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Abstract

Canada exhibits no correlation between income and victimization, rich neighborhoods are less exposed to property crime, rich households are more victimized than their neighbors, and rich households and neighborhoods invest more in protection. We provide a theory consistent with these facts. Criminals within city choose a neighborhood and pay a search cost to compare potential victims, whereas households invest in self-protection. As criminals' return to search increases with neighborhood income, households in rich neighborhoods are likelier to enter a race to greater protection driving crim-inals toward poorer areas. A calibration reproduces the Canadian victimiza-tion and protection pattern by household/neighborhood income.

KEYWORDS

alarms, economics of crime, private protection, property crime, search frictions

JEL CLASSIFICATION

K42, C78, R12

1 INTRODUCTION

As shown in Figure 1, there is no clear-cut relation between household income and property crime in a cross-section of countries. As for investment in private protection, the pattern is clearer: wealthier households invest more. These facts, however, do not account for neighborhood differences. Crime rates and private protection are highly heterogenous at the neighborhood level. This suggests that private protection interacts in complex ways with criminals' decisions. These issues have been mostly neglected by economists, so far, despite their quantitative importance.[1]

Our analysis aims to fill this gap. We first document four key stylized facts on property crimes and protection from detailed data of Canada's General Social Survey. We then build a parsimonious theoretical model able to rationalize these facts. In our model, households decide how much to invest in protection while criminals decide in which neighborhood to operate and how much effort to exert to find a profitable opportunity. We show that strong complementarities emerge between households' and criminals' decisions. As criminals' incentive to search increases with neighborhood income, households in rich neighbourhoods are more likely to enter a rat race to ever greater protection that drives criminals toward poorer areas.

Abbreviations: BE, Break and Entry; CDF, Cumulative Distribution Function; CMA, Census Metropolitan Area; ESN, Equilibrium with a Single-Neighborhood; ETN, Equilibrium with Two-Neighborhoods; EWH, equilibrium with mobile criminals and within-neighborhood heterogeneity; FSA, Forward Sortation Areas; GSS, General Social Survey.

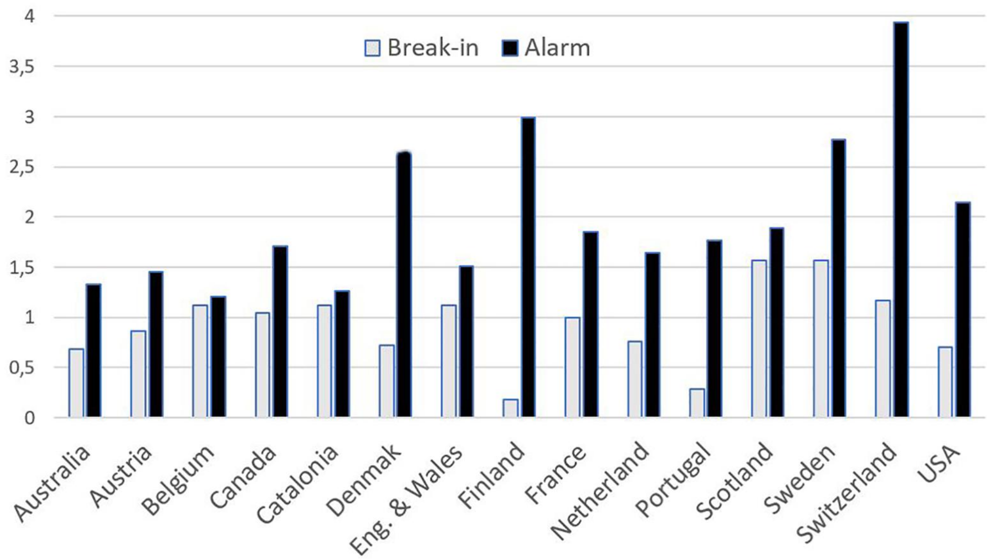

FIGURE 1 Relative victimization and protection for a selection of countries/states. Ratio of above-the-median-income to below-the-median-income household shares. The blue bar is the victimization probability ratio, whereas the red bar is the alarm probability ratio. *Source*: International Crime and Victim Survey, 1996–2000

Section 2 presents the following four facts: (i) Victimization and household income are uncorrelated or slightly positively correlated. Victimization is roughly the same for households below and above the median household income. (ii) Rich neighbourhoods are less victimized than poor ones. The yearly mean victimization rate is about 3.4% and neighbourhoods below the median income experience victimization rates 35% higher than neighbourhoods above the median income. (iii) Conditional on neighborhood income, rich households are more victimized than poor ones. (iv) Rich households as well as rich neighbourhoods invest more in protection. The percentage of households equipped with an alarm is about 30%–35% larger among households above the median income than among households below the median income.

Section 3 describes a theory consistent with these facts. There is a city composed of two neighbourhoods, a rich one and a poor one. The supply of criminals is exogenous at city level, but arbitrage ensure that the returns to crime are equalized at the neighborhood level. Criminals also choose if they pay a search cost to compare different households or simply pick one randomly. Meanwhile, households invest in private protection to reduce the loss in case of break in. The two main mechanisms of the model are as follows. First, criminals' search implies that households make heterogenous protection investment. Households who expect to be compared to each other invest in protection to divert criminals' attention toward neighbors. Second, protection heterogeneity motivates criminals' search. Thus there is strategic complementarity between criminals' search efforts and households' protection investments.

Protection heterogeneity derives from the fact that protection is a positional good. Households make utility gains by being ranked higher in the distribution of protection investment. To see this, suppose all households in a given neighborhoods make the same protection investment, whereas some of the criminals pay the search cost to compare two randomly chosen households. Now, consider the case where two households are scrutinized by a given criminal. As they are alike, the probability of being burglarized is one half for each of them. If one of these households were to invest more by epsilon, then the probability of being burglarized would drop to zero. Therefore a marginal increase in effort would generate a mass gain. Therefore, similar agents make heterogenous investment in equilibrium. Even more, this argument implies that the distribution of protection investment has no mass point.

Our model can generate equilibrium outcomes in line with the set of facts reported in Section 2. In the absence of protection investment, the rich neighborhood is more attractive to criminals. However, its residents also invest more in protection, which repels criminals. Indeed, criminals in the rich neighborhood are more willing to pay the search cost than in the poor neighborhood. Thus rich households expect to be frequently compared to their neighbors. This affects both the mean (people invest more on average) and the dispersion of protection investments (there are still some people who choose not to invest, whereas others invest more). That the rich neighborhood invests more in protection implies low returns to crime, and therefore criminals may be more attracted to the poor neighborhood. Meanwhile, rich households are more victimized than poorer ones in each neighborhood. This equilibrium allocation is illustrated by a parameterization broadly replicating the quantitative facts exposed in Section 2.

Importantly, the starting point of our analysis is that criminals are mobile between different neighborhoods of the same city. Everyone knows whether a neighborhood is wealthy or not and criminals respond to incentive when deciding where to break in. In this light, the strength of our theory is its ability to predict the geography of crime despite criminals are mobile. There is a large set of evidence suggesting that crimes are most often committed near the criminals' home, whereas criminals are more likely to come from lower income families (see, e.g., Burrell & Tonkin, 2020). This fact is actually compatible with our theory. The actual assumption we need is that criminals are sufficiently mobile between neighborhoods so that the return to crime is the same across neighborhoods. In our model, if, as it is likely, more criminals reside in the poor neighborhood, then we can predict that most crime occurs near the criminals' homes.

This paper makes three contributions to the literature. We first contribute to the literature on crime and social interactions. Existing studies have focused, so far, on strategic complementarity between individuals to engage in criminal activities (see Calvó-Armengol et al., 2007; Glaeser et al., 1996; Lazzati & Menichini, 2016; Patacchini & Zenou, 2008; Zenou, 2003). We highlight a novel mechanism through which social interactions affect criminal outcomes. Households invest more in protection when their neighbors also invest in protection. This implies that private decisions have a social multiplier effect.

We develop the first model of protection investment and search for theft opportunities. Protection is modeled in the spirit of Shavell (1991), Helsley and Strange (2005), Hotte and van Ypersele (2008), and Hickey et al., 2021. Adding search allows us to generate protection heterogeneity and endogenous comparison of households within neighbourhoods. Search frictions are a natural ingredient in a market for illegal and informal activities.[2] We build on a specific form of search on goods (initiated by Burdett and Judd (1983)), jobs (Acemoglu & Shimer, 2000; Albrecht et al., 2006; Galenianos & Kircher, 2009) and education (Moen (1999)). These papers share a common feature: a subset of workers deciding on their human capital investment, firms advertising for their vacancy or for their good are compared to each other by another party. This local comparison due to search implies that similar agents make heterogenous decisions (educational investment, posted wage or price). In turn, heterogeneity promotes comparison efforts to yield, in our case, an equilibrium with positive search effort and a non-degenerate distribution of protection investment. Beyond applying standard tools to an original socioeconomic situation, we bring two new elements to a classical search framework. First, we consider two separate marketplaces, that is, neighbourhoods, that are interconnected through the return to property crime. Second, we introduce within-neighborhood heterogeneity. Both elements are needed to make sense of the data.

2 MOTIVATING FACTS

This section documents four Canadian facts on income, property crime and private protection. Namely, (i) income and victimization are uncorrelated (or weakly positively correlated) at household level and (ii) negatively correlated across neighbourhoods, (iii) rich households are more likely to be victimized than their neighbors, and (iv) private protection is positively correlated with income at household and neighborhood levels. We first present our data and then show the four facts.

