



**HAL**  
open science

## An Economic Analysis of Violent Crim

Petros Sekeris, Tanguy van Ypersele

► **To cite this version:**

Petros Sekeris, Tanguy van Ypersele. An Economic Analysis of Violent Crim. *Revue d'économie politique*, 2020, 130 (6), pp.975-999. 10.3917/redp.306.0133 . hal-03607608

**HAL Id: hal-03607608**

**<https://amu.hal.science/hal-03607608>**

Submitted on 14 Mar 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# An Economic Analysis of Violent Crim

Petros G. Sekeris<sup>1</sup> Tanguy van Ypersele<sup>2</sup>

In this article we propose a theoretical model to better comprehend the effect of gun laws on violent property crime. We assume that a violent encounter between a criminal and a victim is costly to both, and we uncover two types of equilibria: a pure strategy violent equilibrium and a mixed strategy equilibrium where the criminal is deterred with strictly positive probability. The effect of a relaxation of gun laws is shown to be conditional on both initial gun laws and on the relative improvement of the victims' defense capacity relative to the criminals' offense capacity. We uncover a potentially inverted U-shaped relationship between gun laws leniency and investments in violent activities which helps reconciling seemingly contradictory empirical findings.

**Crime – Gun Laws – Deterrence**

## *Une analyse économique des crimes violents*

Dans cet article nous développons un modèle théorique permettant de mieux comprendre l'effet des lois sur le contrôle des armes à feu sur les crimes violents liés à l'appropriation de propriété privée. Nous supposons qu'une rencontre violente est coûteuse aussi bien pour un criminel que pour sa victime et nous identifions deux types d'équilibres : un équilibre violent en stratégie pure et un équilibre en stratégie mixte dans le cadre duquel le criminel est dissuadé avec une probabilité strictement positive. L'effet d'un assouplissement de la législation sur les armes à feu dépend du niveau des restrictions légales initiales en la matière, ainsi que du gain d'efficacité relatif de la victime et du criminel. Nous mettons au jour une possible relation en U-inversé entre la souplesse des lois sur les armes à feu et les investissements en activités violentes, ce qui permet de réconcilier des résultats empiriques antérieurs contradictoires.

**Crime – Contrôle des armes à feu – Dissuasion**

*JEL Classification : C72, D74, P48*

---

1. Montpellier Business School (MBS), 2800 Avenue des Moulins, 34185 Montpellier, France. email: p.sekeris@montpellier-bs.com. Petros Sekeris is member of the Entrepreneurship and Innovation Chair, which is part of LabEx Entrepreneurship (University of Montpellier, France) and funded by the French government (Labex Entreprendre, ANR-10-Labex-11-01).

2. Aix-Marseille Univ., CNRS, AMSE, Marseille, France. AMSE-MEGA, 424 chemin du Viaduc, 13080 Aix-en-Provence. France. e-mail: tanguy.vy@univ-amu.fr.  
This work was supported by French National Research Agency Grants ANR-17-EURE-0020.

# 1. Introduction

Crime rates are highly heterogenous across time and space. Besides the recorded important cross-country differences in criminality, there are notable differences across regions, neighborhoods, or even between rural and urban areas (e.g. Munasib *et al.* [2018]). Scholars have made important progress in identifying the causes of this variability by pointing at several factors, including wealth levels, education, or even inherited institutional contexts (Couttenier *et al.* [2017]). The one aspect that has drawn the most attention however, is probably the institutional legal framework and its likely interactions with other explanatory factors (e.g. income, education), with the seminal contribution to the field being the work of Becker [1968].

In that context, and given the American exception in terms of high crime rates and mild gun legislations, a large body of the literature has focused on the effects of gun laws on crime. Figures are staggering: firearm-related incidents account for an average of 36 homicides daily in the U.S. (Edwards *et al.* [2018]), and for more than 33,000 deaths (including suicides) per year in the U.S. (Luca *et al.* [2017]). In 2017 the rate of homicides per 100,000 people in the U.S. was 5.3, when the analogous figures were 1.3 for France, 1.2 for the United Kingdom, 1 for Germany, and 1.8 for Canada (World Bank). Given that homicide rates in the U.S. are closer to the ones of Ecuador, Suriname or Ukraine than to other OECD countries, this American peculiarity has been associated by many to the ease of obtaining guns in the U.S., although no causal link can be drawn without deeper investigation.

The empirical literature on the topic is quite divided as can be witnessed by the titles of two major contributions to the field: "More Guns, Less Crime" by Lott [1998] and "More Guns, More Crime" by Duggan [2001]. The theoretical argument of the "pro-gun" camp rests in their deterrent effect on criminals, while the "anti-gun" proponents point at both the fact that legally purchased guns will often end up in criminal hands (through theft and the black market), while also underlying the increased likelihood of pulling a gun in the face of uncertainty (see Schelling [1960], p. 207). In terms of empirical findings, some scholars show that the *right-to-carry* (RTC) laws allowing gun holders to conceal guns lead to a reduction of crime, thereby pointing at a deterring effect of flexible gun laws (Lott and Mustard [1997]). On the other hand, Duggan [2001], Ayres and Donohue [2003], Cook and Ludwig [2006], or Schneider [2018] propose empirical evidence disproving this deterrence thesis. Studies on the *Stand Your Ground* law which allows the use of deadly force even when retreat is a viable option have shown that homicides tend to increase as a consequence of such laws (Cheng and Hoekstra [2013]; Munasib *et al.* [2018]). Tannenbaum [2020] exploits the exogenous disclosures of gun owners (with maps geolocating the gun owners) to show that criminals are not deterred from knowing that potential victims carry guns. Luca *et al.* [2017] and Edwards *et al.* [2018] study the effect of mandatory waiting periods during the purchase of guns on crime and find contrasted results on homicides since the former show a reducing effect, while the latter find no effect.

These seemingly contradictory results may nevertheless be reconciled. Manski and Pepper [2018] show how findings on the effect of RTC laws are highly dependent on empirical modeling decisions, since results may be reversed using the same database, or when focusing on different points in time. Similarly, Durlauf *et al.* [2016] show that imposing homogeneity in the effect of RTC laws on crime biases the results, thus calling for particular attention to differences in time and space. But neither Durlauf *et al.* [2016], nor Manski and Pepper [2018] are actually able to determine which model is the most appropriate since such a question is inherently a theoretical one, and the on-going debate therefore points to a need to better understand the theoretical consequences of gun laws on crime.

Prior to highlighting our results, it is important to distinguish various types of observed gun-related violence. While the focus of this paper is exclusively on property crime (burglaries, pickpocketing, theft), violent acts like domestic crime, arguments escalating beyond control, suicide (Vitt *et al.* [2018]), or even accidental deaths (Levine and McKnight [2017]) are all likely to be influenced by the availability of guns, and ultimately therefore by the institutional context governing firearms. Edwards *et al.* [2018] for instance, demonstrate empirically that delaying the purchase of guns reduces both homicides and suicides, while Koenig and Schindler [2019] establish a causal link between such waiting period and actual (as opposed to intentions of) guns purchases, in turn leading to a statistically significant drop in homicides. Moreover, Koenig and Schindler present some evidence that it is essentially “impulsive crimes” that are being avoided by this delay in guns’ acquisition. Yet, such non-economic crimes, whether perpetrated with a fire-arm or not, are often rooted in behavioral biases and irrational decision-making processes, as exemplified in Card and Dahl [2011] or Tiihonen *et al.* [2017]. We instead focus on rational decisions surrounding the protection and appropriation of valuable property, that is contexts where behavioral biases are less likely to play a deterministic role.

We propose a theoretical setting that enables us to further our understanding of the effect of gun laws on property crime, and to provide guidance for future empirical research. The model involves a victim and a potential criminal, with the latter attempting to violently steal the former’s property. We show that two types of equilibria can emerge in such settings: a pure strategy equilibrium where the victim is systematically expecting the criminal to steal his property, and a mixed strategy equilibrium where the latter does so with some strictly positive probability, although possibly a small one. The effect of more lenient gun laws is shown to be conditional on both initial conditions and the relative improvement of victims’ defense capacity to the criminals’ offense capacity. More lenient gun laws are assumed to increase both the victims’ and criminals’ capacity for violence. If the *relative* capacity for violence remains unchanged, we show that (i) the aggregate effort of victims and criminals in violence decrease and the probability of an armed confrontation taking place drops, thus yielding support to the pro-gun laws underlining the deterrent effect of lenient gun laws. Quite counter-intuitively, if the relative efficiency of the victim increases, aggregate expenditures in violent activities increase for low efficiency levels of the victim, and they decrease for high efficiency levels of the victim, that is when the criminal is deterred (probabilistically). The former result is well-

known in the contest literature. The increase in aggregate effort is due to the strategic complementarity of the two players' expenditures in the pure-strategy confrontation equilibrium: the now more efficient victim is incentivized to increase his defensive effort, thereby triggering a strategic response of the criminal who increases his own effort too. On the other hand, when the victim is already quite efficient, the criminal refrains from systematically attacking the victim because the latter can indeed afford deterring the criminal at a contained cost. Further increasing the victim's relative capacity then improves the deterrence ability of the victim.

