of the capital stock is used only to compensate for resources depletion. When $\theta$ increases, it decreases the detrimental effect of natural resources on growth caused by the need to keep diminishing resources extraction. That leads to a higher growth rate of the economy. However,contrary to what happens in the market equilibrium, the rate of resources extraction doesn't depend on $\theta$ but only depends on the social rate of time preference. Indeed, the central planner manage resources in a way that reflects its intergenerational preferences : since its preferences are not impacted by a change in $\theta$, there is no reason for the rate of resources extraction to be impacted. Finally, combining (31) and (32), we get:

$$(1+\rho)\frac{d_{t+1}}{c_t} = \frac{A_{t+1}k_{t+1}^{\alpha}x_{t+1}^{\nu}e_{t+1}^{-\theta}(\nu x_{t+1}^{-1} + \phi\theta e_{t+1}^{-1})}{A_t k_t^{\alpha}x_t^{\nu}e_t^{-\theta}(\nu x_t^{-1} + \phi\theta e_t^{-1})} \tag{35}$$

Evaluating the right hand side of (35) at the BGP, we can write:

$$(1+\rho)\frac{d_{t+1}}{c_t} = (1+a)\tilde{\mu}_k\tilde{\mu}_x^{-1} \tag{36}$$

Since $\tilde{\mu}_x \in (0,1)$ is not affected by a change in $\theta$, one can infer that an increase in $\theta$ causes an increase in the right hand side of the last equation, which represents the rate of return of capital (and resources) on the BGP. It means that, on the BGP, an increase in $\theta$ provokes a decrease in present consumption at the benefit of future consumption. Each generation thus consume less while young and a greater share of national income could be devoted to capital accumulation, leading to a higher growth.

## 4. Decentralizing the Ramsey optimal balanced growth path

OLG model with a pollution externality are often associated with two market failures. The first one linked to the pollution, the second one linked to the demographic structure of the OLG economy. Concentrating on BGP, it appears from propositions 1 and 4 that pollution doesn't distort the economy because the flow pollution I am considering evolves at the extraction rate. In addition, because pollution is not persistent, the generation that pollutes suffers from its own pollution in the form of lower income. OLG economies are also known to allow for possible capital overaccumulation, which enables the implementation of Pareto improving policies. Nevertheless, Rhee (1991), and Gerlagh & Keyzer (2001) have shown that OLG economies endowed with finite non-renewable resources are efficient in the Pareto sense, and this results is robust in the model I am using here (Agnani *et al.* , 2005). Finally, in the present paper, the need for public intervention comes from the fact that the present generation doesn't take into account the negative externality its resources use today imposes on future generation in the form of a lower resources stock. The aim of public intervention is thus to solve an intergenerational equity problem.

The aim of the present section is to propose a policy that is able to decentralize the Ramsey optimal allocation for a given social discount rate calibrated by the policy maker to reflect intergenerational fairness. Looking at proposition 1 and 4, it immediately appears that decentralization of the BGP requires to put the market extraction rate at its optimal level $\tilde{q} = \gamma/(1-\gamma)$ letting other growth determinants unchanged.[11] This may be performed using a tax.

Let's assume that the government tax the resources use at a rate $\tau_t$. Equation (15) becomes:

$$p_t + \tau_t = \nu A_t k_t^{\alpha} x_t^{\nu-1} e_t^{-\theta} \tag{37}$$

The tax is redistributed to the young generation as a transfer $g_t$ such that the government

[11] Results obtained in this section only apply to the BGP and should not be translated outside.

budget is balanced:[12]

$$\tau_t x_t = g_t \tag{38}$$

The budgetary constraint of the young agent (5) is thus modified as follows:

$$g_t + w_t = c_t + s_t + p_t m_t \tag{39}$$

**Proposition 6.** If the tax and the resources price increase at the same rate, the balanced growth path of the economy is defined as in proposition 1.