Data — We use cross-sectional individual data from the victimization part of Canada's General Social Survey (GSS). This survey is conducted every 5 years. We consider years 1999, 2004, 2009 and 2014. We supplement the GSS with the Census to compute the mean neighborhood income and neighborhood low income proportion. Years do not exactly match. We consider the following Census years: 1996, 2001, and 2006. The 2014 GSS already contains the needed information. Each GSS is representative at Census Metropolitan Area (CMA) level. Each CMA represents an urban area, ranging from 6 million resident for the Toronto metropolitan area to small urban area of just over 20 thousand residents. Our data set contains 74 distinct CMAs. Each individual is located by a six-digit postal code. The first three digits define Forward Sortation Areas (FSAs), our neighbourhoods. FSA is not a common geographic unit in the Census and so we use a table mapping Dissemination Areas in the Census with FSAs. FSAs are exceptionally shared by several CMAs. When this happens, the FSA is divided into several sub-neighbourhoods belonging to different CMAs. Household income is declared in 10 classes. We attribute the class mean to each income class. We restrict our analysis to urban areas. FSA are composed of an average of ten thousand households. Our data set contains 442 distinct FSAs, adding up to 2097 years FSAs across the 74 CMAs. Economists and criminologists often use FSA as reference areas, both for its convenience and relevance. Our data set contains a total 32,097 household observations.

We use two variables of protection. Respondents declare whether they have ever installed an alarm and if they have ever installed speciality locks or bars. The variable PA take a value of one when there is an alarm and $PA = 0$ otherwise.

We also consider the dummy variable *PA*2, which takes the value one when the household has installed an alarm or bars/locks.

The main variable of victimization *BE* is equal to one when the household experienced a successful or attempted break-in over the past 12 months before the interview date. Mean neighborhood protection $\overline{PA}$ and mean victimization $\overline{BE}$ are computed by aggregation of household observations. We use sample weights for the GSS and arithmetic means for the Census.

In Tables 1, 2 and 3, we show averages conditional on household and neighborhood income. Namely, we sort households on whether their income is below the median income or not, and whether they live in a neighborhood with an average income above or below the median one. In both cases, our income measure are relative to the CMA average income.[3]

Facts — Table 1 shows the mean victimization rate and mean protection investment by household income. Victimization is slightly higher and protection much larger for households above the median income ($Y > Y_{50}$) than for those below the median income ($Y < Y_{50}$). This statement remains true for all measures of property crime and private protection. Table 1 also displays mean figures for the different quartiles of the household income distribution. They confirm that there is no relationship (or a very weak one) between household income and victimization, whereas private protection investment strongly increases with household income.

Table 2 features similar statistics for the different quartiles of the neighborhood income distribution. In all cases, property crime decreases with neighborhood income and private protection increases with it. Households living in neighbourhoods below the median neighborhood income ($\overline{Y} < \overline{Y}_{50}$) are 40% (BE) more exposed to property crime than households leaving in neighbourhoods above the neighborhood median income ($\overline{Y} > \overline{Y}_{50}$).

TABLE 1 The distribution of victimization and protection by household income

	BE	PA	PA2
overall	0.032	0.350	0.506
std. err.	0.001	0.002	0.002
$Y < Y_{50}$	0.031	0.280	0.446
std. err.	0.001	0.003	0.003
$Y > Y_{50}$	0.033	0.410	0.577
std. err.	0.001	0.003	0.003

Notes: The variable Y denotes household income and Y_{50} is the 50th percentiles. Reading: 3.1% of Canadian households bellow the median of the income distribution experienced a break-in or an attempt of break-in in the past 12 months.

Source: Canada's GSSs 13, 18, 23 and 28.

TABLE 2 The distribution of victimization and protection by neighborhood income

	BE	PA	PA2
overall	0.032	0.350	0.506
std. err.	0.001	0.002	0.002
$\bar{Y} < \bar{Y}_{50}$	0.038	0.320	0.479
std. err.	0.001	0.003	0.003
$\bar{Y} > \bar{Y}_{50}$	0.027	0.378	0.530
std. err.	0.001	0.003	0.003

Notes: Neighbourhoods are FSAs in Canada's GSS. The variable Y denotes household income and Y_{50} is the 50th percentiles. Reading: 3.8% of Canadian households leaving in neighbourhoods bellow the median of the neighborhood income distribution experienced a break-in or an attempt of break-in in the past 12 months (BE).

Source: Canada's 1996, 2001 and 2006 census for BE and PA and Canada's GSSs 13, 18, 23 and 28 for income.

TABLE 3 The cross distribution of victimization and protection by neighborhood and household income

	$Y < Y_{50}$	$Y > Y_{50}$	$Y < Y_{50}$	$Y > Y_{50}$
	BE		PA	
$\bar{Y} < \bar{Y}_{50}$	0.036	0.042	0.270	0.380
std. err.	0.002	0.002	0.004	0.005
$\bar{Y} > \bar{Y}_{50}$	0.026	0.027	0.292	0.430
std. err.	0.002	0.001	0.005	0.004

Notes: neighbourhoods are FSAs in Canada's GSS. Reading: 3.6% of Canadian households below the median income and leaving in neighbourhoods bellow the median neighborhood income experienced a break-in or an attempt of break-in in the past 12 months (BE).
Source: Canada's 1996, 2001 and 2006 census for BE and PA and Canada's GSSs 13, 18, 23 and 28 for income.

Tables 1 and 2 show that income provides incentive to protection investment, either at household or neighborhood levels. They also show that wealthier neighbourhoods are much less victimized, whereas household income and victimization are uncorrelated. These facts seem contradictory, but imperfect sorting on income implies they are not. Table 3 displays the mean victimization rate and the mean protection investment by household and neighborhood income. In all neighbourhoods, the rich invest more in protection than the poor and are more victimized than them. This explains why victimization is weakly increasing in household income despite the rich are over-represented in rich and less victimized neighbourhoods.

To summarize, property crime slightly increases with household income and strongly decreases with neighborhood income. Therefore rich households are more victimized than poor ones in both poor and rich neighbourhoods. Meanwhile protection investments increase with household and neighborhood incomes. It is interesting to see that both richer and poorer households are less exposed to crime when living in a rich neighborhood. At the same time, poorer households invest almost the same regardless in which neighborhood they live. This is consistent with private protection generating positive externality at the neighborhood level. Hereafter we provide a possible scenario generating this collection of facts as equilibrium outcomes. Section 3 sketches a theoretical model where criminals choose a neighborhood and how much they compare possible victims, whereas households invest in private protection. It is important to note that those facts are specific to Canada and that other countries may experience different relationships, especially when it comes to household income and victimization.

3 PROPERTY CRIME AND PROTECTION: THEORY

We base our analysis on a static model[4] of a city divided in multiple neighborhoods. We first present the general structure of our model. We then analyze the simplified case of a single-neighborhood city populated by homogenous households. Next, we turn to the case of a two-neighborhood city with mobile criminals. We begin with income heterogeneity across neighborhoods, but homogenous income distribution within each neighborhood. We conclude with the more complex and empirically relevant case where households are heterogenous within and across neighborhoods. Our preferred equilibrium configuration is illustrated by a parameterization replicating the main facts reported in Section 2. All proofs can be found in Appendix C.

3.1 The model

Our city is composed of two neighborhoods indexed by $j \in \{A, B\}$. Each neighborhood is populated by K_j residents. We start with homogenous income distribution within each neighborhood: all residents in neighborhood j have the same income $V_j > 0$. This assumption is relaxed in Section 3.4, where we introduce within-neighborhood heterogeneity. We assume $V_A > V_B$ and refer to neighborhood A as the rich one and to neighborhood B as the poor one.

There is an exogenous number C of criminals with (limited) mobility across neighborhoods. The number of criminals operating in neighborhood j is C_j. Each criminal commits one crime in a given neighborhood. Thus the crime rate in neighborhood j is $c_j = C_j/K_j$. By definition, $C_A + C_B = C$.

Residents independently invest in self-protection $\theta \geq 0$ at a cost of $\gamma\theta V$. Private protection reduces the loss incurred during a theft. The total loss is $\max\{(\alpha - \theta)V, 0\}$, where $\alpha \in (0, 1)$ is the proportion of income stolen in absence of protection. Both protection costs and gains are proportional to income/wealth/property size, which amounts to a normalization assumption.[5] Losses are direct transfers to criminals.[6]

We do not explicitly model the way protection reduces losses. Criminals may have less time to find valuable objects in the house. Alternatively, they may have a larger probability of being caught. Both effects have similar behavioral implications when agents are risk neutral, which explains why we do not distinguish them.

We allow households to employ mixed strategies, where $H_j(\theta)$ is the cumulative distribution function (cdf) guiding protection efforts by a particular household residing in neighborhood j. This function allows for pure strategy θ when $H(\theta) = 1$ and $H(\theta') = 0$ for all $\theta' \neq \theta$. Define $\Theta \subset \mathbb{R}_+$ the support of this distribution.

Individual income and protection are imperfectly observable. A criminal who plans a robbery in neighborhood j is presented with one or two theft opportunities from whom he observes protection and income levels. The cost of obtaining one opportunity is normalized to zero, while the cost of searching for a second opportunity is s. Thus the margin of decision is whether to pay the search cost and be able to compare two houses. Criminals and potential victims are matched randomly.

We denote by C_{j1} the expected number of criminals in neighborhood j who have a single theft option (*single-option criminals*), and by C_{j2} the expected number of criminals who have two options (*double-option criminals*). Similarly, c_{j1} and c_{j2} denote the corresponding criminal-to-household ratios.

We assume the number of criminals willing to chase better opportunities in another location is sufficiently large that the return to crime can be equalized across neighborhoods in equilibrium.

Equilibrium Concept — We consider a sub-game perfect Nash equilibrium. The timing, information sets and strategies used by each agents are as follows:

Stage 1: Criminals decide on search effort and location of activity, whereas households decide on protection level;
Stage 2: Criminals observe protection and income of selected households. Those with two theft opportunities select the most profitable one. If indifferent, criminals select one of the two options with equal probability.