Our simple theoretical framework therefore underlines the importance of clearly delineating the initial relative confrontation abilities of victims and criminals, while also accounting for the expected evolution in relative efficiency following a change in the gun laws legislation. Given the non-linear relationship between access to weapons and crime, our predictions may help reconciling seemingly contradictory results, while also drawing the attention on the need to focus both on the *extensive margin* as well as on the *intensive margin* when studying the effect of gun laws on crime. Indeed, while at the margin a more lenient gun law may indeed have a deterring effect on crime (intensive margin), a drastic toughening of gun laws may nevertheless lead to a larger drop in crime rates, or at least on their intensity (extensive margin).

We next review the related theoretical literature in Section 2, before presenting our model (Section 3) and comparative statics results (Section 4). Section 5 concludes.

## 2. Literature review

Becker's [1968] model constitutes the seminal theoretical contribution to the economics of crime literature. His study focused on the impact of institutional constraints on the equilibrium crime levels, by adopting a "market" approach to modeling crime since economic agents are seen as unable to strategically influence each other in such violent interactions. Likewise, all the early literature on the topic favored a market approach that studied individual incentives in the absence of strategic thinking (*e.g.* Ehrlich [1981, 1998]). Subsequent works, however, adopted a game theoretic approach that enabled us to better comprehend the victims' strategies, and more specifically their incentives to deter crime being directed against them. Likewise, our model considers strategically interacting law-abiding citizens and criminals. Several studies on crime have adopted a game-theoretic approach, but with a different focus to ours. Donohue and Levitt [1998] and Mialon and Wiseman [2005] concentrate on the relationship between the lethality of crime (Donohue and Levitt [1998]) or the relaxation of gun laws (Mialon and Wiseman [2005]) and the likelihood of crime being committed by allowing players to take binary decisions of committing a crime of given (uncertain) intensity or not. Donohue and Levitt [1998] point at a non-linear effect of lethality on the probability of crime, while Mialon and Wiseman [2005] find an inverted U-shape relationship between gun laws flexibility and

victims' well-being. We uncover a similar mechanism to Donohue and Levitt [1998] and we equally identify non-linearities in the victims' well-being as Mialon and Wiseman [2005]. We nevertheless propose a richer framework since, unlike these two studies, we allow players to choose their effort amongst a continuum of potential effort levels rather than restricting this decision to a binary choice between committing or not a crime of a predetermined intensity. Besides giving rise to a richer configuration of equilibria (pure strategy confrontation equilibrium and mixed strategy deterrent equilibrium), our approach also enables us to study the endogenous variation in the intensity of violence rather than merely looking at the likelihood of confrontation or the players' (fixed) utility.

Other models of crime have focused on the role of incomplete information on the victim's belongings (Lacroix and Marceau [1995]), on the role of observability of private protection on the efficiency of the private protection decisions (Hotte and van Ypersele [2008]), on the strategic interaction between criminals and authorities (Marceau [1997]), or between victims' protection investments and government's policing levels (Guha [2003]; Hickey *et al.* [2019]), or on the study of optimal sanctions in such settings (Bac [2010]). Our focus is on the strategic interaction between victims and criminals in a complete information setting, and on the effect of gun laws in such frameworks.

We model the violent confrontation between a potential victim and the criminal as a two player contest *a la* Tullock [1980] where two strategies survive at equilibrium: a confrontation strategy and a deterrence strategy. While a very vast literature has adopted similar setups to model violent encounters (*e.g.* Hirshleifer [1989]; Skaperdas [1992], for early contributions), only a smaller number of contributions have scrutinized the concept of deterrence in such settings. Grossman and Kim [1995] introduce the concept in the contest literature by distinguishing offensive from defensive investments, and by imposing a sequential setup with defensive – that is deterrent – investments taking place first. Our setting considers a single type of investment in effort for violence as well as a simultaneous timing. In settings similar to ours, other works have established the existence of deterrent equilibria, and have demonstrated as is the case in our paper too – that no pure strategy deterrent equilibrium can exist (Neary [1996]; Jacobsson [2009]; and De Luca and Sekeris [2013]). Our contribution is the first however to propose a full characterization of the mixed strategy deterrent equilibrium, thus enabling us to derive richer predictions.

## 3. A two-stage game of crime

### 3.1. The setting

We consider a two-stage game with two players: a victim labelled  $v$ , and a criminal labelled  $c$ . In stage 1, before the two agents encounter, the victim

decides its protection investment  $x_v \in \mathbb{R}_+$ , aimed at protecting its property  $W$ , and the criminal chooses simultaneously his capacity for violence,  $x_c \in \mathbb{R}_+$ , that can be used to appropriate the victim's property.<sup>3</sup> In stage 2 the criminal takes his criminal decision,  $d$ , of attempting to violently appropriate the victim's wealth ( $d=A$ ), or not ( $d=N$ ).

If the criminal refrains from attacking the victim, the players' respective payoffs are given by the following expressions:

$$U_v^N = W - x_v, \quad [1]$$

and

$$U_c^N = -x_c. \quad [2]$$

The confrontation technology is described by a Tullock contest success function (Tullock 1980), such that the probability that the criminal appropriates  $W$  is given by  $\pi_c(x_c, x_v) = \frac{\alpha_c x_c}{\alpha_c x_c + \alpha_v x_v}$ , with the complementary probability designating the likelihood the criminal fails stealing the wealth from the victim. The parameters  $\alpha_i$  represent the efficiency of the opponents' (viz. criminal and victim) capacity for violence, with  $\alpha = \alpha_v / \alpha_c$ , so that the above probability can be written as  $\pi_c(x_c, x_v) = \frac{x_c}{x_c + \alpha x_v}$ . In line with the real world we impose that  $\alpha < 1$ . The appropriation act bears a cost both to the victim and to the criminal that is proportional to the weapons' lethality. For simplicity, we shall assume that the cost of confrontation,  $k_i$ ,  $i = \{c, v\}$  is a function of the efficiency parameters,  $\alpha_c$  and  $\alpha_v$ , such that  $\partial k_i(\alpha_p, \alpha_{-i}) / \partial \alpha_p > 0$  and  $\partial k_i(\alpha_p, \alpha_{-i}) / \partial \alpha_{-i} > 0$ . We thus assume that the cost of violence, which is a short notation capturing both the lethality of the injury and its likelihood, increases directly when increasing one's opponent's fighting efficiency, but also indirectly when increasing one's own efficiency because of the increased likelihood that the opponent will commit a violent act in that instance.<sup>4</sup> Moreover, since  $\alpha < 1$ , we assume that  $k_v > k_c, \forall \alpha$ . For the remainder of this section, we use the short notation  $k_i$  for clarity.

Summarizing this information, the payoff of a victim under violent appropriation is given by:

$$U_v^A = \frac{\alpha x_v}{x_c + \alpha x_v} W - x_v - k_v, \quad [3]$$

3. Contrary to Grossman and Kim's [1995] seminal contribution to deterrence in conflictual settings that considers sequential arming, we assume that players take their decisions simultaneously. Bear in mind that our assumed timing captures the non-observability of the players' respective investments at the time of deciding how to act. Besides the obvious fact that victims cannot observe the criminals' investments in violence, this assumption also reflects the fact that the criminal is very imperfectly aware of the victim's effort in violence because most of the investments, and especially the ones relating to gun-ownership and associated training, are not observable.

4. There is ample evidence that pro-gun laws have led to increase murders and nonnegligent manslaughter of both assailants and victims, e.g. Cheng and Hoekstra [2013], Kalesan *et al.* [2016], and McClellan and Tekin [2017].

While the criminal's payoff then reads as:

$$U_c^A = \frac{x_c}{x_c + \alpha x_v} W - x_c - k_c. \quad [4]$$

We solve for the game's subgame perfect equilibria, and thus proceed backwardly.