*Proof.* Proof is reported in Appendix E. □

Decentralization of the optimal balanced growth path thus requires to find the tax level that enforces $q^* = \tilde{q}$. In the presence of a tax on resources, equation (18) writes :

$$k_{t+1} = A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} \left[ \frac{\beta}{2+\rho} - \frac{\nu(1-q_t)}{q_t} \right] + \frac{\tau_t x_t}{2+\rho} \tag{40}$$

Dividing both side by $k_t$, it leads to:

$$\mu_k - \frac{\tau_t x_t}{(2+\rho)k_t} = A_t k_t^{\alpha-1} x_t^{\nu} e_t^{-\theta} \left[ \frac{\beta}{2+\rho} - \frac{\nu(1-q_t)}{q_t} \right] \tag{41}$$

Since the tax and the resources price increase at the same rate, the fiscal revenues grow at a rate $\mu_{x\tau} = \mu_k$, and $\tau_t x_t / k_t$ is constant on the BGP. Let $\xi$ denote this constant. Evaluating (40) on the BGP leads to:

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}\alpha(2+\rho)q - \xi\alpha q}{\beta q - \nu(1-q)(2+\rho)} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}}{1-q} - (1-\delta) \tag{42}$$

It follows that the optimal tax is such that:

$$\begin{aligned} \tilde{\xi} = -\frac{[(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{\nu-\theta}{1-\alpha}-1} - (1-\delta)][\beta\tilde{q} - \nu(2+\rho)(1-\tilde{q})]}{\alpha\tilde{q}} \\ -(1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{\nu-\theta}{1-\alpha}}(2+\rho) \end{aligned} \tag{43}$$

where it should be recalled that $\tilde{q} = \gamma/(1-\gamma)$.

[12]Considering that the tax is invested and then redistributed to old households leads to the same results.

**Proposition 7.** The optimal balanced growth path may be decentralized using a tax such that $\xi = \tilde{\xi}$.

Calibrating the model with annual discount rates of $a = 0.028$, $\delta = 0.027$, $\rho = 0.016$, $\nu = 0.05$, $\alpha = 0.3$, $\beta = 0.65$, and $\theta = 0.01$, we can characterize $\tilde{\xi}$ for different level of $\gamma$. With the above calibrated values, a negative taxation (i.e. subvention) for resources extraction is required for an annual discount rate of $\gamma > 0.0202$. As previously discussed, a reasonable value for $\gamma$ is such that $0 < \gamma < \rho$. A subvention becomes necessary when the social planner preferences are oriented in favor of close in time generations, such that he or she considers that current generations are too conservative. In such a case, the policy maker should subvention the resources use, provoking a decrease in the rate of growth. In case of very strong social preference in favor of present, the rate of growth may become negative, leading to (optimal) extinction. This little simulation exercise shows that for reasonable annual social discount rate values, policy makers should implement a tax on fossil resources use. In this model with flow pollutants, this tax also helps to fight against pollution and may also be qualified as an environmental tax.

## 5. Conclusion and general discussion

In this paper, it is shown that a flow pollution resulting from resources extraction and affecting current factor productivity could help the economy to reach a non-declining consumption path, because the net resources contribution to growth is low (as in Schou (2000) but also because resources use diminishes and capital accumulation increases with the intensity of pollution effects, thanks to increases in the interest rate and in the resources price rate of growth on the balanced growth path. I also analyze the ability of a tax to decentralize the optimal allocation. The differences between the optimal allocation and the decentralized allocation come from the fact that the central planner takes into consideration the welfare of unborn generations. A tax on resources use redistributed to the young as a transfer allows to decentralize the Ramsey optimal equilibrium, modifying the rate of extraction.