In the next section, we focus on the case involving a unique neighborhood (*single-neighborhood equilibrium*), rendering criminals' location choices irrelevant. We then study the allocation of criminals across the two neighborhoods (*two-neighborhood equilibrium*). To simplify notations, we neglect the neighborhood index j until needed.

Agents' payoffs — Let Ω denote the expected payoff for a given criminal. Allowing for mixed strategies, we have:

$$\Omega = \alpha V - (1-q)\mathbb{E}(\theta)V - q\mathbb{E}(\min\{\theta, \theta'\})V - sq, \tag{1}$$

where $q \in [0, 1]$ is the probability of searching for two opportunities. A theft opportunity is a random draw in the distribution of households' protection investments. When criminals have a single theft opportunity, their expected payoff thus depends on the unconditional mean $\mathbb{E}(\theta)$ of protection investments. When they have two opportunities, they choose the least protected household. Their expected payoff depends on the mean of the minimum protection level in such a case.

The return to search is the expected gain associated with choosing a less protected house:

$$\Gamma = [\mathbb{E}(\theta) - \mathbb{E}(\min\{\theta, \theta'\})]V. \tag{2}$$

Having two theft options is only advantageous when protection levels are heterogenous. When the distribution of protection investments collapses to a single mass point, then $\Gamma = 0$. When protection levels are heterogenous, criminals compares Γ with the search cost. If $\Gamma > s$, a criminal prefers having two options, so $q = 1$. When $\Gamma < s$, a criminal is happy with only one, meaning $q = 0$. Only when $\Gamma = s$, a criminal is willing to use a fully mixed strategy.

Let $W(\theta)$ represents the expected payoff for an household with protection θ. Denote by η_1 and $\eta_2(\theta)$ the expected number of bugleries by single-option and double-option criminals respectively.[7] We have

$$W(\theta) = V - [\eta_1 + \eta_2(\theta)] \max\{(\alpha - \theta), 0\}V - \gamma\theta V. \tag{3}$$

In Appendix B, we shows that if all households use a symmetric mixed strategy $H(\theta)$, then

$$\eta_1 = c_1 = c(1-q), \tag{4}$$

$$\eta_2(\theta) = 2c_2[1 - H(\theta)] = 2cq[1 - H(\theta)]. \tag{5}$$

Single-option criminals randomly sample within the household set. Thus η_1 is equal to the ratio of such criminals to households. As for the expected number of buglaries by double-option criminals, when a given household is compared to another potential victim, the household with the lowest protection level is robbed whereas the other stays safe. Households sample their protection investment in the distribution $H(\theta)$, which will be determined endogenously in equilibrium. Thus, the proportion of households with a protection level above θ is simply $1 - H(\theta)$.

3.2 Single-neighborhood city

In this section, we shutdown criminals' location decisions. This allows us to focus on the interaction between households' protection choices and criminals' search decisions in a simple environment. Thus, the city is composed of a single neighborhood and criminals sample theft opportunities over the full city. We solve for equilibrium protection investment, θ, and search effort, q, for a given number of criminals, C, and corresponding crime rate, $c = C/K$.

Definition 1 An *equilibrium with a single-neighborhood city* (ESN) is a search effort q^* and a cdf H such that

(i) $\theta \in \Theta$ if and only if (iff) $\theta \in \arg\max_{\theta' \geq 0} W(\theta', q^*)$,
(ii) $q^* \in \arg\max_{q \in [0,1]} \Omega(q, H(., q^*))$.

In equilibrium, protection efforts maximize households' well-being, whereas the search effort maximizes criminals' payoffs. If there is a unique value of θ that maximizes individual well-being, then the distribution H is degenerate. Otherwise, the exact distribution results from the equality of payoffs $W(\theta, .)$ over the equilibrium support of the distribution.

There may be a unique equilibrium featuring full protection $\theta^* = \alpha$ by all households and no search $q^* = 0$ by all criminals. If households are never compared to their neighbors, the per unit return on protection is simply cV. With the linear cost function $\gamma\theta V$, the investment in protection is an all-or-nothing decision. When $c \geq \gamma$, the only equilibrium features households choosing to fully protect their house. Facing homogenous theft opportunities, criminals then prefer not to search.

This equilibrium has no empirical content because protection does not change with neighborhood income. Therefore, Assumption 1 below guarantees that households do not protect their house when all criminals only look for a single option.[8]

Assumption 1 $\gamma > \frac{C}{\min\{K_1, K_2\}}$.

We define $x(q) \equiv 2cq/(\gamma - c(1-q))$ as the *protection attractiveness index*. As we will see, this variable is linked to the dispersion of protection investments. It is increasing in both c and q.

Lemma 1 *(Equilibrium distribution of protection investment) In an ESN, the equilibrium distribution of protection investment is such that*

(i) $\Theta = \{0\}$ *and* $H(0) = 1$ *when* $q^* = 0$;
(ii) $H(\theta) = \frac{\theta}{\alpha - \theta} / \frac{\overline{\theta}}{\alpha - \overline{\theta}}$, *for all* $\theta \in \Theta = [0, \overline{\theta}]$, *with* $\overline{\theta} = \alpha x(q^*)$, *when* $q^* > 0$.

Properties (i) and (ii) highlight the complementarity between criminals' search efforts and households' protection investments. Property (i) states that households do not invest when criminals do not make efforts. Of course, protection reduces the magnitude of the loss due to break-in. However, this effect alone is not sufficient for protection to take place in equilibrium. What matters for protection investment is that (some of the) criminals compare households to each other. Then, it is worth investing to divert criminals to neighbors.

Property (ii) shows that there is a continuous equilibrium distribution of protection investment when criminals make search effort. We here elaborate on a result initially due to Stigler in the context of price setting and further developed by Burdett and Judd (1983) in the context of wage determination. Households are actually indifferent between different protection levels. The indifference condition fully characterizes the equilibrium distribution of protection investment. It is equivalent to say that agents play mixed strategies and sample in the equilibrium distribution, or each of them plays a pure strategy and the distribution of resulting investments coincides with the equilibrium distribution.

The cdf of the equilibrium distribution is continuous, which means there is no mass point. Suppose, on the contrary, that there is a mass point in this distribution. Then agents with the corresponding investment have a strictly positive probability, say π, of being compared to a household with the same level of investment. In such a case, they lose $(\alpha - \theta)V$ with 50% chance. Now this is interesting to deviate. While investing marginally more, the loss probability becomes 0 instead of 1/2. Indeed, the criminal will prefer the other household. Thus, an epsilon cost generates a mass gain, that is, $\pi(\alpha - \theta)V/2$. It follows that the distribution must be continuous.

Zero protection always belongs to the support of the equilibrium distribution. On the contrary, let us suppose that the lower bound is strictly positive. Investing less in protection would cost less and would not increase the probability of being burglarized. This contradicts the fact that the lower bound is strictly positive.

The density function $h(\theta) \equiv H'(\theta)$ is strictly increasing in θ. Along the equilibrium distribution, the marginal return to investment must equal the marginal cost. Formally, we have

$$[c_1 + 2c_2(1 - H(\theta))]V + 2c_2 h(\theta)(\alpha - \theta)V = \gamma V. \tag{6}$$

The marginal cost stands in the right-hand side. It is constant and equal to γV. The marginal return stands in the left-hand side. It has two components. The first component is due to the loss reduction effect. A marginal increase in protection reduces the impact of theft by the quantity V. The second component is due to the marginal decrease in probability of being robbed. This gain is proportional to the property loss $(\alpha - \theta)V$. When θ is large, there is not much to lose. This small gain must be compensated by a large increase in occurrence probability. Thus the density h must be increasing in θ.

The equilibrium distribution of protection investment depends on the crime rate, c, and on the search probability, q. Both increase the expected number of burglarized by double-option criminals. Thus, households anticipate being compared with their neighbors more frequently and invest more as a result. The first implication is that the maximum effort $\overline{\theta}$ increases, so that the support of the distribution expands. The second implication is that households assign more weight to higher investments. Thus the distribution becomes more concentrated around the highest values of investment.

To compute the individual return to search, we define the distribution of $\min\{\theta, \theta'\}$ at given search effort, $\tilde{q}$, of other criminals. Its density is equal to $2h(\theta)[1 - H(\theta)]$. Thus,

$$\Gamma = V \int_0^{\overline{\theta}} \{h(\theta)\theta - 2h(\theta)[1 - H(\theta)]\theta\} d\theta. \tag{7}$$

The computation gives

$$\Gamma \equiv \Gamma(x(\tilde{q})) = \alpha V \frac{x(\tilde{q}) - 1}{x(\tilde{q})^2}[(2 - x(\tilde{q}))\ln(1 - x(\tilde{q})) + 2x(\tilde{q})]. \tag{8}$$

The return to search depends on the search effort of the other criminals, $\tilde{q}$, through the protection attractiveness index, $x(\tilde{q})$. It increases with protection investment dispersion, which is non-monotonic in other criminals' search effort, $\tilde{q}$. When criminals search more, the support of the protection distribution $[0, \overline{\theta}]$ expands, but the actual distribution $H(\theta)$ becomes more concentrated at the top. The former effect dominates at low levels of search efforts and is dominated at larger ones. The effects cancel each other at $\hat{x}$, where $\Gamma'\hat{x}) = 0$ and Γ reaches its maximal value.

Hereafter, we suppose that the max value of the return to search is larger than the search cost, s.

Assumption 2 $\Gamma(\hat{x}) > c$.

Thus, search displays external increasing returns when $x(\tilde{q}) < \hat{x}$ and external decreasing returns when $x(\tilde{q}) > \hat{x}$. By external or internal, we mean that the return to search of a given criminal increases or decreases with the search effort of the other criminals. The return to search also increases with the proceeds of crime.

The equilibrium resolution reduces to finding $q^* \in \arg\max_{q\in[0,1]}\{q\Gamma(x(q^*)) - sq\}$, that is, q^* must be an individual best-response to itself.