## 3.2. Equilibrium analysis

### Stage 2

In stage 2 the criminal decides whether or not to commit the crime. Accordingly, he is deterred from committing the crime, and thus prefers implementing his idle strategy ( $N$ ), if the utility he would obtain is higher than the expected utility of violent appropriation ( $A$ ). This will be the case when the following inequality holds:

$$U_c^N(x_c, x_v) = -x_c \geq \frac{x_c}{x_c + \alpha x_v} W - x_c - k_c = U_c^A(x_c, x_v) \quad [5]$$

$$\Leftrightarrow x_v \geq x_v^{det} = \frac{x_c}{\alpha} \left[ \frac{W}{k_c} - 1 \right].$$

### Stage 1

In stage 1, the criminal chooses the utility-maximizing capacity for violence,  $x_c$ , given constraint (5). The criminal has two potential strategies: he can either invest in capacity for violence in anticipation that he will commit a crime, or else he can optimize anticipating that he will not commit the crime. Using superscripts to describe the players' strategies, we accordingly call the former strategy the *appropriation strategy* that – in line with Stage 2 actions – we label  $A$ , and the latter the *idle strategy* that we label  $N$ .

We first focus on the appropriation strategy, which consists in maximizing  $U_c^A(x_c, x_v)$  conditional on (5) not being satisfied, *i.e.* conditional on the criminal not being deterred. This problem reads as:

$$\max_{x_c} U_c^A = \max_{x_c} \left\{ \frac{x_c}{x_c + \alpha x_v} W - x_c - k_c \right\} \text{ s.t. } x_v < \frac{x_c}{\alpha} \left[ \frac{W}{k_c} - 1 \right] \quad [6]$$

Optimizing, we obtain the appropriation best response of the criminal:

$$x_c^A(x_v) = \alpha x_v \left[ \left[ \frac{W}{\alpha x_v} \right]^{1/2} - 1 \right] \text{ s.t. } x_v < \frac{x_c}{\alpha} \left[ \frac{W}{k_c} - 1 \right] \quad [7]$$



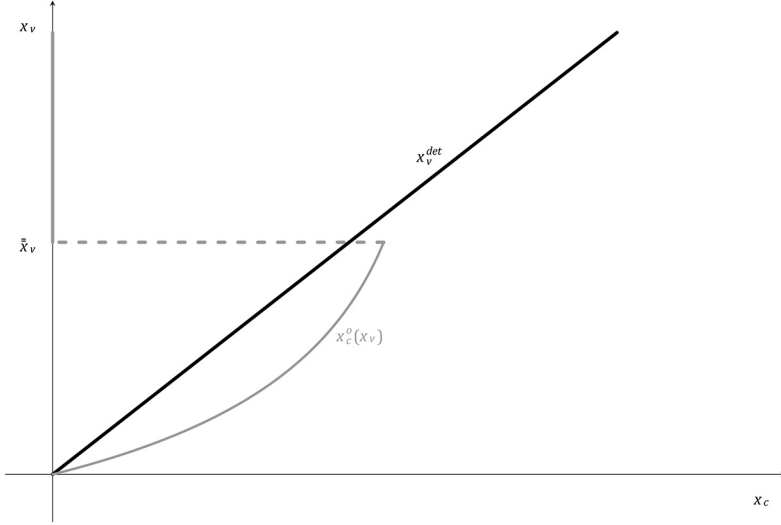
We next need to verify when the constraint in (7) is satisfied. Replacing  $x_c^A(x_v)$  in Condition (5), we deduce that the condition is not violated when:

$$x_v < \bar{x}_v = \frac{[W - k_c]^2}{\alpha W}. \quad [8]$$

In summary then, the appropriation strategy is defined over the interval  $x_v \in [0, \bar{x}_v]$ , and yields a utility of:

$$U_c^A(x_v) = [W^{1/2} - \alpha x_v^{1/2}]^2 - k_c. \quad [9]$$

**Figure 1. Best response function of the criminal.**



The idle strategy on the other hand consists in maximizing  $U_c^N$  as given by (2) subject to Condition (5) being violated. Since it is immediate to see that  $x_c^N(x_v) = 0$ , it follows that this strategy is defined for any  $x_v > 0$ , and it yields zero utility.

Combining the above elements, we can now deduce the optimal strategy of the criminal. Observe that  $U_c^A(0) = W > 0$ . Moreover,  $U_c^A(x_v)$  is strictly decreasing in  $x_v$ , with  $U_c^A(x_v) = 0$  when  $x_v = \bar{x}_v = \frac{[W^{1/2} - [k_c]^{1/2}]^2}{\alpha}$ . Thence  $U_c^A(x_v) > U_c^N$  for  $x_v < \bar{x}_v$ . Moreover, observe that (by construction) in  $x_v = \bar{x}_v$ ,  $U_c^A(\bar{x}_v) = -x_c(\bar{x}_v) < 0 = U_c^N(\bar{x}_v)$ , thus implying that  $\bar{x}_v > \bar{x}_v$ .

Summing up, the criminal's best response is defined as:

$$x_c^o(x_v) = \begin{cases} \alpha x_v \left[ \left[ \frac{W}{\alpha x_v} \right]^{1/2} - 1 \right] & \text{if } x_v < \bar{x}_v \\ 0 & \text{otherwise} \end{cases} \quad [10]$$

To fix ideas, we graphically depict  $x_c^o(x_v)$  on Figure 1. More specifically, we represent  $x_v$  on the  $y$ -axis, thus requiring a reverse reading of  $x_c^o(x_v)$ . According to the above description of  $x_c^o(x_v)$ , the reaction function of the criminal consists in following his appropriation strategy up to the threshold  $\bar{x}_v$ , after which it is optimal to implement the idle strategy. Graphically, the best response of the criminal is thus increasing and convex in the victim's expenditures levels up to  $\bar{x}_v$ , thence reflecting the strategic complementarity of expenditures in violence. For higher values of the victim's protection investment, however, the criminal prefers to totally refrain from investing in violence, and renounces to commit the crime as well.

We now proceed likewise for the victim. From the preceding analysis we know that the victim's utility is given by:

$$U_v = \begin{cases} U_v^A = \frac{\alpha x_v}{x_c + \alpha x_v} W - x_v - k_v & \text{if } \alpha x_v < x_c \left[ \frac{W}{k_c} - 1 \right] \\ U_v^N = W - x_v & \text{otherwise} \end{cases}, \quad [11]$$

where the constraint already reflects an optimization result, since, for any expenditure level  $x_v$ , the victim always prefers avoiding the confrontation and retaining the entire prize  $W$ , rather than having to face the criminal. If the victim expects the criminal to follow his appropriation strategy, the best response of the victim is obtained by maximizing  $U_v^A(x_c, x_v)$  with respect to  $x_v$ , which gives us:

$$x_v^A(x_c) = \frac{x_c}{\alpha} \left[ \left[ \frac{\alpha W}{x_c} \right]^{1/2} - 1 \right] \text{ if } \alpha x_v < x_c \left[ \frac{W}{k_c} - 1 \right]. \quad [12]$$

Substituting  $x_v^A(x_c)$  in the constraint, we deduce that the constraint is satisfied when:

$$x_c > \bar{x}_c = \frac{\alpha k_c^2}{W}. \quad [13]$$

The unique value  $\bar{x}_c$  therefore defines a threshold such that the confrontation strategy is defined over the interval  $x_c \in [\bar{x}_c, +\infty[$  and yields a utility of:

$$U_v^A(x_c, x_v) = \frac{[ [\alpha W]^{1/2} - x_c^{1/2} ]^2}{\alpha} - k_v. \quad [14]$$

The deterrence strategy, on the other hand, consists in maximizing  $U_v^N$  as given by (1) subject to Condition (5) being violated. In other words, the victim then aims at minimizing his capacity for violence expenditures such that  $x_v \geq x_v^{det}$ , and the solution to that problem is obviously given by  $x_v = x_v^{det}$ .

