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Structure and Dynamics of Combustion Waves in Premixed Gases

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Structure and Dynamics of Combustion Waves in Premixed Gases

Paul Clavin

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Lecture XII

Galloping detonations

Lecture 12 : Galloping detonations

12-1. Physical mechanisms

Instability mechanism

Two limiting cases

12-2. General formulation

Constitutive equations

Strong shock in the Newtonian approximation

12-3. Strongly overdriven regimes in the limit $(\gamma - 1) \ll 1$

Distinguished limit

Integral-differential equation for the dynamics

Oscillatory instability

12-4. CJ detonations for small heat release

Reactive Euler equations in 1-D geometry

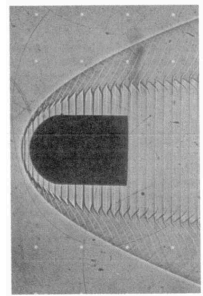
Near CJ regimes for small heat release. Transonic reacting flows

Slow time scale

Asymptotic model for CJ or near CJ regimes

Results for simplified chemical kinetics

XII-1) Physical mechanisms

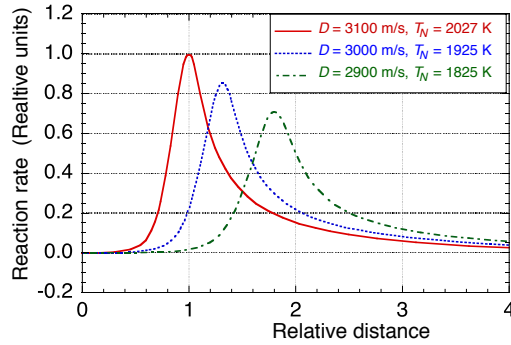


Lehr 1972

Detonation = inner shock followed by an exothermal reaction zone

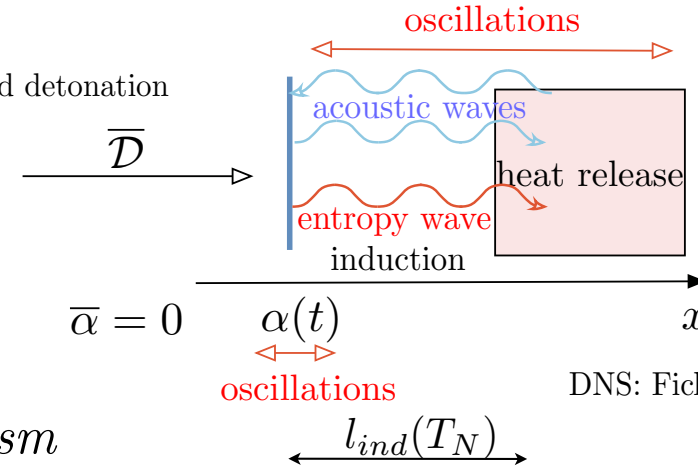
Inner structure = uniform induction zone + zone of heat release

Galloping detonation = oscillatory instability: oscillation of the velocity of the lead shock



$l_{ind}(T_N)$

reference frame of the unperturbed detonation



DNS: Ficket Woods 1966

Instability mechanism

$\delta D = -\dot{\alpha}_t \neq 0 \Rightarrow \delta T_N \Rightarrow \delta l_{ind}$ motion of the heat release zone produces a piston like effect

feedback loop

the **unstable** character depends on the **phase shift**

Two different coupling mechanisms: $\left\{ \begin{array}{l} \text{acoustic waves} \\ \text{entropy wave} \end{array} \right.$

Two limiting cases

Strongly overdriven regimes for $(\gamma - 1) \ll 1$: quasi-isobaric flow, the delay by the acoustic waves is negligible

CJ regime for $q_m/c_p T_u \ll 1$ and $(\gamma - 1) \ll 1$: transonic flow, the entropy wave is negligible

Lecture 12 : Galloping detonations

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Reactive Euler equations in 1-D geometry

$D/Dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla$
 $\frac{1}{a\rho} \times$
 $a^2 = \gamma p / \rho$
 $1-D : D^\pm/Dt \equiv \partial/\partial t \pm (a \pm u)\partial/\partial x$

$\frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \mathbf{u},$
 $\left(\rho \frac{D\mathbf{u}}{Dt} = -\nabla p, \right)$
 $p = (c_p - c_v)\rho T,$

$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u \frac{\partial}{\partial x}$
 $\frac{D\psi}{Dt} = \frac{\dot{w}}{\bar{t}_N},$
 $\frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N},$
 $\Rightarrow \pm \left(\frac{1}{\gamma p} \frac{Dp}{Dt} + \nabla \cdot \mathbf{u} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N} \right)$
 $\Rightarrow \frac{1}{\gamma p} \frac{D^\pm p}{Dt} \pm \frac{1}{a} \frac{D^\pm u}{Dt} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$

$\dot{w}(\psi, T)$
entropy equation

1-D Euler (compressible) eqs.
 $\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] p \pm \frac{1}{a} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$
generalized acoustic eqs.
($\delta p = \pm \rho a \delta u$)
(useful form for the following)