Proposition 1 *(Existence of ESN) Let $\overline{q} > \underline{q}$ be such that $\Gamma(x(\overline{q})) = \Gamma(x(\underline{q})) = s$.*

(i) if $\Gamma(x(1)) < s$ and $x(1) < \hat{x}$, the only ESN is $q^* = 0$;
(ii) if $\Gamma(x(1)) < s$ and $x(1) > \hat{x}$, there are three ESN and $q^* \in \{0, \underline{q}, \overline{q}\}$;
(iii) if $\Gamma(x(1)) > s$, there are three ESN and $q^* \in \{0, \underline{q}, 1\}$.

The proof is based on Figure 2. The return to search and the search cost lie on the vertical axis, whereas the protection attractiveness index lies on the horizontal axis; it is bounded by $x(1) \leq 1$ at the top. The number of equilibria crucially depends on the max value of the protection attractiveness index, $x(1)$.

Note first that there is always an equilibrium with $q^* = \theta^* = 0$. When households do not invest in protection, criminals have no incentive to compare them to each other. In Figure 2, the return to search is 0 when $q = 0$. Thus, it is lower than the search cost, s. Therefore, criminals do not search. This confirms households' choice who cannot divert criminals' attention to alternative households.

Part (i) shows the case where this no-search no -protection equilibrium is unique. It is empirically irrelevant because all households have the same level of protection and endure the same risk of victimization.

Parts (ii) and (iii) feature more interesting configurations where the model admits three equilibria. In part (ii), the additional equilibria correspond to cases where the return to search is equal to the search cost. In Figure 3, the search cost line crosses the return to search locus twice, in $\underline{x}$ and $\overline{x}$. When $x(1) > \overline{x}$, these points correspond to $q = \underline{q}$ and $q = \overline{q}$,

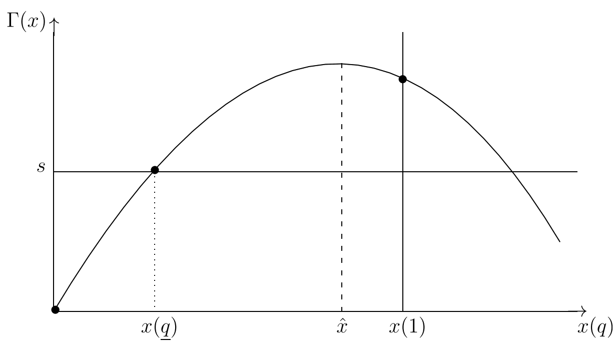

FIGURE 2 Equilibrium search efforts when $x(1) > \hat{x}$

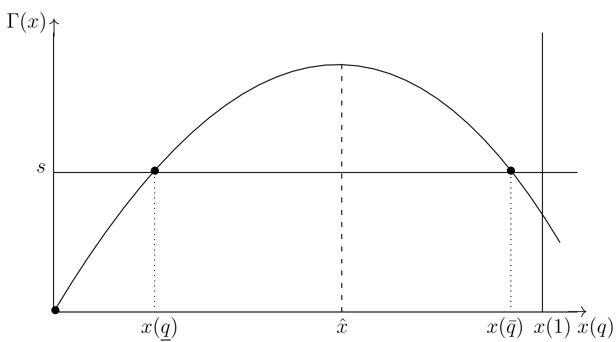

FIGURE 3 Equilibrium search efforts when $x(1) >> \hat{x}$

with $0 < \underline{q} < \overline{q} < 1$. Then criminals search with probability $\underline{q}$ or $\overline{q}$ and households draw their investment from the cdf $H(\theta; \underline{q})$ or $H(\theta; \overline{q})$.

In part (iii), $x(1) < \overline{x}$ and $\overline{x}$ cannot be reached for admissible values of q. The third equilibrium is then $q^* = 1$. In Figure 2, the return to search is larger than the search cost, which implies that $q^* = 1$ is indeed a best-response to itself. In this full-search equilibrium, households sample their investment according to the cdf $H(\theta; 1)$.

The multiplicity of equilibria raises the question of their selection. Pure-strategy equilibria are stable with respect to a trembling-hand argument. Suppose that a small proportion of individuals make a mistake and slightly deviate from $q^* = 0$. They actually choose $0 + \varepsilon$, for small ε. The return to search, Γ, slightly increases as a result but remains below the search cost. Thus, the remaining households still choose $q = 0$ as a best-response. In the same line, the equilibrium where all criminals compare is also robust when it exists.

The case of interior equilibria $\underline{q}$ and $\overline{q}$ depends on whether search has increasing external returns in equilibrium or not. The mixed-strategy equilibrium $\underline{q} \in (0, 1)$ is unstable. Indeed, we have $\Gamma'(\underline{q}) > 0$. If agents tend to overshoot $\underline{q}$, then the return to search becomes larger than the search cost and the best-response to this trembled situation consists in setting $q = 1$. A similar argument prevails in the symmetric case where agents tend to undershoot $\underline{q}$. Lastly, the mixed-strategy equilibrium $\overline{q}$ is stable. This is so because $\Gamma'(\overline{q}) > 0$. Suppose trembling agents choose a slightly larger search probability. Then the return to search becomes lower than the search cost and the best-response consists in setting a lower search probability

Let $x(q) \equiv x(q, c)$ to highlight the dependence with respect to the crime rate, c. The protection attractiveness index, $x(q, c)$, increases with c. At given search effort, households are compared to their neighbors with higher probability, which promotes protection investment. In turn, the rise in c decreases the unstable equilibrium $\underline{q}$, that is, $d\underline{q}/dc < 0$, and increases the stable equilibrium $\overline{q}$, that is, $d\overline{q}/dc$. Limiting our attention to stable equilibria, search efforts increase with crime incidence.

3.3 | Two-neighborhood city

Criminals' location decisions—We now expand our model by dividing our city into two neighborhoods. This raises the question of criminals' mobility between neighborhoods. As mentioned in the introduction, individuals committing property crimes often tend to choose a location close to their place of residence. It is very likely that a significant proportion of the criminals are tied to the neighborhood where they live. Hereafter, we assume that the number of mobile criminals is sufficiently large, so that the returns to crime may be equalized across neighborhoods. Mobile criminals choose a neighborhood based on their expected payoff. They do so on the basis of their expectations about the crime rate, the search effort made by the other criminals, and households' protection investments.

Definition 2 An *equilibrium with a two-neighborhood city* (ETN) is a pair of search efforts (q_A^*, q_B^*), a collection of cdf $\{H_A, H_B\}$ and an allocation of criminals across neighborhoods (C_A^*, C_B^*) such that, for $j = A, B$,

(i) $\theta \in \Theta_j$ iff $\theta \in \arg\max_{\theta' \geq 0} W_j(\theta', q_j^*)$;

(ii) $q_j^* \in \arg\max_{q \in [0,1]} \Omega_j(q, H(., q_j^*, C_j^*/K_j))$;

(iii) $C_i^* > 0$ iff $i \in \arg\max_{j \in \{A,B\}} \Omega_j(q_j^*, H(., q_j^*, C_j^*/K_j))$;

(iv) $C_A^* + C_B^* = C$.

An ETN is composed of two ESN (parts (i) and (ii)) with the additional requirement that the number of criminals by location is endogenous. Thus, part (iii) requires that criminals enter neighborhoods with the highest payoffs. Finally, part (iv) states that the total supply of criminals is fixed.

When criminals enter both locations, the return to crime must be the same in each location. Otherwise, all criminals operate in the same neighborhood. For example, if the wealthier neighborhood feature no search and no protection, the return to crime is simply αV_A. Then, the return to crime is higher there than in the poor neighborhood no matter what. [9]

In neighborhood j, the search effort for a given number of criminals is $q^*(C_j/K_j)$. Thus, the neighborhood-j specific return to crime is

$$\Omega_j = \alpha V_j - \mathbb{E}\left(\theta \,|\, q^*\left(C_j/K_j\right), C_j/K_j\right) V_j + q^*\left(C_j/K_j\right)\left(\Gamma\left(x\left(q^*\left(C_j/K_j\right), C_j/K_j\right), V_j\right) - s\right). \tag{9}$$

Proposition 3 *(Existence of ETN) Assume* $x(1, C/K) < x_{\max}$, *and let* E_{TN} *be the set of ETN with typical element* $\{(q_A^*, C_A^*), (q_B^*, C_B^*)\}$. *The following properties hold:*

(i) There always exists an ETN, that is, $E_{MC} \neq \varnothing$;
(ii) If $C_A^*/K_A < C_B^*/K_B$, *then* $q_A^* > q_B^*$.

Part (i) states that there always exists an ETN where all criminals enter the rich neighborhood, residents do not protect themselves, and criminals do not search. Indeed, Proposition 1 shows that there always exists an ESN with no search and no protection. All criminals enter the rich neighborhood in this case, because our model does not feature any other source of congestion. In line with the phenomenon of multiple victimization, a household can be broken into more than one time. In the particular case where the neighborhoods are equally rich, the allocation of criminals across neighborhood is indeterminate.

There may exist other types of ETN where criminals enter in both neighborhoods and search in at least one of them. The reason why all criminals do not enter the same neighborhood is because location choices convey a congestion externality. Congestion operates through protection investments. As explained below Proposition 1, the distribution of protection investments widens with the crime rate. This reduces the return to crime and discourages further entry.

Parts (ii) describe a key property of such equilibria. If the poor crime rate exceeds the rich one, then it must be that criminals search more in the rich neighborhood. This is important in our quest to rationalize the set of stylized facts displayed in Section 2. There we show that rich neighborhoods are actually less victimized than poor ones. This is compatible with our model provided criminals search more in rich neighborhoods. Then households protect more in this neighborhood, which makes property crime a less rewarding activity and implies some of the criminals choose to stay in the poor neighborhood.