The associated utility of the deterrence strategy is then equal to:

$$U_v^N(x_c, x_v) = W - \frac{x_c}{\alpha} \left[ \frac{W}{k_c} - 1 \right] \quad [15]$$

Combining the above elements, we can now deduce the optimal strategy of the victim. Over the interval  $x_c \in [0, \bar{x}_c]$  the victim always opts for the deterrence strategy, while for values of  $x_c > \bar{x}_c$ , he opts for the confrontation strategy if  $U_v^A(x_c, x_v) > U_v^N(x_c, x_v)$ . Comparing  $U_v^N(x_c, x_v)$  to  $U_v^A(x_c, x_v)$ , we obtain:

$$U_v^A(x_c, x_v) > U_v^N(x_c, x_v) \Leftrightarrow \left[ [\alpha W]^{1/2} - x_c^{1/2} \right]^2 - \alpha [k_v + W] + x_c \left[ \frac{W}{k_c} - 1 \right] > 0 \quad [16]$$

Which simplifies to:

$$-2 [\alpha W x_c]^{1/2} - \alpha k_v + x_c \frac{W}{k_c} > 0 \quad [17]$$

Since the above expression describes a parabola, upon deriving the roots, we obtain that  $U_v^A(x_c, x_v) > U_v^N(x_c, x_v)$  for  $x_c \in ]-\infty, x_c^- [ \cup ] \bar{x}_c, +\infty [$ , with  $x_c^- = \frac{\alpha}{W} [k_c]^2 \left[ 1 - \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right]^2$  and  $\bar{x}_c = \frac{\alpha}{W} [k_c]^2 \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right]^2$ . Notice, however, that  $x_c^- < 0$ , while  $\bar{x}_c > \bar{x}_c$ . We thence conclude that the deterrence strategy is optimal for any  $x_c \leq \bar{x}_c$ .

The victim's equilibrium strategy  $x_v^o(x_c)$  is therefore defined as:

$$x_v^o(x_c) = \begin{cases} \frac{x_c}{\alpha} \left[ \frac{W}{k_c} - 1 \right] & \text{if } x_c \leq \bar{x}_c \\ \frac{x_c}{\alpha} \left[ \left[ \frac{\alpha W}{x_c} \right]^{1/2} - 1 \right] & \text{otherwise} \end{cases} \quad [18]$$

Notice lastly, that  $x_v^A(\bar{x}_c) < x_v^N(\bar{x}_c)$  since by comparing these two expressions we have:

$$\begin{aligned} x_v^A(\bar{x}_c) &< x_v^N(\bar{x}_c) \\ \Leftrightarrow \left[ \frac{\alpha W}{\bar{x}_c} \right]^{1/2} &< \frac{W}{k_c} \\ \Leftrightarrow \frac{[k_c]^2 \alpha}{W} &< \bar{x}_c = \frac{\alpha [k_c]^2}{W} \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right]^2 \\ \Leftrightarrow 1 &< \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right] \end{aligned}$$

In other words, the victim's reaction function jumps downwards at the discontinuity point  $x_c = \bar{x}_c(x_v)$ .

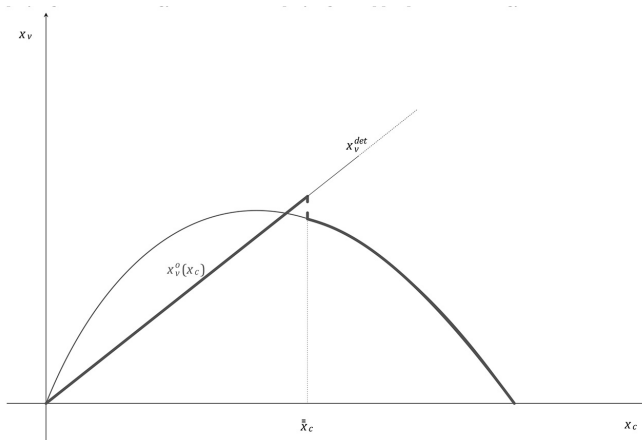
We depict on Figure 2 the victim's best response function which consists in deterring the criminal for values smaller to  $\bar{x}_c$  and in confronting the criminal for values higher to this threshold.

Having derived the players' best responses, we now focus on the game's equilibria. Consider first the following lemma:

**Lemma 1** *The game admits at most one pure strategy equilibrium, where the crime is always committed.*

**Proof.** Observe first that if the victim plays his deterrence strategy at equilibrium, there can be no pure strategy equilibrium since by definition of  $x_v^{det}(x_c)$ , the criminal's best response is to either play his idle strategy whereby  $x_c^N(x_v) = 0$ , or his appropriative strategy  $x_c^A(x_v)$ . In the former case,  $x_v^{det}(0) = 0$ , and because of the CSF's discontinuity in  $(0,0)$ , the criminal's best response is to produce a strictly positive effort  $x_c^A(0) > 0$ . In the latter case, it is necessary that  $x_c^A(x_v^{det}(x_c)) > x_v$ , for otherwise  $x_v > \bar{x}_v$ , and the criminal is deterred. Consider next a potential equilibrium where the criminal plays his appropriative strategy and the victim his confrontation strategy. Observe that the two players' best responses being strictly concave, they can cross only twice, in  $(0,0)$  and for a combination of some strictly positive values. The former crossing is not a fixed point, however, because of the discontinuity of the CSF in  $(0,0)$ . The latter crossing is a fixed point and could constitute a pure strategy of the game. Lastly, the criminal playing his idle strategy and the victim playing the appropriative strategy cannot be an equilibrium either since  $x_c^N(x_v) = 0$ , and  $x_v^A(0) = 0$ . Yet, because of the CSF's discontinuity in  $(0,0)$ , the criminal will always have incentives to deviate by playing his appropriative strategy if  $x_v = 0$ .

**Figure 2.** Best response function of the victim.



We next derive the two equilibria the game admits: the (unique) pure strategy equilibrium, and the mixed strategy equilibrium.

### 3.2.1. Pure strategy equilibrium

Consider first the unique pure strategy equilibrium where the criminal always commits the crime.

**Result 1** *The game admits a unique equilibrium such that  $x_c^* = x_v^* = \frac{W\alpha}{[\alpha + 1]^2}$  and  $d^* = A$  if and only if*

$$\frac{W}{k_c} > [\alpha + 1] \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right].$$

**Proof.** From Lemma 1 we know that the game admits only one pure strategy equilibrium. Notice first that the only fixed point where the criminal plays his appropriate strategy,  $x_c^A(x_v)$ , and the victim plays his confrontation strategy,  $x_v^A(x_c)$ , is such that  $x_c^* = x_v^* = \frac{W\alpha}{[\alpha + 1]^2}$ . Observe next that the condition in Result 1 is tantamount to imposing  $x_c^* > \bar{x}_c$ , so that it is optimal for the victim to play his confrontation strategy in response to  $x_c^*$ . Next, we demonstrate that  $x_c^* > \bar{x}_c \Rightarrow x_v^* < \bar{x}_v$ , which, in turn, implies that the criminal's best response to  $x_c^*$  is given by his appropriate strategy. Observe indeed that  $x_v^* < \bar{x}_v$  can be written as  $\frac{W}{k_c} > [\alpha + 1]^2$ . Using the condition of Result 1, it is therefore sufficient to show that:

$$[\alpha + 1] \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right] > [\alpha + 1]^2,$$

Or,

$$\left[ 1 + \frac{k_v}{k_c} \right]^{1/2} > \alpha,$$

which follows straightforwardly from  $\alpha_v \leq \alpha_c \Leftrightarrow \alpha \leq 1$ . This ensures that  $x_c^* = x_v^* = \frac{\alpha W}{[\alpha + 1]^2}$  is a pure strategy equilibrium of this game.

To rule out the existence of other – mixed strategy – equilibria, observe that over the interval  $x_c \in [x_c^*, +\infty[$ ,  $x_v^*(x_c) = x_v^A(x_c)$ . We can easily show that  $\frac{\partial x_v^*(x_c)}{\partial x_c} < 0$  for any  $x_c > x_c^*$ , so that  $x_v^*(x_c) < x_v^*$  over that range of parameters. And since  $x_c^*(x_v) = x_c^A(x_v)$  for  $x_v < x_v^*$ , this rules out the existence of equilibria

in that specific space since there exists a unique pure strategy equilibrium. Next, over  $x_c \in [0; x_c^*]$  there can be no equilibrium if the maximal value of  $x_v^*(x_c)$  is strictly smaller than  $\bar{x}_v$ . Let us proceed by contradiction. Since  $x_v^* < \bar{x}_v$ , assume that the highest value that  $x_v^N(x_c)$  can take over that interval is larger than  $\bar{x}_v$ . Since  $x_v^N(x_c)$  is monotonically increasing in  $x_c$ , the highest value it can attain lies at the discontinuity, *i.e.* in  $x_c = \bar{x}_c$ . We are then assuming that  $x_v^{det}(\bar{x}_c) > \bar{x}_v$ . Using the notation  $Y = [1 + [1 + k_v/k_c]^{1/2}]^2$  and  $x^{det} = x_v^{det}(\bar{x}_c)$  this implies:

$$\frac{\alpha k_c^2}{W} Y \frac{1}{\alpha} \left[ \frac{W}{k_c} - 1 \right] > \frac{[W^{1/2} - k_c^{1/2}]^2}{\alpha},$$