Continuous set of feed back loops

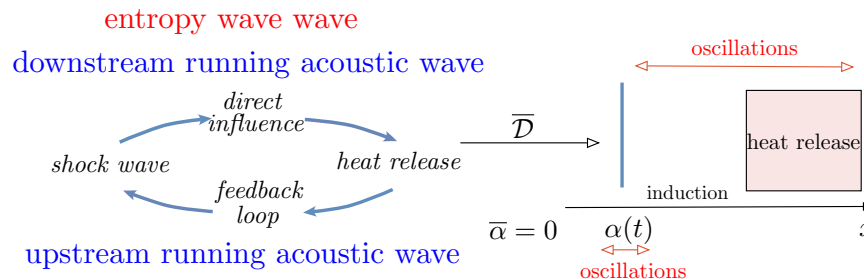
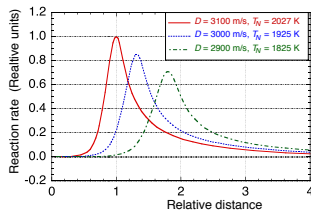
upstream running acoustic wave

$$\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u - a) \frac{\partial}{\partial x} \right] p - \frac{1}{a} \left[\frac{\partial}{\partial t} + (u - a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

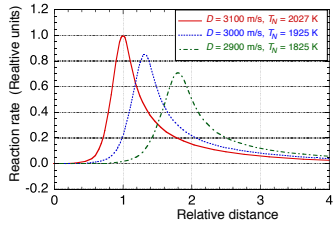
downstream running acoustic wave

$$\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u + a) \frac{\partial}{\partial x} \right] p + \frac{1}{a} \left[\frac{\partial}{\partial t} + (u + a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

acoustic waves
entropy wave



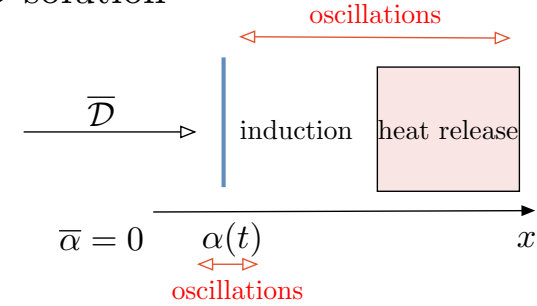
Galloping detonation = pulsating instability of the 1-D solution



velocity of the shock oscillates

reaction rate oscillates

$$\mathcal{D}(t) = \bar{\mathcal{D}} - \dot{\alpha}_t$$



Reduced equations

Reactive Euler equations

$$\frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \mathbf{u}, \quad \rho \frac{D\mathbf{u}}{Dt} = -\nabla p, \quad p = (c_p - c_v)\rho T, \quad \frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma - 1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}, \quad \frac{D\psi}{Dt} = \frac{\dot{w}(\psi, T)}{\bar{t}_N}, \quad \bar{t}_N \equiv \tau_r(\bar{T}_N)$$

$D/Dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla$

Reduced mass weighted distance from the shock (useful for **unsteady 1-D** problems)

$$x = \alpha(t) \quad \text{instantaneous shock position} \quad x \equiv \frac{1}{\rho_u \bar{\mathcal{D}} \bar{t}_N} \int_{\alpha(t)}^x \rho(x', t) dx', \quad t \equiv \frac{t}{\bar{t}_N}, \quad \bar{t}_N \equiv \tau_r(\bar{T}_N)$$

reaction time at the Neumann state of the unperturbed solution

$$\frac{\partial}{\partial x} = \frac{\rho}{\rho_u} \frac{1}{\bar{\mathcal{D}} \bar{t}_N} \frac{\partial}{\partial x}, \quad \bar{t}_N \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) = \frac{\partial}{\partial t} + m(t) \frac{\partial}{\partial x}, \quad \text{where} \quad m(t) \equiv \left[\frac{\rho(x, t)[u(x, t) - \dot{\alpha}_t]}{\rho_u \bar{\mathcal{D}}} \right]_{x=\alpha(t)} = 1 - \frac{\dot{\alpha}_t}{\bar{\mathcal{D}}}$$

$\dot{\alpha}_t \equiv d\alpha/dt$

$$\begin{cases} u|_{x=0}(t) - \dot{\alpha}_t = u_N(t) \\ \rho_N(t)u_N(t) = \rho_u(\bar{\mathcal{D}} - \dot{\alpha}_t) \end{cases}$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + m(t) \frac{\partial}{\partial x}, \quad m(t) = 1 - \frac{\dot{\alpha}_t}{\bar{\mathcal{D}}}$$

$$\frac{D}{Dt} \left(\frac{\rho_u}{\rho} \right) = \frac{\partial}{\partial x} \left(\frac{u}{\bar{\mathcal{D}}} \right) \Leftrightarrow \frac{D}{Dt} \left(\frac{\bar{\rho}_N}{\rho} \right) = \frac{\partial}{\partial x} \left(\frac{u}{\bar{u}_N} \right),$$

$$\frac{D}{Dt} \left(\frac{u}{\bar{\mathcal{D}}} \right) = -\frac{\partial}{\partial x} \left(\frac{p}{\rho_u \bar{\mathcal{D}}^2} \right), \quad p = (c_p - c_v)\rho T,$$

$$\frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma - 1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} = \frac{q_m}{c_p T} \dot{w}, \quad \frac{D\psi}{Dt} = \dot{w},$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + m(t) \frac{\partial}{\partial x} \quad \frac{D}{Dt} \left(\frac{\rho_u}{\rho} \right) = \frac{\partial}{\partial x} \left(\frac{u}{\bar{D}} \right) \Leftrightarrow \frac{D}{Dt} \left(\frac{\bar{\rho}_N}{\rho} \right) = \frac{\partial}{\partial x} \left(\frac{u}{\bar{u}_N} \right),$$

$$m(t) = 1 - \frac{\dot{\alpha}_t}{\bar{D}}, \quad \frac{D}{Dt} \left(\frac{u}{\bar{D}} \right) = - \frac{\partial}{\partial x} \left(\frac{p}{\rho_u \bar{D}^2} \right), \quad p = (c_p - c_v) \rho T,$$

$$\frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma - 1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} = \frac{q_m}{c_p T} \dot{w}, \quad \frac{D\psi}{Dt} = \dot{w},$$