Crime, wealth and protection—Figure 4 illustrates the case where the poor neighborhood B features a higher crime rate than the rich neighborhood A. When few criminals operate in a neighborhood, Proposition 1 states that in the unique ESN criminals do not search and households do not invest in protection. With no search, crime has constant returns. This explains the horizontal part of the return to crime Ω_j in each neighborhood, with $\Omega_A > \Omega_B$. When the number of criminals is sufficiently large, Proposition 3 also states that another ETN occurs where criminals search. Then, crime has decreasing returns, as more criminals stimulate protection investments.

Figure 4 displays two ETN. The no-search equilibrium where all criminals locate in neighborhood A, and an equilibrium where criminals are more numerous in the rich neighborhood and pay the search cost there, whereas criminals do not compare potential victims in the poor neighborhood.

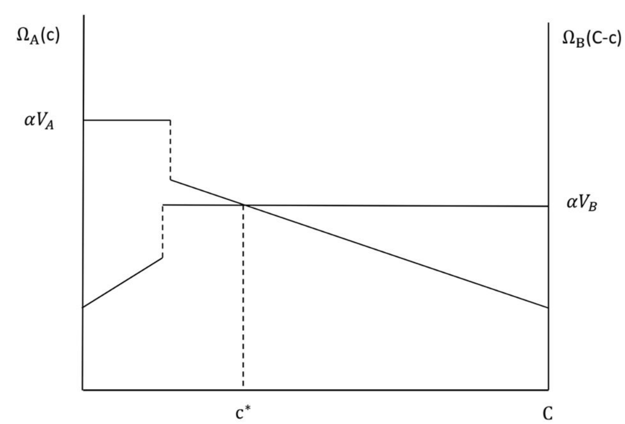

FIGURE 4 Interior equilibrium with mobile criminals. Population sizes are $K_A = K_B = 1$. There is an equilibrium with equal returns to crime, that is, $\Omega_A = \Omega_B$, positive numbers of criminals in both neighborhoods, that is, $c_A^* \in [0, C]$, more crime in the poor neighborhood, that is, $c_A^* < C/2$, maximum search in the rich neighborhood, and no search in the poor one, that is, $q_A^* = 1 - q_B^* = 1$

3.4 Within-neighborhood heterogeneity

We introduce within-neighborhood heterogeneity to account for the whole set of facts displayed in Section 2. In each neighborhood j, there are a fraction μ_j of poor households and the remaining fraction $1-\mu_j$ of rich households, with $\mu_A < \mu_B$. The poor have income $V^P = V$, whereas the rich have $V^R = (1+\delta)V$, $\delta > 0$. Wealth heterogeneity affects households' investments and criminals' search.

Definition 3 *An equilibrium with mobile criminals and within-neighborhood heterogeneity* (EWH) is a pair of search efforts (q_A^*, q_B^*), a collection of cdf $\{H_A^R, H_A^P, H_B^R, H_B^P\}$ and an allocation of criminals across neighborhoods (C_A^*, C_B^*) such that, for $j = A, B$,

(i) $\theta \in \Theta_j$ iff $\theta \in \arg\max_{\theta' \geq 0} W_j(\theta', q_j^*)$;

(ii) $q_j^* \in \arg\max_{q\in[0,1]} \Omega_j(q, H_j^R(., q_j^*, C_j^*/K_j), H_j^P(., q_j^*, C_j^*/K_j))$;

(iii) $C_j^* > 0$ iff $C_j \in \arg\max_{j\in\{A,B\}} \Omega_j(q_j^*, H_j^R(., q_j^*, C_j^*/K_j), H_j^P(., q_j^*, C_j^*/K_j))$;

(iv) $C_A^* + C_B^* = C$.

Definition 3 generalizes Definition 2 to two types of households. Therefore, parts (ii) and (iii) are slightly modified and distinguish the distributions of protection investment by neighborhood and household type.

Neglecting the j index, a criminal's expected payoff is still

$$\Omega = \mathbb{E}(\alpha V - \theta V) + q\Gamma - sq, \tag{10}$$

where the return to search is

$$\Gamma = \mathbb{E}[\max\{\alpha V - \theta V, \alpha V' - \theta' V'\}] - \mathbb{E}(\alpha V - \theta V). \tag{11}$$

The main difference with the homogenous case is that mean operators must account for income heterogeneity.

The expected numbers of burglaris by double-option criminals, η_2^R and η_2^P, depend on household type. A type-i household of protection θ is preferred to a household of similar type with protection θ' if and only if $\theta < \theta'$. This event occurs with probability $\Pr(\theta' > \theta \mid i) = 1 - H^i(\theta)$. When the households have different types, the type-i household is chosen if and only if $(\alpha - \theta)V^i > (\alpha - \theta')V^{-i}$. This event occurs with probability $\Pr(\theta' > \theta V^i/V^{-i} - \alpha(V^i - V^{-i})/V^{-i} \mid -i) = 1 - H^{-i}[\theta V^i/V^{-i} - \alpha(V^i - V^{-i})/V^{-i}]$. It follows that

$$\eta_2^R(\theta) = 2c_2\left[1 - (1-\mu)H^R(\theta) - \mu H^P(\theta(1+\delta) - \alpha\delta)\right], \tag{12}$$

$$\eta_2^P(\theta) = 2c_2\left[1 - (1-\mu)H^R(\theta/(1+\delta) + \alpha\delta/(1+\delta)) - \mu H^P(\theta)\right]. \tag{13}$$

As in the homogenous case, households play mixed strategies. The distribution of protection investment results from payoff equality over the support of the distribution.

There are two possible types of equilibria within a neighborhood: "segregated victimization" and "mix victimization", which we now describe.

Segregated victimization — This case occurs when $x < \bar{x} \equiv \alpha\delta/(1+\delta)$. The rich are much richer than the poor and always preferred to them by criminals. For $i = R, P$,

$$H^i(\theta) = \frac{\theta}{\alpha - \theta}\,\frac{\alpha - \overline{\theta}^i}{\overline{\theta}^i}, \tag{14}$$

for all $\theta \in [0, \overline{\theta}^i]$, with $\overline{\theta}^R = \alpha(1-\mu)x/(1-\mu x)$ and $\overline{\theta}^P = \alpha\mu x$. These distributions are very similar to the case of homogenous income described in Section 3.2. In particular, the support of each distribution widens when the protection attractiveness index, x, increases.

Using H^R and H^P, we obtain the return to search and the return to crime:

$$\Gamma_o = \frac{\alpha V}{x^2}\left[-2x[(1-\mu)(\delta - x(1+\delta-\mu)) + 1 - x\mu] + (x-2)\left[(1+\delta)(1-x)\log(1-x) - (\delta - x(1+\delta-\mu))\log(1-\mu x)\right]\right], \tag{15}$$

$$\Omega_o = \max_{q\in[0,1]}\left\{\frac{\alpha V}{x^2}\left[-2qx\left[(1-\mu)(\delta - x(1+\delta-\mu)) + 1 - x\mu\right] + [q(x-2)-x]\left[(1+\delta)(1-x)\log(1-x) - (\delta - x(1+\delta-\mu))\log(1-\mu x)\right]\right] - sq\right\} \tag{16}$$

Mix victimization — This case happens when $x \geq \bar{x}$. Poor households who do not protect much are preferred to rich households who invest a lot in protection. Then,

$$H^R(\theta) = \begin{cases} \dfrac{\theta}{\alpha-\theta}\dfrac{1-x}{(1-\mu)x} & \text{if } \theta \in [0,\bar{x}] \\ \dfrac{\theta(\delta - x(1+\delta)(1-\mu)) - \alpha\delta\mu x}{(\alpha-\theta)(\delta - x(1+\delta)\mu)}\dfrac{1-x}{(1-\mu)x} & \text{if } \theta \in \left[\bar{x}, \overline{\theta}^R\right] \end{cases}, \tag{17}$$

$$H^P(\theta) = \frac{\theta}{\alpha-\theta}\frac{\alpha - \overline{\theta}^P}{\overline{\theta}^P} \quad \text{if } \theta \in \left[0, \overline{\theta}^P\right], \tag{18}$$

with $\overline{\theta}^R = \alpha x$ and $\overline{\theta}^P = \alpha[x - \delta(1-x)]$. The rich distribution, H^R, has two parts. Below the threshold $\bar{x}$, strategic interaction only involves rich households, which explains the formula's simplicity. Above the threshold, the formula accounts for cases where very protected rich are preferred to poor with low protection.

The return to search and the return to crime are

$$\Gamma_e = \alpha V\frac{1-x}{x^2}[-2x + (x-2)\log(1-x)], \tag{19}$$

$$\Omega_e = \max_{q\in[0,1]}\left\{\frac{\alpha V}{x^2}(1-x)(1+\delta)[-2x + qx + (q(x-2)-x)\log(1-x)] - sq\right\}. \tag{20}$$

With mix victimization, the return to search is independent from the poor proportion, μ. Any increase in this proportion translates into lower competition for the rich who invest less as a result.

The model may admit different equilibrium configurations: segregated versus mix-victimization in each neighborhood, interior versus bounding search efforts, that is, $q^* \in (0, 1)$ versus $q^* = 0$ or $q^* = 1$, and interior versus extreme allocation of criminals between neighborhood, that is, $C_A^* > 0$ and $C_B^* > 0$ versus $C_A^* = 0$ or $C_A^* = C$.

However, these different configurations have different empirical implications. In particular, only some of them can predict the set of facts shown in Section 2. Let BE_i denote the property crime rate in neighborhood i, whereas BE_{ij} denotes the property crime rate for group j in this neighborhood. Let also $\theta_0 \geq 0$ be the level of protection above which households install an alarm. The proportion of households of type i in neighborhood j who install an alarm is $\text{PA}_{ij} = 1 - H_j^i(\theta_0)$. The neighborhood proportion is PA_i.