And after simplifications:

$$\left[ \frac{\left[ \frac{W}{k_c} \right]^{1/2} + 1}{\left[ \frac{W}{k_c} \right]^{1/2} - 1} \right] \alpha Y > \frac{W}{k_c}.$$

Bearing in mind the parameter restriction for the result to hold true, it is also necessary that the following restriction be satisfied:  $\frac{W}{k_c} > [\alpha + 1]Y$ . Combining those two conditions then, we reach a contradiction if the following inequality cannot be satisfied:

$$\left[ \frac{\left[ \frac{W}{k_c} \right]^{1/2} + 1}{\left[ \frac{W}{k_c} \right]^{1/2} - 1} \right] \alpha Y > [\alpha + 1]Y.$$

Re-arranging and simplifying, this expression reduces to:

$$\frac{W}{k_c} < [2\alpha + 1]^2.$$

Simplifying we obtain:

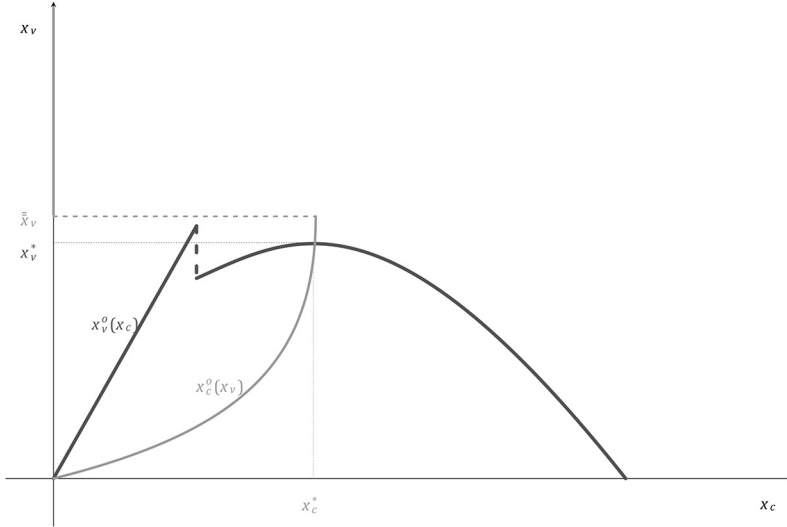
$$[1 + k_v/k_c] < 2\alpha$$

And since  $k_v \geq k_c$  and  $\alpha \leq 1$ , we reach a contradiction.

We represent graphically this pure strategy equilibrium on Figure 3.

We next complete the characterization of the equilibrium by inquiring into the parameter restrictions violating the condition for obtaining the (unique) confrontation pure strategy equilibrium presented in Result 1. Our second result considers that the discontinuity in the victim's best response function occurs for larger values than the ones compatible with a confrontation pure strategy equilibrium.

**Figure 3. The unique pure strategy equilibrium.**



**Result 2** *The game admits a mixed strategy equilibrium such that  $x_v^* = \frac{[W^{1/2} - k_c^{1/2}]^2}{\alpha}$ ,  $x_c^{N*} = 0$ ,  $x_c^{A*} = k_c^{1/2} [W^{1/2} - k_c^{1/2}]$ , and the probability of the criminal committing the crime is given by  $p^* = \frac{2}{\alpha} \left[ 1 - \frac{k_c^{1/2}}{W^{1/2}} \right]$  if and only if:*

$$\frac{W}{k_c} < [\alpha + 1] \left[ 1 + \left[ 1 + \frac{k_v}{k_c} \right]^{1/2} \right].$$

**Proof.** The constraint in the above result implies that  $\frac{\alpha W}{(\alpha + 1)^2} < \bar{x}_c$ , ruling out the existence of a pure strategy equilibrium. For any  $x_c \geq \bar{x}_c$  the victim's best response is to play its confrontation strategy, whereby there can be no fixed point since this would violate the equilibrium uniqueness in Result 1. For any  $x_c < \bar{x}_c$  the victim's best response is to play his deterrence strategy. Observe next that the criminal's best response exhibits a *single* discontinuity in  $\bar{x}_v$ , irrespective of the location of the discontinuity, *i.e.* whether the discontinuity occurs for values lower or larger than  $\alpha W / [\alpha + 1]^2$ , the value of the optimal capacity for violence at the pure strategy equilibrium identified in Result 1. This implies that there exists exactly one mixed strategy equilibrium where the criminal mixes between his appropriation and idle strat-

egies. The equilibrium capacity for violence of the victim,  $x_v^*$ , must be such that the criminal is indifferent between his two strategies. This reads as:

$$EU_c^A(x_v^*) = EU_c^N(0) \Leftrightarrow [W^{1/2} - [\alpha x_v^*]^{1/2}]^2 - k_c = 0 \Leftrightarrow x_v^* = \frac{[W^{1/2} - k_c^{1/2}]^2}{\alpha}.$$

For the victim to choose this level of capacity for violence at equilibrium, however, he should have no incentives to deviate. The victim's expected utility under a mixed strategy equals:

$$\begin{aligned} EU_v &= p^* EU_v^A(x_c^A(x_v^*), x_v^*, p^*) + (1 - p^*) EU_v^N(x_c^N(x_v^*), x_v^*, p^*) \\ &= p^* \frac{\alpha x_v^*}{\alpha x_v^* + x_c^A(x_v^*)} W - x_v^* - p^* k_v + [1 - p^*] W \end{aligned}$$

Replacing for  $x_c^A(x_v^*)$ , as given by (7) allows us to re-write the above expression as:

$$EU_v = p^* (\alpha W x_v^*)^{1/2} - x_v^* - p^* k_v + [1 - p^*] W$$

The mixed strategy equilibrium then ought to satisfy the following condition:

$$\left. \frac{dEU_v}{dx_v} \right|_{x_v=x_v^*} = 0 = p^* \left[ \frac{\alpha W}{x_v^*} \right]^{1/2} - 1 \left\{ p^* = \frac{2}{\alpha} \left[ I - \frac{k_c^{1/2}}{W^{1/2}} \right] \right\}$$

This mixed strategy equilibrium is represented graphically on Figure (4).

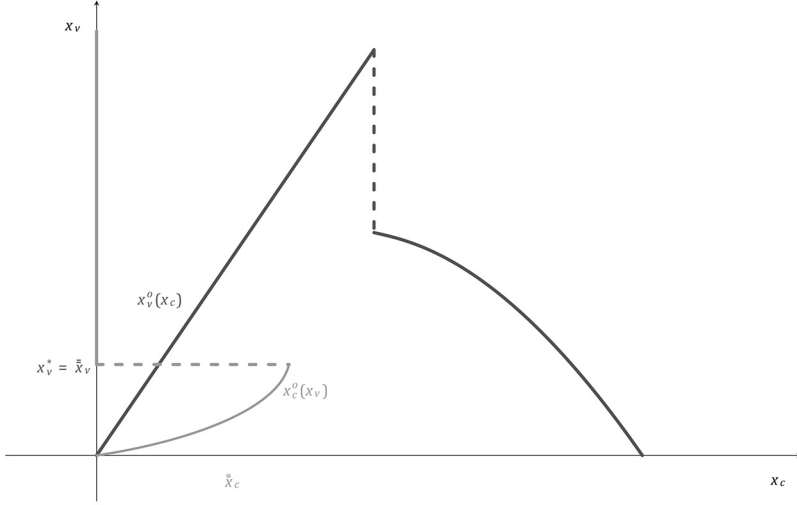
Having characterized the game's equilibria, we now inquire into the effect of the right to carry guns on the game's equilibria.

## 4. The effect of gun laws

In this section we study the effect of gun laws on the game's equilibria, namely on the likelihood of crime being committed, on the equilibrium levels of guns, and on the victim's utility. Relaxing the gun laws increases both  $\alpha_v$  and  $\alpha_c$ , with an indeterminate effect on  $\alpha = \alpha_v / \alpha_c$ . Indeed, while non-criminals may commonly be thought of improving their relative efficiency in a possible encounter with a criminal, more lenient gun laws also imply an easier access to guns by criminals through theft or via a higher supply of guns on the black market (Webster *et al.* [2012]). As such, we shall consider all possible cases to propose a thorough analysis of the possible consequences of gun laws legislations. To sum up, if we denote by  $g$  the easeness of the gun laws, we shall assume that  $\frac{d\alpha_i}{dg} > 0$  with  $i = \{v, c\}$ .