$m(t)$ unknow

$$\dot{\alpha}_t / \bar{D} = 1 - m(t)$$

adiabatic compression

Boundary conditions

Neumann state $x = 0$: $\rho = \rho_N(t)$, $p = p_N(t)$, $T = T_N(t)$ $\rho_N(t)(u - \dot{\alpha}_t) = \rho_u \bar{D} m(t)$ $\psi = 0$
 expressed in terms of $m(t)$ by the RH conditions

$$\frac{u_N}{\bar{D}} = \frac{\rho_u}{\rho_N} = \frac{(\gamma - 1)M_u^2 + 2}{(\gamma + 1)M_u^2}, \quad \frac{p_N}{p_u} = \frac{2\gamma M_u^2 - (\gamma - 1)}{(\gamma + 1)}, \quad M_u = \bar{M}_u m(t)$$

$$\frac{T_N}{T_u} = \frac{[2\gamma M_u^2 - (\gamma - 1)][(\gamma - 1)M_u^2 + 2]}{(\gamma + 1)^2 M_u^2}, \quad M_N^2 = \frac{(\gamma - 1)M_u^2 + 2}{2\gamma M_u^2 - (\gamma - 1)}$$

Burnt gas $x \rightarrow \infty$: $\left\{ \begin{array}{l} \text{overdriven regimes: } u = \bar{u}_b \\ \text{CJ regime: } p - \bar{p}_b = \bar{\rho}_b \bar{a}_b (u - \bar{u}_b) \end{array} \right.$ i.e. outgoing acoustic waves (radiation condition)

Analytical solutions are obtained in limiting cases

Strong shock in the Newtonian approximation

$$\bar{M}_u \gg 1, \quad (\gamma - 1) \ll 1 \Rightarrow \bar{M}_N^2 \approx \frac{\gamma - 1}{2} + \frac{1}{\bar{M}_u^2} \ll 1$$

Distinguished limit: $\bar{M}_u^2 \gg 1, \quad (\gamma - 1)M_u^2 = O(1)$

$$\epsilon^2 \equiv \bar{M}_N^2 \ll 1, \quad \bar{M}_u^2 = O(1/\epsilon^2), \quad (\gamma - 1) = O(\epsilon^2)$$

small parameter

$$\bar{u}_N / \bar{D} = \bar{\rho}_u / \bar{\rho}_N \approx \epsilon^2, \quad \bar{p}_N / \bar{p}_u \approx \bar{M}_u^2 = O(1/\epsilon^2), \quad \bar{a}_N / \bar{D} \approx \epsilon, \quad (\bar{a}_N / \bar{a}_u)^2 \approx [2 + (\gamma - 1)\bar{M}_u^2] / 2 = O(1)$$

$$\left. \begin{array}{l} \bar{\rho} \bar{u} - \bar{\rho}_N \bar{u}_N = 0 \\ \frac{d\bar{p}}{dx} + \bar{\rho} \frac{d\bar{u}}{dx} = 0 \end{array} \right\}$$

and $a_N^2 / \gamma = p_N / \rho_N \Rightarrow$

$$(\bar{p} / \bar{p}_N - 1) = -\epsilon^2 (\bar{u} / \bar{u}_N - 1),$$

Quasi-isobaric approximation of the shocked gas

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Strongly overdriven detonations in the limit $(\gamma - 1) \ll 1$

(Clavin He 1996)

Distinguished limit

$$\epsilon^2 \equiv \bar{M}_N^2 \ll 1, \quad \bar{M}_u^2 = O(1/\epsilon^2), \quad (\gamma - 1) = O(\epsilon^2)$$

$$q_N \equiv q_m/c_p \bar{T}_N = O(1) \Leftrightarrow \text{strongly overdriven regime}$$

$$M_{u_{CJ}} = \sqrt{Q} + \sqrt{Q+1} \quad \text{where } Q \equiv \frac{\gamma+1}{2} \frac{q_m}{c_p T_u}$$

$$\bar{T}_N/T_u = O(1) \Rightarrow M_{u_{CJ}} = O(1) \Rightarrow M_u \gg M_{u_{CJ}}$$

negligible adiabatic compression

$$\left. \begin{aligned} \frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma-1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} &= \frac{q_m}{c_p T} \dot{w}, \\ \delta p/p &= O(\epsilon^2) \end{aligned} \right\} \Rightarrow \left\{ \begin{aligned} \frac{\partial T}{\partial t} + m(t) \frac{\partial T}{\partial x} &= \frac{q_m}{c_p} \dot{w}(\psi, T), \\ \frac{\partial \psi}{\partial t} + m(t) \frac{\partial \psi}{\partial x} &= \dot{w}(\psi, T) \end{aligned} \right.$$

$$x = 0: \quad \psi = 0, \quad T = T_N(t)$$

$$T_N/T_u \approx [(\gamma - 1)M_u^2 + 2]/2 \quad \text{where } M_u^2 = \bar{M}_u^2 (1 - \dot{\alpha}_t/\mathcal{D})^2$$