Proposition 4 *(Properties of EWH) There may be an EWH such that:*

(i) $BE_B > BE_A$, $BE_{AR} > BE_{AP}$, $BE_{BR} > BE_{BP}$;
(ii) $PA_B < PA_A$, $PA_{AR} > BE_{AP}$, $BE_{BR} > BE_{BP}$.

Properties (i) and (ii) describe the qualitative features of victimization and protection by neighborhood and income as reported in Section 2. We now prove Proposition 4. The proof consists of a parameterization leading to an equilibrium satisfying properties (i) and (ii). To go beyond an example without any empirical relevance, this parameterization replicates the main features of the Canadian income-victimization nexus as reported by Section 2 in Tables 1–3. We focus on households who experienced a break-in or an attempt of break-in, BE, and households who have installed an alarm, PA, our most natural measures protection.

The parameterization is based on a mix-victimization equilibrium and interior search efforts in both neighborhoods, that is, $(q_A^*, q_B^*) \in (0,1)^2$. As discussed in Proposition 3, we must have $0 < q_B^* < q_A^* \leq 1$. Moreover, our model features two groups of agents, which advocates for having a mix-victimization equilibrium: in reality, households slightly below the median income are sometimes preferred to households slightly above it when similar households invest differently.[10]

The technical aspects of the parameterization are provided in the Appendices. Three parameters are set to specific values before the structural estimation procedure. First, δ is set to 50%. This implies that the mean income of above-the-median households is about 2.5 times the mean income of below-the-median households, roughly what can be found in Canada's GSS. Second, V is normalized to 1 and α is set to 1/2. This is so because we cannot simultaneously identify V, α and s, that is, several combinations of these parameters can generate the same model outcomes. The choice $\alpha = 1/2$ is done to guarantee the existence of mix-victimization equilibria. Such equilibria occur when the protection attractiveness index $x_i = 2c_i^* q_i^* / \left(\gamma - c_i^* \left(1 - q_i^*\right)\right) > \bar{x} \equiv \alpha\delta/(1+\delta)$ for $i \in \{A, B\}$.

Once combined with the average investment, about 30%, $\alpha = 1/2$ implies that criminals get 20% of household income in case of break-in. This is too much, a result certainly due to the small number of parameters in this model.

The other parameters are computed by means of the method of moments. We have more moments than parameters, which means we cannot perfectly fit the different moments. Instead we choose the parameter configuration that minimizes the sum of squared errors between model outcomes and empirical moments.

Table 4 describes the parameter set, key model variables and model fit. The poor proportion is 36% in the rich neighborhood and 66% in the poor one. Lastly, the rich neighborhood is slightly more peopled than the poor one. The average neighborhood-specific search costs, sq_A^* and sq_B^*, amount to 7% of the income that can be stolen, αV, in the rich neighborhood and 3% in the poor one. The rich spend about 4% of αV^R in protection, against 3% of αV for the poor.

The protection attractiveness index, x, is 0.928, slightly below one, its max value. Criminals make larger search efforts in the rich neighborhood than in the poor one, $q_A^* = 0.865$ against $q_B^* = 0.341$. Consequently, the rich invest more in protection in the rich neighborhood and, therefore, are less likely to be preferred to poor households by the criminals than in the poor neighborhood, that is, the probability of being preferred to a poor household is $z_A^* = 0.591$ in the rich neighborhood against $z_B^* = 0.671$ in the poor one. Overall, the model predicts victimization better than protection. In particular, the chosen parameterization amplifies protection differentials between rich and poor households.

TABLE 4 Calibration of an equilibrium with interior search and mix victimization in both neighborhoods

	Parameters								
	V	α	δ	s	k	γ	μ_A	μ_B	θ_0
	1.0	0.5	1.5	0.040	0.522	0.053	0.355	0.658	0.393
	Model variables								
	x	q_A	q_B	z_A	z_B				
	0.928	0.865	0.341	0.591	0.671				
	Crime fit								
	BE	BE_A	BE_B	BE_{AR}	BE_{AP}	BE_{BR}	BE_{BP}	BE_R	BE_P
model	0.032	0.027	0.038	0.028	0.024	0.041	0.036	0.032	0.032
data	0.032	0.027	0.038	0.027	0.026	0.042	0.036	0.033	0.031
	Protection fit								
	PA	PA_A	PA_B	PA_{AR}	PA_{BR}	PA_{AP}	PA_{BP}	PA_R	PA_P
model	0.403	0.493	0.305	0.661	0.187	0.531	0.187	0.618	0.187
data	0.350	0.378	0.320	0.430	0.292	0.380	0.271	0.410	0.280

Notes: z_A and z_B are the neighborhood-specific probabilities that a random rich household is preferred to a random poor household; σ_c and σ_θ are, respectively, the standard errors of victimization and protection moments. The min variable is the value of the optimization criterion. See Appendix E for details on the calibration. The empirical variables BE and PA are described in Section 2.

The methodology to set the different parameters is explained in the Appendices. Variables z_A and z_B are the average probabilities that a rich household is preferred to a poor one in the rich neighborhood and in the poor one, respectively.

4 CONCLUSION

We develop a model of property crime and private protection allocation within cities. Our analysis is based on four Canadian facts: household income and victimization are uncorrelated or weakly positively correlated, rich neighborhoods are less victimized, rich households are more victimized than their neighbors, and rich households and neighborhoods invest more in private protection. In our theory, criminals choose a neighborhood and whether to make a search effort to compare potential victims. In turn, households choose how much they invest in private protection. Such investments reduce criminals' gains and divert their attention to less protected neighbors. Households in a rich neighborhood are more likely to enter in a rat race to protection characterized by strong incentives to self protect and to search and a weak incentive to enter the neighborhood. When sufficiently fierce, the rat race leads criminals to prefer poorer and less protected neighborhoods. A parameterization of our model features the Canadian pattern of victimization by household and neighborhood income as equilibrium outcomes.

Our model can be enriched in different ways. First, we assume the number of criminals is exogenous at city level. This number could increase with the return to crime. Such an extension would allow us to study the impact of public policies aimed at reducing overall property crime. Second, we suppose all agents are risk neutral, whereas risk aversion is certainly a key driver of protection investment. We could study the resulting demand for insurance. Third, protection reduces losses inflicted to households by criminals. In addition it could increase their probability of being caught. Lastly, we abstract from households' residential choices. They should respond to the geography of property crime.

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ENDNOTES

[1] The supply factors of criminal activities have been extensively studied, for example, the size of the police force (Levitt, 1997), the incarceration rate (Levitt, 1996), overall inequality (Brush, 2007; Freeman, 1999; Nilsson, 2004), wage inequality (Machin & Meghir, 2004), unemployment (Fougère et al., 2009), and geographic location (Glaeser & Sacerdote, 1999; Hémet, 2020). Less has been done to understand the demand side of crime. Early analyzes on protection can be found in Cook (1986), Shavell (1991) and Hotte and van Ypersele (2008).

[2] There already exist search models of crime, but they do not focus on protection (see, e.g., Burdett et al., 2003, 2004; Engelhardt et al., 2008; Galenianos et al., 2012; Huang et al., 2004).

[3] Our household income is defined as $Y = \ln(\text{household income}) - \ln(\text{CMA average household income})$. The neighborhood average income is defined in a similar way.

[4] This leaves out some important dynamic features of criminality, like recidivism and criminals occupational choices. Our model focuses on what types of neighborhood criminals operate in and what types of households they target. In that sense, it does not matter if the perpetrator is a recidivist or not. For more discussions on dynamic aspects of crime, see Imrohoroglu et al. (2004).

[5] There may be economies of scale in protection investment. A barking dog for instance protects small and big houses in the same manner. Such economies of scale imply that the rich have additional incentive to invest in protection, which would strengthen the force in our model leading the rich to protect more than the poor on average. However, other forms of protection investment do not benefit from such economies of scale, like motion detectors, which costs increase with the property size.

[6] The model could easily be extended to cases where losses differ from gains. Insurance coverage could imply smaller losses, while psychological costs associated with crime and breakage would implies higher losses. Similarly, the market value of stolen goods is most of the time lower than the replacement values.

[7] For tractability, we assume that the value of subsequent burglaries are the same as the first one. This is consistent with a world where households replace stolen items. Cases of multiple victimization are quite rare within our model. Imagine the case where victimization is purely random, even with a high burglary rate of 5%, the chance to be victimized twice is only 0.25% and drops to 0.012 5% for three incidents.

[8] In the Supplementary Appendix, we discuss the implications when this assumption is not satisfied.

[9] This equilibrium is not consistent with the facts reported by Section 2 whereby victimization decreases and private protection increases with neighborhood income.

[10] The only drawback of mix-victimization equilibria is that there is equal protection for the poor in the two neighbourhoods, whereas, in the GSS, the poor protect more when residing in the rich neighborhood.

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APPENDICES

A | International crime and victim survey

Figure 3 is built from the International Crime and Victim Survey. This survey is conducted in a large number of countries. It randomly samples households and the questions are addressed to one particular member of each household. Thus the sampling unit is the household. We consider OECD countries/states surveyed in 1996 and/or 2000.

We use two sets of questions. The first set is about property crime:

> Over the past five years, did anyone actually get into your house or flat without permission and steal or try to steal something? I am not including here thefts from garages, sheds or lock-ups.

and

> When did this happen? Was this... (1) this year; (2) last year; (3) before then; (4) don't know/can't remember

We divide the population into two groups: households above median income and households below it. In each subgroup, we compute the proportion of persons/households who respond "yes" to the first question, and "(1) this year" to the second one. We finally compute the ratio of these proportions.

The second set consists of one question about private protection:

> In order to help us understand why some homes are more at risk of crime than others, could I ask you a few questions about the security of your houses? Is your house protected by the following (multiple answers are allowed): 1) a burglar alarm, 2) special door locks, 3) special window or door grilles, 4) a dog that would detect a burglar, 5) a high fence, 6) a caretaker of security guard, 7) a formal neighborhood watch scheme, 8) friendly arrangements with neighbors to watch each others houses, 9) not protected by any of these, 10) respondent refuses to answer

In each income group, we compute the share of households who declare they are protected by an alarm. We then compute the ratio of the two shares.