**Figure 4. The mixed strategy equilibrium.**



We proceed sequentially by first assuming that the type of equilibrium does not change following a (marginal) change in the guns' legislation, before inspecting the effect of a change in  $\alpha$  on the type of equilibrium.

#### 4.1. Pure strategy confrontation equilibrium

The equilibrium capacity for violence in the pure strategy confrontation equilibrium is integrally a function of  $\alpha$ . We therefore differentiate the pure strategy equilibrium capacity for violence of player  $i$  ( $i = \{c, v\}$ ) with respect to  $g$  to obtain:

$$\frac{\partial x_i^*}{\partial g} = \frac{\partial \left\{ \frac{\alpha W}{(\alpha + 1)^2} \right\}}{\partial \alpha} \frac{\partial \alpha}{\partial g} = \frac{(1 - \alpha)W}{(\alpha + 1)^3} \frac{\partial \alpha}{\partial g} \quad [19]$$

We thus obtain that in a confrontation equilibrium, if the relative efficiency of the victim rises (decreases) with a gun laws relaxation, both the victim and the criminal (whose equilibrium investments in capacity for violence are equal) are incentivized to increase (decrease) their investments in capacities for violence. This is a widely established result in the literature on contests and on conflict: making the players' fighting efficiency more symmetric incentivizes the less efficient player (*i.e.* the victim) to increase his investment in capacity for violence, thereby triggering a strategic reaction of the criminal whose incentives to invest in capacity for violence are now higher.

We can therefore state the following result:

**Proposition 1** *If the criminal is never deterred at equilibrium, pro-gun laws increase the level of violence if they make victims relatively more efficient, while they decrease the level of violence otherwise.*

Focusing then on the victim's expected utility, it is easy to show that it equals:

$$U_v^A = \frac{\alpha^2}{(1+\alpha)^2} W - k_v(\alpha_c, \alpha_v).$$

Assuming that  $\frac{d\alpha_c}{dg} = \lambda \frac{d\alpha_v}{dg}$ , with  $\lambda > 0$ , we can differentiate the first term of the above expression with respect to  $g$  to obtain:

$$d\left\{\frac{\alpha^2}{(1+\alpha)^2}\right\}/dg = \frac{2\frac{\alpha}{\alpha_c}}{(1+\alpha)^3} \frac{d\alpha_v}{dg} - \frac{2\frac{\alpha^2}{\alpha_c}}{(1+\alpha)^3} \frac{d\alpha_c}{dg}$$

Since  $\frac{d\alpha_c}{dg} = \lambda \frac{d\alpha_v}{dg}$ , the above expression simplifies to,

$$\frac{d\left\{\frac{\alpha^2}{(1+\alpha)^2}\right\}}{dg} = \frac{2\frac{\alpha}{\alpha_c}}{(1+\alpha)^3} \frac{d\alpha_v}{dg} [1 - \alpha\lambda].$$

Observing then that  $d\alpha = \frac{1-\alpha\lambda}{\alpha_c} d\alpha_v$ , the above expression reads as:

$$\frac{d\left\{\frac{\alpha^2}{(1+\alpha)^2}\right\}}{dg} = \frac{2\alpha}{(1+\alpha)^3} \frac{d\alpha}{dg}.$$

It thus follows that the effect of a gun laws relations on the equilibrium utility of the victim is given by:

$$\frac{2\alpha}{(1+\alpha)^3} \frac{d\alpha}{dg} - \frac{dk_v}{dg}.$$

with  $\frac{dk_v}{dg} = \frac{dk_v}{d\alpha_c} \frac{d\alpha_c}{dg} + \frac{dk_v}{d\alpha_v} \frac{d\alpha_v}{dg} > 0$ . The sign of this expression is always negative if  $d\alpha/dg < 0$ , while otherwise it is indeterminate. Indeed, if a gun laws relaxation,  $dg > 0$ , maps in an increase of the criminals' relative efficiency,  $d\alpha/dg < 0$ , the victim will naturally be penalized on a double count by having a lower probability of successfully defending his property and by facing a larger risk of damage. In the opposite case, the victim will see his probability of successfully defending his property go up, at the cost of a larger risk of damage, thus implying an indeterminate effect on his wellbeing.

## 4.2. Mixed strategy equilibrium

We next turn to the mixed strategy equilibrium described in Result 2. Taking the total differential of  $x_v^*$  we obtain:

$$\frac{dx_v^*}{dg} = -\frac{x_v^*}{\alpha} \frac{d\alpha}{dg} - \frac{dk_c}{dg} \frac{[W^{1/2} - k_c^{1/2}]}{\alpha k_c^{1/2}}.$$

We observe that if  $d\alpha/dg > 0$  so that the relative efficiency of the victim increases with more lenient gun laws, then the victim will expend less resources in capacity for violence at equilibrium. This follows from the increased deterrent capacity of the victim in such instances. If, on the other hand  $d\alpha/dg < 0$ , the opposite could be observed provided the increased expected damage of confrontation for the criminal is not too high. In other words, if the deterrent capacity of the victim decreases and the expected damage to the criminal from a more violent confrontation is not too high, the victim will have to expend more resources in violent activities at equilibrium.

We can differentiate  $x_c^{A*}$  with respect to  $g$  to obtain:

$$\frac{dx_c^{A*}}{dg} = -\frac{1}{2k_c^{1/2}} [W^{1/2} - 2k_c^{1/2}] \frac{dk_c}{dg},$$

with the sign of this expression following from the requirement that  $p^* < 1$ , which is equivalent to  $[2 - \alpha]W^{1/2} < 2k_c^{1/2}$ , thus implying that, *à fortiori*,  $W^{1/2} < 2k_c^{1/2}$ .

Having shown that the criminal's equilibrium effort will unequivocally decrease but that the victim's equilibrium effort could increase following a gun laws relaxation, we then inspect the effect of such a policy on the expected aggregate equilibrium efforts,  $p^*x_c^* + x_v^*$  which is given by:

$$p^*x_c^* + x_v^* = \frac{[W^{1/2} - 2k_c^{1/2}]^2}{\alpha} \left[ \frac{2k_c^{1/2}}{W^{1/2}} + 1 \right].$$

Differentiating this expression with respect to  $g$  then yields:

$$\frac{d\{p^*x_c^* + x_v^*\}}{dg} = -\frac{3[W^{1/2} - 2k_c^{1/2}]}{\alpha W^{1/2}} \frac{dk_c}{dg} - \frac{[W^{1/2} - 2k_c^{1/2}]^2}{\alpha} \left[ \frac{2k_c^{1/2}}{W^{1/2}} + 1 \right] \frac{d\alpha}{dg},$$

which reveals that if the victim's relative efficiency increases, the total effort for violence decreases, while in the event the victim's relative efficiency decreases, we could witness an increase in aggregate violence.

Finally, inspecting the effect of a change in gun laws on the equilibrium probability of confrontation,  $p^*$ , we observe that:

$$\frac{dp^*}{dg} = -\frac{p^*}{\alpha} \frac{d\alpha}{dg} - \frac{1}{\alpha W^{1/2} k_c^{1/2}} \frac{dk_c}{dg}.$$

And we thus obtain that in the event a gun laws relaxation increases the relative efficiency of the victim, the probability of confrontation unambiguously decreases. Otherwise, if the added expected damage from gun laws relaxation is not too large, then we could observe an increase in the probability of confrontation.

This enables us to claim the following result:

**Proposition 2** *In a mixed strategy equilibrium where the criminal commits the crime probabilistically, if pro-gun laws increase the relative efficiency of the victim this leads to a decrease of both the level of violence and the probability of an armed confrontation. Otherwise, the level of violence will increase. The probability of armed confrontation increases if the marginal damage of guns is low.*

The intuition behind these results is the following. By increasing the relative efficiency of the victim's technology, pro-gun laws enable the victim to deter the criminal with a lower effort. The strategic complementarity will generate a reduction in the criminal's equilibrium effort too. Since a higher victim's confrontation efficiency will increase the marginal benefit of confrontation to the victim, and since this effect is further reinforced by the reduction in the players' equilibrium efforts, which will make the outcome more sensitive to further changes in confrontation effort, for the victim to be kept indifferent between its two strategies the confrontation probability will have to decrease. If, however, the criminal becomes relatively more efficient following a relaxation of gun laws, the effects can be reversed, and the probability of confrontation could increase following a relaxation of gun laws.