The solution yields $T(x,t)$ and $\psi(x,t)$ in terms of $m(t) = 1 - \dot{\alpha}_t/\mathcal{D}$

Distinguished limit

$$(\gamma - 1)\beta_N = O(1)$$

 \Leftrightarrow

$$\beta_N = O(1/\epsilon^2)$$

$$\beta_N \equiv \left[\frac{T}{\dot{w}} \frac{\partial \dot{w}}{\partial T} \right]_{T=T_N}$$

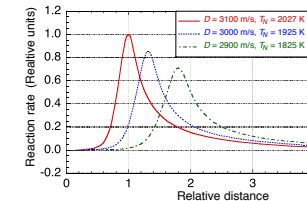
$$m(t) = 1 - \dot{\alpha}_t/\bar{D}, \quad \dot{\alpha}_t/\bar{D} = O(1/\beta_N) \Rightarrow \begin{cases} \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} = \frac{q_m}{c_p} \dot{w}(\psi, T), \\ \frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} = \dot{w}(\psi, T) \end{cases} \quad x = 0 : T = T_N(t), \psi = 0, \\ \frac{D}{Dt} \approx \frac{\partial}{\partial t} + \frac{\partial}{\partial x}$$

$$(T_N - \bar{T}_N)/\bar{T}_N = O(1/\beta_N) \Rightarrow \Theta_N(t) \equiv \beta_N(T_N(t) - \bar{T}_N)/\bar{T}_N = O(1), \quad \dot{w} - \bar{\dot{w}} = O(1)$$

Steady state solutions

$$\begin{cases} \frac{dT}{dx} = \frac{q_m}{c_p} \dot{w}(\psi, T), \\ \frac{d\psi}{dx} = \dot{w}(\psi, T) \end{cases} \quad x = 0 : T = T_N, \psi = 0, \quad T_N = \text{cst.} \neq \bar{T}_N \\ \Theta_N \equiv \beta_N(T_N - \bar{T}_N)/\bar{T}_N$$

$$T = \mathcal{T}(\Theta_N, x), \quad \psi = \mathcal{Y}(\Theta_N, x), \quad \Omega(\Theta_N, x) \equiv \dot{w}(\mathcal{T}, \mathcal{Y})$$

*Unsteady solution* (retarded functions)

$$T(x, t) = \mathcal{T}(\Theta_N(t - x), x), \quad \psi(x, t) = \mathcal{Y}(\Theta_N(t - x), x), \quad \dot{w} = \Omega(\Theta_N(t - x), x)$$

Distinguished limit

$$(\gamma - 1)\beta_N = O(1) \iff \beta_N = O(1/\epsilon^2)$$

$$m(t) = 1 + O(1/\beta_N) \quad \left[\frac{\partial}{\partial t} + m(t) \frac{\partial}{\partial x} \right] \approx \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right]$$

Rankine-Hugoniot

$$\frac{u_N}{\bar{u}_N} \approx 1 + \frac{\dot{\alpha}_t}{\bar{u}_N}$$

$$\left. \begin{aligned} \frac{T_N}{T_u} &= \frac{[2\gamma M_u^2 - (\gamma - 1)][(\gamma - 1)M_u^2 + 2]}{(\gamma + 1)^2 M_u^2} \\ M_u^2 &= \bar{M}_u^2 (1 - \dot{\alpha}_t/\bar{D})^2 \end{aligned} \right\} \Rightarrow (T_N(t) - \bar{T}_N)/\bar{T}_N \approx -(\gamma - 1)\dot{\alpha}_t/\bar{u}_N \ll 1, \quad \Rightarrow \frac{\dot{\alpha}_t}{\bar{u}_N} = -\frac{\Theta_N(t)}{\beta_N(\gamma - 1)}$$

Quasi-isobaric approximation in the shocked gas + Continuity

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right] \frac{T}{\bar{T}_N} \approx \left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right] \frac{\bar{\rho}_N}{\rho} = \frac{\partial}{\partial x} \left(\frac{u}{\bar{u}_N} \right)$$

Energy

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right] \frac{T}{\bar{T}_N} = \frac{q_m}{c_p \bar{T}_N} \dot{w} \quad \frac{u_b}{\bar{u}_N} - \frac{u_N}{\bar{u}_N} = \frac{q_m}{c_p \bar{T}_N} \int_0^\infty \dot{w}(x, t) dx$$

$$\dot{w} = \Omega(\Theta_N(t - x), x)$$

Nonlinear integral-equation

$$\int_0^\infty \Omega(\bar{\Theta}_N, x) dx = 1 \quad \boxed{1 + b\Theta_N(t) = \int_0^\infty \Omega(\Theta_N(t - x), x) dx,} \quad b^{-1} = \beta_N(\gamma - 1) \frac{q_m}{c_p \bar{T}_N} = O(1)$$

(Clavin He 1996)

strongly overdriven detonations in the Newtonian limit

quasi-isobaric approximation in the shocked gases

+ energy eq. for T

heat release per unit mass

progress variable

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}\right) \left[\ln T - \frac{(\gamma-1)}{\gamma} \ln p \right] = \frac{q_m}{c_p T} \frac{\dot{w}(T, Y)}{t_r} \quad \left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r}\right) Y = \frac{\dot{w}(T, Y)}{t_r}$$