The present paper may be extended in several directions. One of them would be to dispense with the assumption that pollution is a flow and would introduce pollution as a stock. While working with flow pollution is interesting, the introduction of stock pollution would provide new insights as it allows to tackle with climate change issues. If one assume (as we do) that pollution is caused by the extraction and use of a non-renewable resources necessary to production, and accumulates in the atmosphere as in John & Pecchenino (1994), *i.e.* with a natural decay, we could expect that pollution would describe a inverse U-shaped curve across time. Indeed, in the first periods of time, great quantities of resources would be used in the production, generating large emissions, so that the stock of pollution would increase. When time goes by, resources would be used in ever smaller amount. At some point in time, the natural decay would exceed new emissions so that the pollution stock would start to decrease. An environmental Kuznets curve is thus likely to appear in a growing economy. A ceiling of pollution could then be introduced in the model to capture climate change aspects. The aim of a policy fighting climate change would be to reduce pollution concentration and to spread resources use, and then pollution, across time, to ensure that the ceiling would be not exceed. Future research should address this issue.

## Appendix A. Proof of the Market Clearing Condition

Distribution of revenues from production gives:

$$y_t = w_t + (r_t + \delta)k_t + p_t x_t$$

The production is used to consume, to save, and to absorb capital depreciation. We have:

$$y_t = c_t + d_t + s_t - k_t + \delta k_t$$

Thus:

$$\begin{aligned}
&w_t + (r_t + \delta)k_t + p_t x_t = c_t + d_t + s_t - k_t + \delta k_t \\
&(1 + r_t)k_t + p_t(x_t + m_t) = d_t \\
&(1 + r_t)k_t + p_t m_{t-1} = d_t
\end{aligned}$$

From equation (6), the last equation is true if $k_t = s_{t-1}$.

## Appendix B. Proof of Proposition 1

The proof uses continuously definition 1.

- A BGP implies a constant extraction rate. Thus, equation (2) implies $\mu_m = 1 - q$.
- Equation (1) implies $\mu_x = \mu_m$.
- Since emissions are modeled as a linear function of resources use, $\mu_e = \mu_x$.
- The technological level increases at an exogenous rate $a$ such that $\mu_a = 1 + a$.
- Evaluating (19) in $t+1$ and $t$ and taking the ratio, we can write:

$$\frac{(1+a)\mu_{k,t+1}^{\alpha}\mu_{x,t+1}^{\nu-\theta-1} - 1 + \delta}{(1+a)\mu_{k,t}^{\alpha}\mu_{x,t}^{\nu-\theta-1} - 1 + \delta} = (1+a)\mu_{k,t+1}^{\alpha-1}\mu_{x,t+1}^{\nu-\theta}$$

  Evaluating on the BGP, we obtain:

$$(1+a)\mu_k^{\alpha-1}\mu_x^{\nu-\theta} = 1$$

  which implies:

$$\mu_k = (1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}$$

- The market clearing condition $s_t = k_{t+1}$ implies $\mu_s = \mu_k$ on the BGP.
- Evaluating the ratio of production (12) in $t+1$ and $t$ on the BGP: $\mu_y = (1+a)\mu_k^{\alpha}\mu_x^{\nu-\theta} = \mu_k$.
- From the firms FOC for resources use (15), one can compute $\mu_p = (1+a)\mu_k^{\alpha}\mu_x^{\nu-\theta-1} = \frac{\mu_k}{\mu_x}$.
- From the firms FOC for labor (16) $\mu_w = \mu_k$.
- The non-arbitrary condition for households (9) gives $\mu_p = 1 + r$. Thus, the interest rate should be constant and $\mu_r = 1$.
- Since the interest rate is constant, Evaluating the ratio of the Euler equation in $t+1$ and $t$ gives $\mu_c = \mu_d$.

- Reintroducing the Euler equation (8) in the IBC (7) one obtain:

$$w_t = \frac{c_t(2+\rho)}{1+\rho}$$

On the BGP, taking the ratio of the last expression in $t+1$ and in $t$, one obtain $\mu_c = \mu_w$.