Turning to the effect of a relaxation of gun laws on the victim's expected utility, we first rewrite the victim's indirect utility as:

$$U_v^* = p^* [\alpha W x_v^*]^{1/2} - x_v^* - p^* k_v + [1 - p^*] W.$$

Totally differentiating w.r.t.  $g$ , we obtain:

$$\frac{dU_v^*}{dg} = [(\alpha W x_v^*)^{1/2} - k_v - W] \frac{dp^*}{dg} + \frac{\partial U_v^*}{\partial x_v^*} \frac{dx_v^*}{dg} - p^* \frac{dk_v}{dg}.$$

The second term in the above expression is nil by definition of  $p^*$ , thus implying that a more lenient guns law will unambiguously reduce the victim's utility if  $dp^*/dg < 0$ , which can only occur if  $d\alpha/dg < 0$ . If  $dp^*/dg > 0$ , on the other hand, the victim's expected utility could increase if the guns law relaxation does not increase too much the expected damage ( $dk_v/dg$ ), weighted by the likelihood of a confrontation taking place,  $p^*$ .

### 4.3. The effect of gun laws on deterrence

Consider lastly that the constraint defining the type of equilibrium is binding, so that, after re-writing the condition in Result 1 we have:

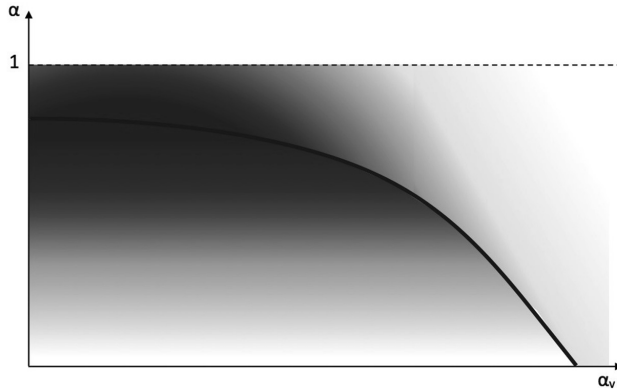
$$W = [\alpha + 1] \left[ k_c + \left[ k_c [k_c + k_v] \right]^{\frac{1}{2}} \right],$$

and the game therefore admits a unique pure strategy confrontation equilibrium prior to relaxing the guns law. An increase in  $\alpha_v$  then leads to higher expected damage  $k_c(\alpha_c, \alpha_v)$  and  $k_v(\alpha_c, \alpha_v)$ , thus implying that the RHS increases, and the game switches to a mixed strategy equilibrium where deterrence takes place with strictly positive probability.

### 4.4. Graphical representation and discussion

We can combine the elements derived in this section and visualize graphically on Figure 5 the effect of gun laws on the expected aggregate equilibrium effort in violence, that is  $x_c^* + x_v^*$  in the pure strategy equilibria space, and  $p^* x_c^* + x_v^*$  in the mixed strategy equilibria space. On the  $x$ -axis we depict the efficiency of the victim's capacity for violence, and on the  $y$ -axis the players' relative efficiency,  $\alpha_v / \alpha_c$ . The darker shades on the figure reflect higher levels of aggregate efforts. The negative-sloped dark curve separates the graph in two zones. To the south-west of this frontier, the game admits a unique pure-strategy confrontation equilibrium, while to the north-east of the frontier the game admits a mixed strategy equilibrium.

**Figure 5.** The effect of gun laws relaxation on aggregate equilibrium effort in violence.



Having shown that in the pure strategy equilibrium aggregate effort in violence is increasing in  $\alpha$  alone, we obtain visually darker colors the closer we get to the separating frontier. In the mixed-strategy equilibrium, however, the results with respect to  $\alpha$  are reversed, higher values of  $\alpha_v$  and of  $\alpha$  map in lower aggregate efforts in violence.

This graph allows to visualize the potentially ambiguous effect of gun laws on aggregate expenditures in violence. In the event gun laws improve the victim's and the criminal's relative efficiencies in same proportions so as to leave their relative efficiency unchanged, the aggregate expenditures will either remain unchanged or decrease for sufficiently large efficiency levels. Bearing in mind, however, that higher efficiencies go hand in hand with more harmful encounters, the relaxation of gun laws will decrease the victims' well-being up to a point where the deterrent effect of higher guns availability initiates a sufficiently important decrease in the probability of the crime being committed, and provided the marginal damage of weapons  $dk_v$  is not too high. In other words, while more lenient gun laws could prove beneficial to victims, this will only be true locally since the victim's utility is  $U$ -shaped in  $\alpha_v$ .

Proponents of gun laws are likely to claim that a more realistic scenario involves more lenient gun laws increasing both the absolute *and* relative efficiencies of the victim,  $\alpha_v$  and  $\alpha$ . Even in such cases, however, our findings are highly ambiguous as one can observe on Figure 5. Indeed, moving from the south-west part of the graph towards its north-east part, we observe that aggregate expenditures in violence first increase and then decrease. The increasing part reflects the higher incentives for criminals to invest in violence when facing more dangerous victims, while the decreasing part is driven by deterrence becoming more effective with the rise in the expected costs of confrontation.

What the above analysis thus reveals is a non-linear relationship between both, violent effort and victims' well-being, and the degree of leniency of gun laws. This is quite an important aspect to underline since empirical studies focusing on marginal effects (*e.g.* the impact of *right-to-carry* laws or the impact of *stand your ground laws*) may very well demonstrate a drop in guns-related violence, when a drastic coarsening of gun laws (*e.g.* forbidding guns) could have proved much more effective in reducing crime. In other words, while focusing on marginal effects is instructive, empirical studies should also attempt to explore more systematically the extensive margin given the non-linearities we uncover in this paper. Second, our theory also allows to reconcile seemingly contradictory results since the marginal effect of a gun law will depend both on the relative efficiency of victims compared to criminals (*e.g.* degree of organization of criminal networks, guns training of victims, etc...), as well as on whom the reform most benefits to. Empirical evidence establishes that more lenient gun laws equally benefit criminals (*e.g.* Webster *et al.* [2012]), and this is likely to lead to an intensification of gun-related violence if the balance is initially tilted in favor of criminals as is the case in contexts with highly strict gun laws.

## 5. Concluding remarks

In this paper we have developed a simple theoretical setting to help us better understand the effect of gun laws, and thus of institutions, on gun-related violence. Our model features a (potential) victim and a criminal attempting to steal the former's property. We show that the effect of gun laws on the equilibrium level of violence is non-linear and dependent on the initial relative efficiency of players, as well as to whom the change in guns legislation benefits the most. When victims are relatively inefficient in confronting criminals, our model predicts that the criminal will never be deterred from committing a crime, but that the efforts exerted by both the victim and the criminal will be low. This situation typically characterizes polities with harsh gun laws, whereby the level of injury of a possible encounter is also low. Relaxing gun laws will then increase the level of violence if that equally maps in an increase in the relative efficiency of (potential) victims because the criminals react strategically to the expected increase in the victims' effort for violence by spiraling up their own effort too. When the initial relative efficiency of victims is already high, however, as one would expect in polities with lenient gun laws, further relaxing guns legislation will likely produce a decrease in both the players' violent effort and the likelihood of a crime being committed. In such instances, the resulting increase in victims' efficiency will deter criminals from committing crimes, and even more so if the expected damage from a confrontation increases more.

Our results may help rationalize a series of empirical regularities, while also reconciling seemingly contradictory empirical findings. Several studies point for instance at the deterrent effect of more lenient gun laws in rural areas as opposed to the resulting increase in violence in urban areas (e.g. Durlauf *et al.* [2016]), attributing these differences to a series of factors. Higher "prizes" and lower probabilities of arrest both explain higher likelihood of criminal acts (Glaeser and Sacerdote [1999]). Indeed, in rural areas social networks are denser, thereby resulting in less "anonymity" and higher chances of getting arrested for potential criminals (Decker [1979]), but also implying less social control in urban areas (Chamlin and Cochran [2004]). An alternative possible explanation of such findings that we uncover is that the initial relative efficiency of victims in rural areas with a more developed gun-culture (e.g. Felson and Pare [2010]; Gresham and Demuth [2020]) is higher than in urban areas, thus implying that in the former areas criminals are to a large extent already deterred from committing crimes prior to the law change. By making gun laws more flexible, this deterrent effect will then intensify, whereas in urban centers we will observe an escalation of violence because of the two sides' mutual expectations of higher effort in violence. Likewise, the literature has identified racial patterns in gun ownership, with blacks being more likely to carry guns in U.S. (Felson and Pare [2010]; Kalesan *et al.* [2016]). Our theory then helps reconcile these facts with the reduced effect of SYG laws on increased crime among blacks (McClellan and Tekin [2017]), by attributing this reduced effect to the initially higher degree of gun ownership among potential (black) victims.