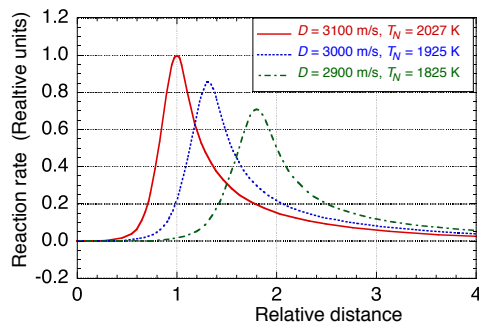
\dot{w} : non-dimensional reaction rate

chemical kinetics

mass

$$\frac{1}{\rho} \frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} = 0$$

$$\rho T \approx \text{cst.}$$



unsteady / steady distribution of the rate of heat release

$$\Omega(\Theta_N(t-x), x)$$

$$\Theta_N(t) = \frac{\beta(T_N(t) - \bar{T}_N)}{\bar{T}_N} \quad x \equiv \frac{1}{t_r \rho_u \bar{D}} \int_{\alpha(t)}^x \rho dx \quad t \equiv \frac{t}{t_r}$$

steady state

$$\Omega(\Theta_N, x)$$

+

conservation of mass

 and boundary conditions

Rankine-Hugoniot at $x = 0$: $\frac{D(t) - \bar{D}}{\bar{D}} \Leftrightarrow \Theta_N(t)$ $x \rightarrow \infty$: boundedness

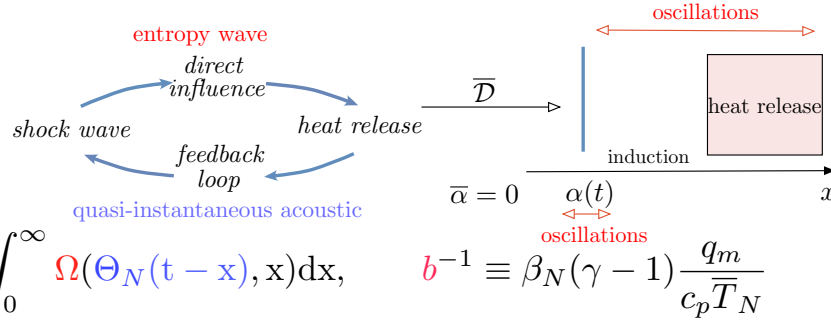
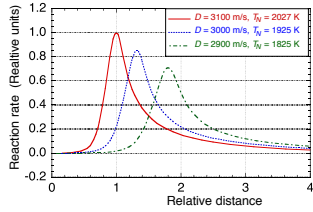
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integral equation for $\Theta_N(t)$

$$1 + b\Theta_N(t) = \int_0^\infty \Omega(\Theta_N(t-x), x) dx,$$

$$b^{-1} \equiv \beta_N(\gamma - 1) \frac{q_m}{c_p \bar{T}_N}$$

strongly overdriven detonations in the Newtonian limit
quasi-isobaric approximation in the shocked gas



$$1 + b\Theta_N(t) = \int_0^\infty \Omega(\Theta_N(t-x), x) dx, \quad b^{-1} \equiv \beta_N(\gamma - 1) \frac{q_m}{c_p \bar{T}_N}$$

Arrhenius $\Omega(\Theta_N, x) \approx e^{\Theta_N} \bar{\Omega}(e^{\Theta_N} x), \quad l/\bar{l} = e^{-\Theta_N}$

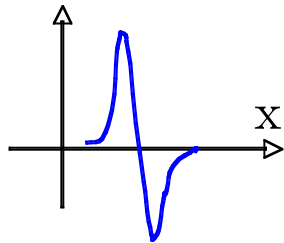
$\Omega(\Theta_N, x) \approx e^{\Theta_N} \bar{\Omega}(e^{\Theta_N} x)$

$\int_0^\infty \Omega(\Theta_N, x) dx = 1$

Linearization. Normal mode analysis

$\Theta_N(t) \propto e^{\sigma t}$
 $\sigma = \hat{s} + i\hat{\omega}$

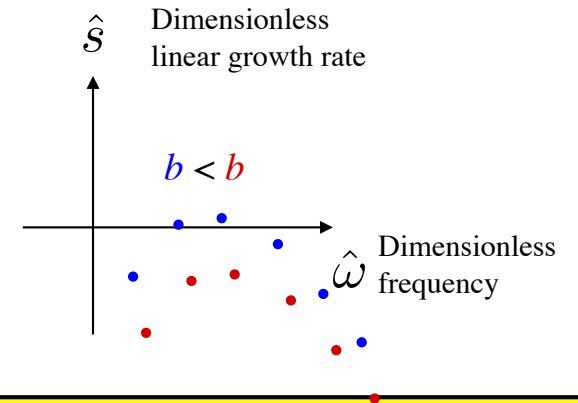
$\Omega'_N(x) \equiv d[x\bar{\Omega}(x)]/dx$



$b = \int_0^\infty \Omega'_N(x) e^{-\sigma x} dx$

$\hat{\omega} = O(1) \Leftrightarrow \omega = O(\bar{t}_N)$

frequency of oscillation
of order of the transit time



Poincaré-Andronov (Hopf) bifurcation

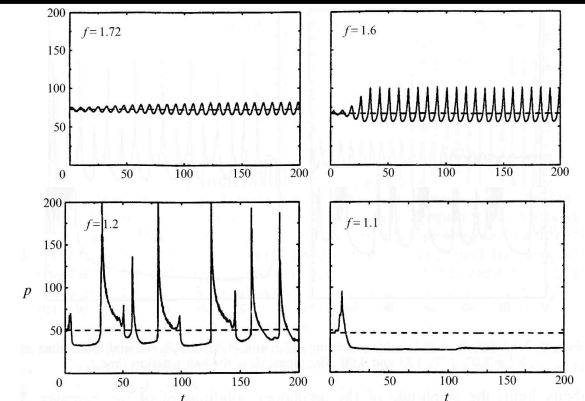
high thermal sensitivity, β_N
and/or

stiffness of the distribution of heat release $\Omega(\Theta_N, x)$

promote the instability

intermittency

nonlinear effects: stochasticity + dynamical quenching



OK with DNS

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Reactive Euler equations in 1-D geometry