It is now necessary to characterize the constant BGP extraction rate $q$. Equation (18) may be rewritten as:

$$\mu_{k,t+1} = \left[\frac{\beta}{2+\rho} - \frac{\nu(1-q)}{q}\right] A_t k_t^{\alpha-1} x_t^{\nu} e_t^{-\theta}$$

which gives:

$$\mu_{k,t+1} = \left[\frac{\beta}{2+\rho} - \frac{\nu(1-q)}{q}\right] \frac{1}{\alpha}(r_{t+1}+\delta)$$

Using equation (9) and evaluating on the BGP, we can obtain:

$$\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}\alpha(2+\rho)q}{\beta q - \nu(1-q)(2+\rho)} = \frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}}{1-q} - (1-\delta) \tag{B.1}$$

It may now be established that $q^*$ is solution to the preceding nonlinear equation. We denote $LHS$ and $RHS$ the left and right hand sides of (20). $RHS(q)$ is defined on $[0;1[$ with $RHS(0) = (1+a)^{\frac{1}{1-\alpha}} - (1-\delta)$. Since $\lim_{q\to 1} RHS(q) = +\infty$, $RHS(q)$ admits a vertical asymptote in q=1. Moreover, $\frac{\partial RHS(q)}{\partial(q)} > 0$ and $\frac{\partial RHS(q)^2}{\partial^2(q)} > 0$ imply an increasing and convex function. $LHS(q)$ is defined on$[0;\hat{q}[\cup]\hat{q};1]$ with $\hat{q} = \frac{\nu(2+\rho)}{\beta+\nu(2+\rho)}$. $LHS(q) < 0 \,\forall\, q < \hat{q}$ which do not allow the possibility of an equilibrium extraction rate given that $RHS(q) > 0 \,\forall\, q \in [0;1[$. Since $\lim_{q\to\hat{q}^+} = +\infty$ and $\lim_{q\to 1} = 0$ it exists a unique $q^*$ such that (20) is satisfied. This situation is represented in Figure B.1. It should be noted that the economy contracts if $1-(1+a)^{\frac{1}{\theta-\nu}} < \hat{q}$.

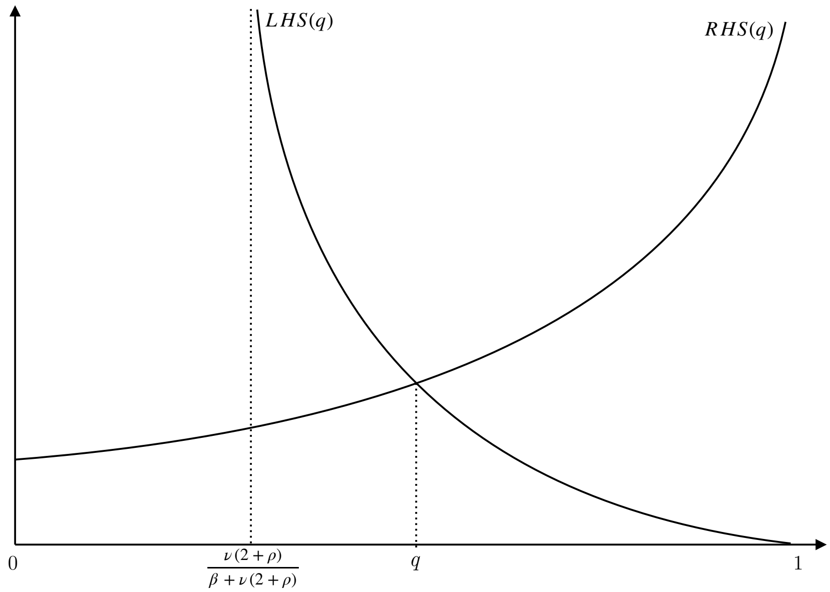


Figure B.1: Characterization of the competitive equilibrium extraction rate

## Appendix C. Proof of Proposition 3

The BGP of the economy is characterized by $\mu = (1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}$.