B | Expected number of burglaries

Hereafter, we refer to theft options as matches. A criminal is matched with zero, one, or two households. All households draw their protection level from the continuously differentiable cdf H. The number of expected burglaries by single-option criminals is equal to the ratio of such criminals to households. Thus $\eta^1 = c_1$, irrespective of household's type. We now study the expected number of burglaries by double-option criminals η_2.

Consider a household whose protection level is θ. With probability

$$\Pi(t) = \binom{C_2}{t}\left(\frac{2}{K}\right)^t\left(1-\frac{2}{K}\right)^{C_2-t}, \tag{B1}$$

the victim is matched with t double-option criminals. When the victim is compared to another household, s/he is chosen by the criminal if and only if the victim's protection level θ is lower than the other household's protection level θ'.

Let us write $(\theta_1, ...\theta_t)$ the t order of the sub-sample of households who are matched with criminals who are also matched with the victim, that is, $\theta_1 < \theta_2... < \theta_t$. The victim is burglarized by $t - g$ criminals matched with another household if and only if $\theta \in [\theta_g, \theta_{g+1}]$. The joint density distribution (x, y) of $(g, g + 1)$ order statistics is given by

$$\frac{t!}{(g-1)!(t-g-1)!}H(x)^{g-1}(1-H(y))^{t-g-1}h(x)h(y). \tag{B2}$$

Therefore the probability that $\theta \in [\theta_g, \theta_{g+1}]$ is given by

$$p(g) = \int_\theta^{\overline{\theta}}\int_0^\theta \frac{t!}{(g-1)!(t-g-1)!}H(x)^{g-1}(1-H(y))^{t-g-1}h(x)h(y)\ dx\ dy = \frac{t!}{g!(t-g)!}H(\theta)^g(1-H(\theta))^{t-g}. \tag{B3}$$

as $\int H(x)^{g-1}h(x)dx = H(x)^g/g$ and $\int (1 - H(y))^{t-g-1}h(y)dy = (1 - H(y))^{t-g}/(g - t)$.

Thus the expected number of burglaries by double-option criminals is

$$\eta_2 = \sum_{t=0}^{C_2}\Pi(t)\sum_{g=0}^{t}(t-g)p(g) = 2c_2[1-H(\theta)]. \tag{B4}$$

C | Proofs

Proof of Proposition 1 Part (i) When $\Theta = \{0\}$, the return to search is 0. Therefore criminals do not search. Alternatively, when $q = 0$, households cannot divert criminals' attention to their neighbors. Therefore, $\Theta = \{0\}$. This reasoning proves that $\Theta = \{0\}$ and $q^* = 0$ is alway an ESN.

Part (ii) We have

$$W(\theta,.) = \{1-[c(1-q)+2cq(1-H(\theta))](\alpha-\theta)-\gamma\theta\}V. \tag{C1}$$

There cannot be a degenerate distribution of protection investment. On the contrary, suppose that the distribution is degenerate, and so the equilibrium investment is θ_0. This implies that $W(\theta, .) < W(\theta_0, .)$ for all $\theta \neq \theta_0$. Let $\theta = \theta_0 + \varepsilon > \theta_0$. We have

$$W(\theta) - W(\theta_0) = -(\gamma - c(1-q))\varepsilon V + cq(\alpha-\theta_0)V, \tag{C2}$$

which is positive for ε sufficiently small. It follows that the distribution H cannot be degenerate.

In a mixed-strategy equilibrium, households must be indifferent between the different protection investments. The cdf $H(\theta)$ is obtained by solving $W^i(\theta, .) = W^i(0, .)$. This gives point (ii) of lemma 1. At the upper bound of the support, $H(\overline{\theta}) = 1$. This gives $\overline{\theta}$.

Proof of Proposition 2 The function $\Gamma : [0,1] \rightarrow \mathbb{R}$ has the following properties: $\Gamma(0) = \Gamma(1) = 0$, $\Gamma(x) > 0$ for all $x \in (0, 1)$, and $\Gamma'(x) \gtreqless 0$ iff $x \lesseqgtr x_{\max}$.

Let E_{SN} denote the set of ESN, that is, $q^* \in E_{SN}$ if and only if $q^* \in \arg\max_{q\in[0,1]} \{q\Gamma(x(q^*)) - sq\}$. We have $1 \in E_{SN}$ if $\Gamma(x(1)) \geq s$, $\underline{q} \in E_{SN}$, $\underline{q} \in (0,1)$, if $\Gamma(x(\underline{q})) = s$, and $0 \in E_{IM}$ if $\Gamma(x(0)) < s$.

Figure 2 represents the return to search Γ as a function of x. We already know that x strictly increases with q from 0 to $x(1) < 1$. When $x(1) < x_{\max}$, the return to search strictly increases with q. The result follows: if $\Gamma(x(1)) < s$, then $\Gamma(x(q)) < s$ for all q and $E_{SN} = \{0\}$. If $\Gamma(x(1)) > s$, there is a unique $\underline{q} \in (0,1)$ such that $\Gamma(x(\underline{q})) = s$, and we have $E_{SN} = \{0, \underline{q}, 1\}$. When $x(1) > x_{\max}$, the reasoning is very similar. However, there is something new because $\Gamma(x_{\max}) > s$ and we may have $\Gamma(x(1)) < s$. In this case, there are $(\underline{q}, \overline{q})$, $0 < \underline{q} < \overline{q} < 1$, such that $\Gamma(x(\underline{q})) = \Gamma(x(\overline{q})) = s$.

Parts (i) to (iii) follow.

Proof of Proposition 3 Let E_{TN} be the set of ETN.

Part (i). $\{(0, C), (0, 0)\} \in E_{TN}$. We already know that $0 \in E_{SN}$ and so requirements (i) and (ii) of Definition 2 are satisfied. Moreover, we have $\Omega_A^* = \alpha V_A > \Omega_B^* = \alpha V_B$ and condition (iii) is satisfied. Finally, condition (iv) trivially holds.

Part (ii). Suppose on the contrary that $q_A^* \leq q_B^*$. Then $\Omega_A^* > \Omega_B^*$, which violates condition (iii) of Definition 2.

D | Distribution of protection investment with household heterogeneity

We refer to the separating case as a separating equilibrium of the protection subgame played by the residents of a neighborhood. Similarly, we refer to the pooling case as a pooling equilibrium of this game.

Proposition A1 *(Separating and pooling equilibria) Let $q > 0$ be given and $\overline{\delta} = \frac{x(q)}{1-x(q)}(1-\mu)$. The following properties hold:*

(i) If $\delta > \overline{\delta}$, there is a segregated-victimization equilibrium such that

$$H^i(\theta) = \frac{\theta}{\alpha - \theta} \frac{\alpha - \overline{\theta}^i}{\overline{\theta}^i}, \tag{D1}$$

with $\overline{\theta}^R = \alpha \frac{x(q)}{1-\mu x(q)}$ and $\overline{\theta}^P = \mu\alpha x(q)$;

(ii) If $\delta \leq \overline{\delta}$, there is a mix-victimization equilibrium such that

$$H^R(\theta) = \begin{cases} \dfrac{\theta}{\alpha-\theta}\dfrac{1-x(q)}{(1-\mu)x(q)} \equiv H_\ell^R(\theta) \text{ if } \theta \in \left[0, \dfrac{\alpha\delta}{1+\delta}\right) \\ \dfrac{\theta}{\alpha-\theta}\dfrac{1-x(q)}{(1-\mu)x(q)}\dfrac{\alpha\delta\mu(1-\mu x(q)) + \theta(1-\mu)1 - x(q)(1+\delta)\mu}{1+\mu\delta - x(q)(1+\delta)\mu} \equiv H_u^R(\theta) \text{ if } \theta \in \left[\dfrac{\alpha\delta}{1+\delta}, \overline{\theta}^R\right] \end{cases}, \tag{D2}$$

$$H^P(\theta) = \begin{cases} \dfrac{\theta}{\alpha-\theta}\dfrac{1-x(q)}{(1-\mu)x(q)}\dfrac{(1+\delta)(1-x(q)\mu)}{1+\mu\delta - x(q)(1+\delta)\mu} \equiv H_\ell^P(\theta) \text{ if } \theta \in [0, \overline{\theta}^R(1+\delta) - \alpha\delta) \\ \dfrac{\alpha(1+\delta)}{\mu x(q)}\dfrac{1-x(q)}{\alpha-\theta} + \dfrac{\mu x(q) - 1}{\mu x(q)} \equiv H_u^P(\theta) \text{ if } \theta \in \left[\overline{\theta}^R(1+\delta) - \alpha\delta, \overline{\theta}^P\right] \end{cases} \tag{D3}$$

with $\overline{\theta}^R = \alpha x(q) - \alpha(1-x(q))\delta\frac{2-\mu(1+x(q))}{(1-\mu)(1-\mu x(q))}$ *and* $\overline{\theta}^P = \alpha x(q) - (1-\delta x(q))$.

Proof. Part (i). Suppose that there is a segregated-victimization equilibrium, that is, $V^R(\alpha - \overline{\theta}^R) > \alpha V^P$, which is equivalent to $\overline{\theta}^R < \alpha\delta/(1+\delta)$. Then, we follow Proposition 1 and we obtain (D1). We finally check that this is indeed a segregated-victimization equilibrium when $\delta > \overline{\delta}$.