Near CJ regimes for small heat release. Transonic reacting flows

Slow time scale

Asymptotic model for CJ or near CJ regimes

Results for simplified chemical kinetics

Reactive Euler equations in 1-D geometry

$$D/Dt \equiv \partial/\partial t + \mathbf{u} \cdot \nabla$$

$\frac{1}{a\rho} \times$

$$a^2 = \gamma p / \rho$$

$$1-D : D^\pm/Dt \equiv \partial/\partial t \pm (a \pm u)\partial/\partial x$$

$$\frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \mathbf{u}, \quad \left(\rho \frac{D\mathbf{u}}{Dt} = -\nabla p, \right) \quad p = (c_p - c_v)\rho T,$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u \frac{\partial}{\partial x}$$

$$\frac{D\psi}{Dt} = \frac{\dot{w}}{\bar{t}_N}$$

$$\frac{1}{T} \frac{DT}{Dt} - \frac{(\gamma - 1)}{\gamma} \frac{1}{p} \frac{Dp}{Dt} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N},$$

$$\Rightarrow \pm \left(\frac{1}{\gamma p} \frac{Dp}{Dt} + \nabla \cdot \mathbf{u} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N} \right)$$

$$\Rightarrow \frac{1}{\gamma p} \frac{D^\pm p}{Dt} \pm \frac{1}{a} \frac{D^\pm u}{Dt} = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

entropy equation

$\dot{w}(\psi, T)$

1-D Euler (compressible) eqs.

$$\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] p \pm \frac{1}{a} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

generalized acoustic eqs.
($\delta p = \pm \rho a \delta u$)

(useful form for the following)

CJ detonations for small heat release

(Clavin Williams 2002)

Near CJ regimes for small heat release. Transonic reacting flows

CJ and overdriven regimes

$$M_{u_{CJ}} = \sqrt{Q} + \sqrt{Q+1} \quad Q \equiv \frac{(\gamma+1)}{2} \frac{q_m}{c_p T_u} \quad f \equiv \frac{(\bar{M}_u - \bar{M}_u^{-1})^2}{4Q}$$

CJ regime: $f = 1$, overdriven regime: $f > 1$

Small heat release approximation (transonic regimes)

$$Q \ll 1$$

small parameter: $\epsilon^2 \equiv Q \ll 1$

$$M_{u_{CJ}}^2 \approx 1 + 2\epsilon$$

overdriven regime near CJ: $f = O(1)$

$$\bar{M}_u^2 \approx 1 + 2\epsilon\sqrt{f} \quad \bar{M}_u^2 - M_{u_{CJ}}^2 \approx 2\epsilon(\sqrt{f} - 1)$$



ϵ in p.9 \neq ϵ in p.5

$$q_N \equiv q_m/c_p T_N = O(\epsilon^2)$$

$$f = O(1)$$

Rankine-Hugoniot conditions

$$(M_u^2 - 1) \ll 1$$

see p.6 lecture X

$$\frac{T_N}{T_u} \approx 1 + (\gamma - 1)(M_u^2 - 1),$$

$$\frac{p_N}{p_u} \approx \frac{\rho_N}{\rho_u} \approx 1 + (M_u^2 - 1),$$

$$\psi = 0$$

Steady state

$$(\bar{M}_u^2 - 1) \approx (1 - \bar{M}_N^2) \approx 2\epsilon\sqrt{f}$$

heat release

compressible effect

$$\frac{\bar{T} - \bar{T}_N}{T_u} \approx \epsilon^2 \psi - (\gamma - 1)\epsilon\sqrt{f}[1 - \sqrt{1 - (\psi/f)}],$$

$$(1 - \bar{M}_b^2) \approx 2\epsilon\sqrt{f - 1}$$

$$\frac{(\gamma + 1)}{2\gamma} \frac{(\bar{p} - p_u)}{p_u} = \frac{(\gamma + 1)}{2} \bar{M}_u^2 \frac{(\bar{\mathcal{D}} - \bar{u})}{\bar{\mathcal{D}}} \approx \epsilon\sqrt{f} \bar{M}_u [1 + \sqrt{1 - (\psi/f)}],$$



crossover temperature : $T_u < T^* < T_N$ separation of scale: $\tau_{coll}/\bar{t}_N \ll (\bar{M}_u - 1) \Rightarrow e^{-E/k_B T_N} \ll \epsilon$