- Keeping the extraction rate constant, an increase in $\theta$ implies an increase in $\mu$ since:

$$\frac{\partial \mu}{\partial \theta} = -\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}}\ln(1-q)}{1-\alpha} > 0$$

- Keeping $\theta$ constant, an increase in $q$ implies a decrease in $\mu$ since:

$$\frac{\partial \mu}{\partial q} = -\frac{(1+a)^{\frac{1}{1-\alpha}}(1-q)^{\frac{\nu-\theta}{1-\alpha}-1}(\nu-\theta)}{1-\alpha} < 0$$

- Equation (20) may be rewritten as:

$$L(q) \equiv (1-q)^{\frac{\nu-\theta}{1-\alpha}} = \frac{(1-\delta)[\nu(1-q)(2+\rho)-\beta q]}{(1+a)^{\frac{1}{1+\alpha}}(2+\rho)\alpha q-(1+a)^{\frac{1}{1-\alpha}}\beta q(1-q)^{-1}+(1+a)^{\frac{1}{1-\alpha}}\nu(2+\rho)} \equiv R(q)$$

We then have:

$$\lim_{q \to 0} L(q) = 1$$
$$\lim_{q \to 1} L(q) = 0$$
$$\frac{\partial L(q)}{\partial q} < 0$$
$$\frac{\partial^2 L(q)}{\partial q^2} < 0$$

and

$$\lim_{q \to 0} R(q) = \frac{1-\delta}{(1+a)^{\frac{1}{1-\alpha}}}$$
$$\lim_{q \to 1} R(q) = 0$$
$$\lim_{q \to \hat{\hat{q}}^-} R(q) = -\infty$$
$$\lim_{q \to \hat{\hat{q}}^+} R(q) = +\infty$$

with

$$\hat{\hat{q}} = \frac{\alpha\rho + 2\alpha - \beta + \sqrt{(-\alpha\rho - 2\alpha + \beta + \rho\nu + 2\nu)^2 - 4(\alpha\rho + 2\alpha)(-\rho\nu - 2\nu)} - \rho\nu - 2\nu}{2(\alpha\rho + 2\alpha)}$$

Since $\partial L(q)/\partial\theta > 0$, an increase in $\theta$ implies a decrease in $q$. This situation is represented in figure C.2 where the dashed line represents an increase in $\theta$.

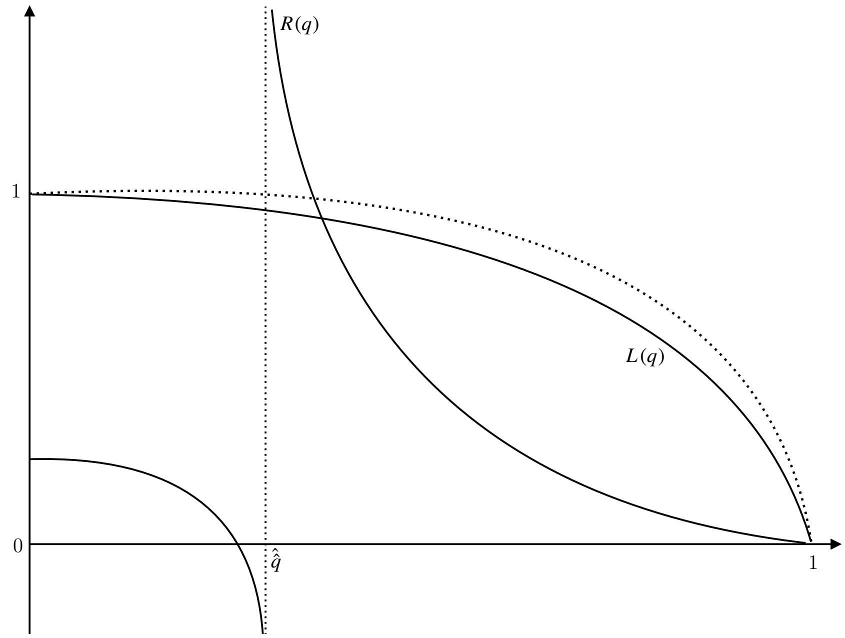


Figure C.2: Effect of an increase of $\theta$ on $q$

To summarize, an increase in $\theta$ increases growth directly, and indirectly due to a decrease in the extraction rate $q$.