Part (ii). Suppose that there is a mix-victimization equilibrium with $\overline{\theta}^P > \overline{\theta}^R(1+\delta) - \alpha\delta$. This means that $\overline{\theta}^R$ is such that $V^R\left(\alpha - \overline{\theta}^R\right) < \alpha V^P$, which is equivalent to $\overline{\theta}^R > \alpha\delta/(1+\delta)$. Expected household payoffs are as follows:

$$W^R(\theta) = \begin{cases} \left[1 - c_1(\alpha-\theta) - 2c_2(\alpha-\theta)\left[1-(1-\mu)H_\ell^R(\theta)\right] - \gamma\theta\right]V^R \text{ if } \theta \in \left[0, \dfrac{\alpha\delta}{1+\delta}\right) \\ \left[1 - c_1(\alpha-\theta) - 2c_2(\alpha-\theta)\left[1-(1-\mu)H_u^R(\theta) - \mu H_\ell^P((1+\delta)\theta - \alpha\delta)\right] - \gamma\theta\right]V^R \text{ if } \theta \in \left[\dfrac{\alpha\delta}{1+\delta}, \overline{\theta}^R\right] \end{cases} \tag{D4}$$

$$W^P(\theta) = \begin{cases} \left[1 - c_1(\alpha-\theta) - 2c_2(\alpha-\theta)\left[1-(1-\mu)H_u^R\left(\dfrac{\theta+\alpha\delta}{1+\delta}\right) - \mu H_l^P(\theta)\right] - \gamma\theta\right]V^P \text{ if } \theta \in \left[0, \overline{\theta}^R(1+\delta) - \alpha\delta\right) \\ \left[1 - c_1(\alpha-\theta) - 2c_2(\alpha-\theta)\mu\left[1-H_u^P(\theta)\right] - \gamma\theta\right]V^P \text{ if } \theta \in \left[\overline{\theta}^R(1+\delta) - \alpha\delta, \overline{\theta}^P\right] \end{cases} \tag{D5}$$

The functions H^R and H^P are continuous, monotonically increasing, with $H^R(0) = 0$, $H^P(0) = 0$, $H^R(\overline{\theta}^R) = 1$ and $H^P(\overline{\theta}^P) = 1$. Thus they are two cdf. Moreover, W^R and W^P are constant over their respective support. Finally, $\overline{\theta}^R > \alpha\delta/(1+\delta)$ iff $\delta \leq \overline{\delta}$. ■

As claimed in the text, $\overline{\theta}^i$ is strictly increasing in c and q in both types of equilibrium. Moreover, the victimization probability is always larger for the rich than for the poor. This statement is obvious in the segregated-victimization case where the rich are always preferred to the poor. In the mix-victimization case, suppose that a criminal is randomly matched with a poor and a rich household. The probability that the rich household is selected is

$$H_u^R\left(\frac{\alpha\delta}{1+\delta}\right) + \int_{\frac{\alpha\delta}{1+\delta}}^{\overline{\theta}^R}\int_{(1+\delta)\overline{\theta}^R - \alpha\delta}^{\overline{\theta}^P} h^P(\theta^P)h^R(\theta^R)d\theta^R d\theta^P = \frac{1}{2} + \delta\frac{\alpha - \overline{\theta}^P}{\overline{\theta}^R} > 1/2. \tag{D6}$$

E | Calibration

As stated in the body text, we normalize income to unity, that is, V = 1, and the max proportion of income that criminals can steal to 50%, that is, α = 0.5. We also set δ to 1.5.

With heterogenous victimization and interior search efforts, the return to crime and the return to search do not depend on the neighborhood-specific poor proportion. We have $\Gamma_o^A(x) = \Gamma_o^B(x) = \Gamma_e(x)$ and $\Omega_e^A(x) = \Omega_e^B(x) = \Omega_e(x)$ for all x ∈ [0, 1]. Therefore we can choose x and set the marginal search cost s = Γ_e(x). It follows that $x_A^* = x_B^* = x$.

The definitions of x_A^* and x_B^* give us two relationships between γ, q_A^* and c_A^* on the one hand, and γ, q_B^* and c_B^* on the other hand. Indeed, $x_A^* = 2c_A^* q_A^*/\left(\gamma - c_A^*\left(1-q_A^*\right)\right)$ and $x_B^* = 2c_B^* q_B^*/\left(\gamma - c_B^*\left(1-q_B^*\right)\right)$. We set c_A^* and c_B^* to the Canadian values. This leaves us with two equations and three unknowns, γ, q_A^* and q_B^*.

For a given pair $\left(c_A^*, c_B^*\right)$, the overall victimization rate is $c = kc_A^* + (1-k)c_B^*$, where $k = K_A/(K_A + K_B)$ is the share of individuals in the rich neighborhood. Therefore we set c to its empirical value and fix $k = \left(c - c_B^*\right)/\left(c_A^* - c_B^*\right)$ to ensure that this is also the equilibrium one.

The following vector of parameters remains: $(x, \mu_A, \mu_B, \gamma)$. We target victimization rates for rich and poor households in rich and poor neighborhoods. As for protection, the difficulty consists of matching a continuous prediction, that is, the level of protection, with a categorical outcome, that is, the household's proportion with an alarm. Let $\theta_0 \geq 0$ be the level of protection above which households install an alarm. The corresponding households' proportions are $1 - H_j^i(\theta_0)$, for household type $i = R, P$ and neighborhoods $j = A, B$. These theoretical proportions must be compared with the empirical values.

The victimization probabilities are as follows:

$$BE_{AR} = \left(1 - q_A^*\right)c_A^* + 2q_A^*c_A^*(\mu_A z_A + (1 - \mu_A)/2), \tag{E1}$$

$$BE_{AP} = \left(1 - q_A^*\right)c_A^* + 2q_A^*c_A^*((1 - \mu_A)(1 - z_A) + \mu_A/2), \tag{E2}$$

$$BE_{BR} = \left(1 - q_B^*\right)c_B^* + 2q_B^*c_B^*(\mu_B z_B + (1 - \mu_B)/2), \tag{E3}$$

$$BE_{BP} = \left(1 - q_B^*\right)c_B^* + 2q_B^*c_B^*((1 - \mu_B)(1 - z_B) + \mu_B/2), \tag{E4}$$

$$BE_R = ((1 - \mu_A)kc_{AR} + (1 - \mu_B)(1 - k)c_{BR})/((1 - \mu_A)k + (1 - \mu_B)(1 - k)), \tag{E5}$$

$$BE_P = (\mu_A kc_{AP} + \mu_B(1 - k)c_{BP})/(\mu_A k + \mu_B(1 - k)), \tag{E6}$$

where $z_i = 1/2 + \delta\left(1 - x_i^*\right)/\left(2x_i^*(1 - \mu_i)\right)$ is the probability that a random rich household of neighborhood i is preferred to a random poor household by a typical criminal.

The corresponding proportions with an alarm are

$$PA_{AR} = 1 - H_R(\theta_0, \mu_A), \tag{E7}$$

$$PA_{BR} = 1 - H_R(\theta_0, \mu_B), \tag{E8}$$

$$PA_{AP} = PA_{BP} = PA_P = 1 - H_P(\theta_0), \tag{E9}$$

$$PA_{AR} = [(1 - \mu_A)kPA_{AR} + (1 - \mu_B)(1 - k)PA_{BR}]/ \tag{E10}$$

$$[(1 - \mu_A)k + (1 - \mu_B)(1 - k)], \tag{E11}$$

$$PA_A = \mu_A PA_{AP} + (1 - \mu_A)PA_{AR}, \tag{E12}$$

$$PA_B = \mu_B PA_{BP} + (1 - \mu_B)PA_{BR}, \tag{E13}$$

$$PA = kPA_A + (1 - k)PA_B, \tag{E14}$$

where

$$H_R(\theta, \mu) = \begin{cases} \dfrac{1 - x}{(1 - \mu)x}\dfrac{\theta}{\alpha - \theta} \text{ if } 0 \leq \theta < \alpha\delta/(1 + \delta) \\ \dfrac{1 - x}{(1 - \mu)x}\dfrac{-\delta\theta + x[\alpha\delta\mu + (1 - \mu)(1 + \delta)\theta]}{[x + (x - 1)\delta](\alpha - \theta)} \text{ if } \alpha\delta/(1 + \delta) \leq \theta \leq \alpha x \end{cases}, \tag{E15}$$

$$H_P(\theta, \mu) = \frac{(1 - x)(1 + \delta)}{x - \delta + x\delta}\frac{\theta}{\alpha - \theta} \text{ if } 0 \leq \theta \leq \alpha(x - \delta + x\delta). \tag{E16}$$

The minimization criterion is based on victimization and protection moments. As for victimization moments, we have

$$\text{crit}_c = \sum_{i=R,P;j=A,B} \left(BE_{ij} - \overline{BE}_{ij}\right)^2, \tag{E17}$$

where $\overline{BE}_{ij}$ is computed from BE, the empirical proportion that experienced a break-in or an attempt of break-in the last 12 months.

As for protection moments, we have

$$\text{crit}_\theta = \sum_{i=R,\,P;j=A,B} \left(PA_{ij} - \overline{PA}_{ij}\right)^2, \tag{E18}$$

where $\overline{PA}_{ij}$ is computed from PA, the empirical share of households who have installed an alarm.

The minimization criterion is

$$\text{crit} = \text{crit}_c + \lambda\text{crit}_\theta. \tag{E19}$$

We set the weight λ to 10^{-5}. As explained in Section 3.4, this allows us to finely reproduce victimization moments, letting parameter θ_0 adjust to fit protection moments.

The parameter vector $(x, \mu_B, \gamma, \theta_0)$ is found by scanning the parameter over the space over a grid of 100 elements for each parameter. Therefore there are four parameters for 13 moments, of which 8 are linearly independent. The parameter space accounts for the three constraints described in Section 3.4. In particular, the return to search, Γ, must be decreasing, which we ensure by scanning x above the value maximizing this return.

The overall value of the criterion is about 1.0×10^{-5}. The standard deviation of victimization moments is $\sigma_c = 0.001$ against $\sigma_\theta = 0.134$ for protection moments.