Distinguished limit

$$(\gamma - 1) = O(\epsilon) \Rightarrow (T - T_N)/T_N = O(\epsilon^2)$$

Non-dimensional equations $t \equiv \frac{t}{\bar{t}_N}$, $x \equiv \frac{x}{a_u \bar{t}_N}$, $\check{u} \equiv \frac{u}{a_u}$, $\check{\pi} \equiv \frac{1}{\gamma} \ln \left(\frac{p}{p_u} \right)$, $\check{\theta} \equiv \frac{(T - \hat{T}_u)}{\hat{T}_u}$

anticipating $\check{\theta} = O(\epsilon^2)$ $a/a_u = 1 + O(\epsilon^2)$ $1 - \check{u} = O(\epsilon)$ $\check{\pi} = O(\epsilon)$

the variation of a is negligible in $\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] p \pm \frac{1}{a} \left[\frac{\partial}{\partial t} + (u \pm a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$

$$\begin{aligned} \left[\frac{\partial}{\partial t} + (1 + \check{u}) \frac{\partial}{\partial x} \right] (\check{\pi} + \check{u}) &= \epsilon^2 \dot{w}, & \left[\frac{\partial}{\partial t} + \check{u} \frac{\partial}{\partial x} \right] [\check{\theta} - (\gamma - 1)\check{\pi}] &= \epsilon^2 \dot{w} \\ \left[\frac{\partial}{\partial t} - (1 - \check{u}) \frac{\partial}{\partial x} \right] (\check{\pi} - \check{u}) &= \epsilon^2 \dot{w}, & \left[\frac{\partial}{\partial t} + \check{u} \frac{\partial}{\partial x} \right] \psi &= \dot{w}, \end{aligned}$$

Slow time scale

$$M_u^2 - 1 = O(\epsilon) \Rightarrow \text{transonic flow: } u/a = 1 + O(\epsilon)$$

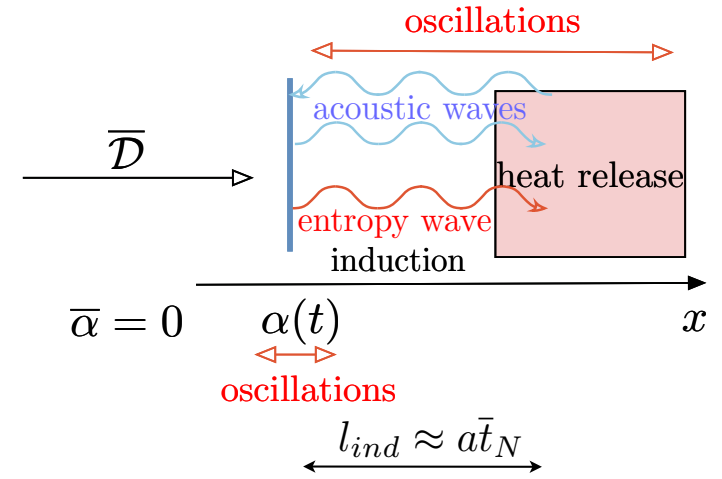
time scale of the *downstream* propagating acoustic wave : $l_{ind}/a = \bar{t}_N$

$$\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u+a) \frac{\partial}{\partial x} \right] p + \frac{1}{a} \left[\frac{\partial}{\partial t} + (u+a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

time scale of the *upstream* propagating acoustic wave : $l_{ind}/(a-u) \approx \bar{t}_N/\epsilon$

$$\frac{1}{\gamma p} \left[\frac{\partial}{\partial t} + (u-a) \frac{\partial}{\partial x} \right] p - \frac{1}{a} \left[\frac{\partial}{\partial t} + (u-a) \frac{\partial}{\partial x} \right] u = \frac{q_m}{c_p T} \frac{\dot{w}}{\bar{t}_N}$$

longest delay in the feed back loop $\approx \bar{t}_N/\epsilon$



Scaling

Period of oscillation = $O(\bar{t}_N/\epsilon)$

non dimensional time of order unity

$\tau \equiv \frac{t}{\bar{t}_N/\epsilon} = \epsilon t \quad \leftarrow \quad t \equiv t/\bar{t}_N$

instantaneous position of the lead shock wave $x = \alpha(\epsilon t/\bar{t}_N)$

$a \equiv \alpha(\tau)/(a_u \bar{t}_N) \leftarrow$ non dimensional position

non dimensional variable of order unity μ, π, θ :

$\check{u} \equiv \frac{u}{a_u} = 1 + \epsilon \mu, \quad \check{\pi} \equiv \frac{1}{\gamma} \ln \left(\frac{\hat{p}}{\hat{p}_u} \right) = \epsilon \pi, \quad \check{\theta} \equiv \frac{(T - \hat{T}_u)}{T_u} = \epsilon^2 \theta$

reference frame of the moving shock : $\tau \equiv \epsilon t,$

$$\xi \equiv x - a(\tau), \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial t} = \epsilon \left(\frac{\partial}{\partial \tau} - \dot{a}_\tau \frac{\partial}{\partial \xi} \right)$$

⚠ u and μ are flow velocities in the lab frame

$$x \equiv x/(a_u \bar{t}_N) \nearrow$$

$$\begin{aligned} \left[\frac{\partial}{\partial t} + (1 + \check{u}) \frac{\partial}{\partial x} \right] (\check{\pi} + \check{u}) &= \epsilon^2 \dot{w}, & \left[\frac{\partial}{\partial t} + \check{u} \frac{\partial}{\partial x} \right] [\check{\theta} - (\gamma - 1)\check{\pi}] &= \epsilon^2 \dot{w} \\ \left[\frac{\partial}{\partial t} - (1 - \check{u}) \frac{\partial}{\partial x} \right] (\check{\pi} - \check{u}) &= \epsilon^2 \dot{w}, & \left[\frac{\partial}{\partial t} + \check{u} \frac{\partial}{\partial x} \right] \psi &= \dot{w}, \end{aligned}$$