## Appendix D. Proof of Proposition 4

- A BGP implies a constant extraction rate. Thus, equations (26) and (27) imply $\tilde{\mu}_x = \tilde{\mu}_m = 1 - \tilde{q}$.
- Since emissions are modeled as a linear function of extracted resources $\tilde{\mu}_e = \tilde{\mu}_x$.
- The low of motion of technology implies $\mu_a = 1 + a$.
- The ratio of the intergenerational optimality condition (30) evaluated in $t + 1$ and in $t$ gives on the BGP $\tilde{\mu}_c = \tilde{\mu}_d$.
- The BGP ratio of the production function (22) in $t + 1$ and in $t$ implies $\tilde{\mu}_y = (1 + a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{\nu-\theta}$.
- Taking the ratio of (23) in $t + 1$ and $t$, it appears that a BGP requires $\tilde{\mu}_c = \tilde{\mu}_k$.

- The intertemporal resources allocation optimality condition (32) evaluated on the BGP gives:

$$(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{\nu-\theta-1} - 1 + \delta = \alpha A_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^{\nu} e_{t+1}^{-\theta}$$

  Taking the ratio of the last expression in $t+1$ and in $t$ and evaluating on the BGP, one obtain:

$$1 = (1+a)\tilde{\mu}_k^{\alpha-1}\tilde{\mu}_x^{\nu-\theta}$$

  Thus:

$$\tilde{\mu}_k = (1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{\nu-\theta} = \tilde{\mu}_y$$

  and

$$\tilde{\mu}_k = (1+a)^{\frac{1}{1-\alpha}}(1-\tilde{q})^{\frac{\nu-\theta}{1-\alpha}}$$

- Using (30), (31) and (32), one have $(1+a)\tilde{\mu}_k^{\alpha}\tilde{\mu}_x^{\nu-1-\theta} = (1+\gamma)\tilde{\mu}_c$. Since $\tilde{\mu}_c = \tilde{\mu}_k$, it can be established that $\tilde{q} = \frac{\gamma}{1-\gamma}$.

## Appendix E. Proof of Proposition 6

Equations (1), (2), (4), (10),(11), (12), (14), (16) and (17) are not modified by the tax. Thus, one can conclude from Appendix A that $\mu_x = \mu_m = \mu_e = 1-q$, $\mu_a = (1+a)$, $\mu_s = \mu_w = \mu_y = \mu_k$ and $\mu_r = 1$. The introduction of the tax and the transfer don't modify the Euler equation (8) or the arbitrary condition between capital and resources (9) in households investment decisions. Thus $\mu_p = 1 + r_{t+1}$ and $\mu_c = \mu_d$.

- If the resources price and the tax level increase at the same rate, taking the ratio of equation (37) in $t+1$ and in $t$ gives:

$$\frac{\mu_p(p_t+\tau_t)}{p_t+\tau_t} = (1+a)\mu_k^{\alpha}\mu_x^{\nu-\theta-1}$$

  Thus, one can conclude that:

$$\mu_p = \frac{\mu_y}{\mu_x}$$

- The IBC with the transfer writes:

$$w_t + g_t = c_t + \frac{d_{t+1}}{1+r_{t+1}} - \frac{p_{t+1}m_t}{1+r_{t+1}} + p_t m_t$$

Substituting equations (8), (16) and (38) and taking into account that the resources price increases at the interest rate, one can writes:

$$\beta A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} + \tau_t x_t = \frac{c_t(2+\rho)}{1+\rho}$$

Taking the ratio of the last expression and evaluating on the BGP, one obtain:

$$\mu_c = \frac{\mu_k \beta A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} + \mu_p \mu_x \tau_t x_t}{\beta A_t k_t^{\alpha} x_t^{\nu} e_t^{-\theta} + \tau_t x_t}$$

Since $\mu_p \mu_x = \mu_k$, one have $\mu_c = \mu_k$.

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