Such analysis is confined to the *intensive margin* of gun laws, but from a broader perspective it is certainly important to equally consider the *extensive margin*. In that respect our model is very instructive in light of the inverted *U*-shaped relationship tying violent effort to gun laws relaxation. This provides a theoretical basis for understanding why the U.S. whose gun laws are the most flexible amongst OECD countries, also stands out as an exception in terms of high crime rates per capita. We are then able to reconcile the fact that gun laws at the margin may seem to deter crime, when from a broader viewpoint the most effective way of reducing crime intensity would be to drastically restrict access to guns.

Rather than viewing this contribution as a comprehensive theoretical setup for understanding the relationship between guns legislations and crime, we instead believe that our model points at some important theoretical mechanisms, while possibly neglecting other equally important channels. This study is a reminder that theory is essential for structuring empirical approaches, and that more theoretical contributions are essential in the study of crime.

## References

- AYRES I., DONOHUE J. [2003], Shooting down the 'more guns, less crime' hypothesis, *Stanford Law Review*, 55(4), 1193-1312.
- BAC M. [2010], The interaction between potential criminals' and victims' demands for guns, *Journal of Public Economics*, 94, 337-343.
- BECKER G. [1968], Crime and punishment: an economic approach, *Journal of Political Economy*, 76(2), 169-217.
- CARD D., DAHL G. B. [2011], Family violence and football: The effect of unexpected emotional Cues on violent behavior, *Quarterly Journal of Economics*, 126(1), 103-143.
- CHAMLIN M. B., COCHRAN J. K. [2004], An excursus on the population size-crime relationship, *Western Criminology Review*, 5(2), 119-130.
- CHENG C., HOEKSTRA M. [2013], Does strengthening self-defense law deter crime or escalate violence? evidence from expansions to castle doctrine, *The Journal of Human Resources*, 48(3), 821-854.
- COOK P. J., LUDWIG J. [2006], The social costs of gun ownership, *Journal of Public Economics*, 90:379-39.
- COUTTENIER M., GROSJEAN P., SANGNIER M. [2017], The wild west *IS* wild: The homicide resource curse, *Journal of the European Economic Association*, 15(3), 558-585.
- DE LUCA G., SEKERIS P. G. [2013], Deterrence in contests, *Economica*, 80, 171-189.
- DECKER S. [1979], The rural county sheriff: An issue in social control, *Criminal Justice Review*, 4(2), 97-111.
- DONOHUE J. J., LEVITT S. D. [1998], Guns, violence, and the efficiency of illegal markets, *American Economic Review, papers and proceedings*, 88(2), 463-467.
- DUGGAN M. [2001], More guns, more crime, *Journal of Political Economy*, 109(51), 1086-1114.



- DURLAUF S. N., NAVARRO S., RIVERS D. A. [2016], Model uncertainty and the effect of shall-issue right-to-carry laws on crime, *European Economic Review*, 81, 32-67.
- EDWARDS G., NESSON E., ROBINSON J. J., VARS F. [2018], Looking down the barrel of a loaded gun: The effect of mandatory handgun purchase delays on homicide and suicide, *Economic Journal*, 128(616), 3117-3140.
- EHRlich I. [1981], On the usefulness of controlling individuals: An economic analysis of rehabilitation, incapacitation and deterrence, *American Economic Review*, 71(3), 307-322.
- EHRlich I. [1998], Crime, punishment, and the market for offenses, *Journal of Economic Perspectives*, 10(1), 43-67.
- FELSON R. B., PARE P.-P. [2010], Gun Cultures or Honor Cultures? Explaining Regional and Race Differences in Weapon Carrying, *Social Forces*, 88(3), 1357-1378.
- GLAESER E. L., SACERDOTE B. [1999], Why is there more crime in cities?, *Journal of Political Economy*, 107(S6), S225-S258.
- GROSSMAN H. I., KIM M. [1995], Swords or plowshares? a theory of the security of claims to property, *The Journal of Political Economy*, 103 (6), 1275-1288.
- GRESHAM M., DEMUTH S. [2020], Who owns a handgun? An analysis of the correlates of handgun ownership in young adulthood, *Crime & Delinquency*, 66(4), 541-571.
- GUHA B. [2003], Guns and crime revisited, *Journal of Economic Behavior & Organization*, 94, 1-10.
- HICKEY R., MONGRAIN S., ROBERTS J., VAN YPERSELE T. [2019], Private protection and public policing, *Melbourne Institute Working Paper Series*, wp2019n04.
- HIRSHLEIFER J. [1989], Conflict and rent-seeking success functions: Ratio vs. difference models of relative success, *Public Choice*, 63, 101-112.
- HOTTE L., VAN YPERSELE T. [2008], Individual protection against property crime: decomposing the effects of protection observability, *Canadian Journal of Economics*, 41(2), 537-563.
- JACOBSSON A. K. S. [2009], War and peace—cyclical phenomena?, *Public Choice*, 141(3/4), 467-480.
- KALESAN B., MOBILY M. E., KEISER O., FAGAN J. A., GALEA S. [2016], Firearm legislation and firearm mortality in the usa: a cross-sectional, state-level study, *The Lancet*, 387, 1847-1855.
- KOENIG C., SCHINDLER D. [2019], Impulse Purchases, Gun Ownership and Homicides: Evidence from a Firearm Demand Shock, *CESifo Working Paper*, No. 7833.
- LACROIX G., MARCEAU N. [1995], Private protection against crime, *Journal of Urban Economics*, 37, 72-87.
- LOTT J. R. [1998], *More Guns Less Crime: Understanding Crime and Gun Control Laws*, The University of Chicago Press.
- LOTT J. R., MUSTARD D. B. [1997], Crime, deterrence, and right to carry concealed handguns, *Journal of Legal Studies*, 26(1), 1-68.
- LUCA M., MALHORTA D., POLIQUIN C. [2017], Handgun waiting periods reduce gun deaths, *Proceedings of the National Academy of Science*, 114(46), 12162-12165.
- MCCLELLAN C., TEKIN E. [2017], Stand your ground laws, homicides, and injuries, *The Journal of Human Resources*, 52(3), 621-653.
- MANSKI C. F., PEPPER J. V. [2018], How do right-to-carry laws affect crime rates? Coping with ambiguity using bounded-variation assumptions, *Review of Economics and Statistics*, 100(2), 232-244.

- MARCEAU N. [1997], Competition in crime deterrence, *Canadian Journal of Economics*, 30(4a), 844-854.
- MIALON H. M., WISEMAN T. [2005], The impact of gun laws: A model of crime and self-defense, *Economics Letters*, 88(2), 170-175.
- MUNASIB A., KOSTANDINI G., JORDAN J. L. [2018], Impact of the stand your ground law on gun deaths: evidence of a rural urban dichotomy, *European Journal of Law and Economics*, 45, 527-554.
- NEARY H. M. [1996], To fight or not to fight: equilibrium structure in a conflict model when conflict is costly, *Discussion Paper. University of British Columbia*, no. 96-14.
- SCHELLING T. [1960], *The Strategy of Conflict*. Cambridge, Mass., Harvard University Press.
- SCHNEIDER R. [2018], Crime and political effects of a concealed carry ban in Brazil, available at SSRN 3311194.
- SKAPERDAS S. [1992], Cooperation, conflict, and power in the absence of property rights, *American Economic Review, papers and proceedings*, 82(4), 720-739.
- TANNENBAUM D. I. [2020], Does the disclosure of gun ownership affect crime? Evidence from New York, *Journal of Public Economics*, 181(C).
- TIIHONEN J., HALONEN P., TIIHONEN L., KAUTIAINEN H., STORVIK M., CALLAWAY J. [2017], The association of ambient temperature and violent crime, *Scientific Reports*, 7, 6543.
- TULLOCK G. [1980]. Efficient Rent Seeking, in: Buchanan J.M., Tollison R.D., Tullock G. (eds), *Toward a Theory of the Rent-Seeking Society*, chapter, College Station, Texas A& M University press, 97-112.
- WEBSTER D. W., VERNICK J. S., BULZACCHELLI M. T., VITTES K. A. [2012], Temporal association between federal gun laws and the diversion of guns to criminals in milwaukee. *Journal of Urban Health*, 89(1), 87-97.
- WORLD BANK [2020], <https://data.worldbank.org/>.