\Rightarrow

$$\begin{aligned} \frac{\partial(\pi + \mu)}{\partial \xi} &= 0 & \frac{\partial(\theta - h\pi - \psi)}{\partial \xi} &= 0, \\ \left[\frac{\partial}{\partial \tau} + (\mu - \dot{a}_\tau) \frac{\partial}{\partial \xi} \right] (\pi - \mu) &= \dot{w}, & \frac{\partial \psi}{\partial \xi} &= \dot{w} \end{aligned}$$

Arrhenius law : $\dot{w}(\psi, \theta) = (1 - \psi)e^{\beta_\epsilon(\theta - \bar{\theta}_N)}$ with

$\beta_\epsilon \equiv \frac{E}{k_B T_N} \epsilon^2 = O(1)$

$$h \equiv (\gamma - 1)/\epsilon = O(1)$$

$$\dot{a}_\tau \equiv da(\tau)/d\tau$$

Asymptotic model for CJ or near CJ regimes

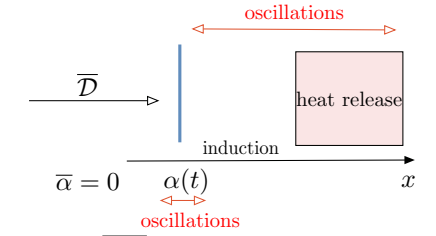
$$\epsilon^2 \equiv q_m/c_p T_u \ll 1, \quad (\gamma - 1) = O(\epsilon), \quad E/k_B T_N = O(1/\epsilon^2) \quad (\text{Clavin Williams 2002})$$

$$\frac{\partial(\pi + \mu)}{\partial \xi} = 0 \quad \frac{\partial(\theta - h\pi - \psi)}{\partial \xi} = 0,$$

$$\left[\frac{\partial}{\partial \tau} + (\mu - \dot{a}_\tau) \frac{\partial}{\partial \xi} \right] (\pi - \mu) = \dot{w}, \quad \frac{\partial \psi}{\partial \xi} = \dot{w}$$

$$\dot{a}_\tau \equiv da(\tau)/d\tau$$

$$M_u = (\bar{\mathcal{D}} - \dot{\alpha}_t)/a_u = \bar{M}_u - \epsilon \dot{a}_\tau$$



Boundary conditions at the Neumann state

$$h \equiv (\gamma - 1)/\epsilon = O(1) \quad f \equiv \text{overdrive factor}$$

$$\frac{T_N}{T_u} \approx 1 + (\gamma - 1)(M_u^2 - 1),$$

$$\xi = 0 : \quad \theta \equiv \theta_N = 2h(\sqrt{f} - \dot{a}_\tau), \quad \pi \equiv \pi_N = 2(\sqrt{f} - \dot{a}_\tau)$$

$$\frac{p_N}{p_u} \approx \frac{\rho_N}{\rho_u} \approx 1 + (M_u^2 - 1)$$

$$M_u^2 - 1 \approx 2(\bar{M}_u - 1) - 2\epsilon \dot{a}_\tau$$

$$\theta_N - h\pi_N = 0$$

$$(\bar{M}_u - 1) \approx \epsilon \sqrt{f}$$

$$\xi = 0 : \quad \mu \equiv \mu_N = -\sqrt{f} + 2\dot{a}_\tau \quad \text{and} \quad \psi = 0,$$

$$\mu_N + \pi_N = \sqrt{f}$$

$$\rho_u(\bar{\mathcal{D}} - \dot{\alpha}_t) = \rho_N(u|_{x=\alpha} - \dot{\alpha}_t)$$

⚠ u and μ are flow velocities in the lab frame

$$\mu + \pi = \sqrt{f}$$

$$\theta = h\sqrt{f} - h\mu + \psi$$

The problem is reduced to solve **two equations** for μ and ψ

$$\left[\frac{\partial}{\partial \tau} + (\mu - \dot{a}_\tau) \frac{\partial}{\partial \xi} \right] \mu = -\frac{\dot{w}}{2}, \quad \frac{\partial \psi}{\partial \xi} = \dot{w}(\psi, \theta)$$

$$\xi = 0 : \quad \mu = -\sqrt{f} + 2\dot{a}_\tau \quad \text{and} \quad \psi = 0,$$

Boundary condition in the burnt gas

$$\xi \rightarrow \infty : \quad \psi = 1, \quad \mu = \bar{\mu}_b = -\sqrt{f - 1}$$

yields an integral equation for $\dot{a}_\tau(\tau)$

Nonlinear equation for a transonic reacting flow

$$\dot{w}(\psi = 1) = 0$$

$$\left[\frac{\partial}{\partial \tau} + (\mu - \dot{a}_\tau) \frac{\partial}{\partial \xi} \right] \mu = -\frac{\dot{w}}{2}, \quad \frac{\partial \psi}{\partial \xi} = \dot{w}(\psi, \theta)$$

$$\theta = h\sqrt{f} - h\mu + \psi$$

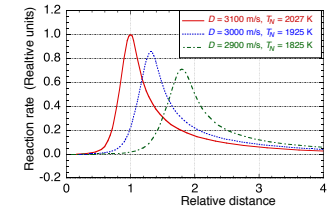
$$\xi = 0: \quad \mu = -\sqrt{f} + 2\dot{a}_\tau \quad \text{and} \quad \psi = 0,$$

$$\xi \rightarrow \infty: \quad \psi = 1, \quad \mu = \bar{\mu}_b = -\sqrt{f-1}$$

Result for simplified chemical kinetics

Simplification:

The reaction rate depends only on $T_N(t)$



The stability analysis is similar to that of strongly overdriven regimes !

Similar integral equation but with a delay controlled by the upstream running acoustic wave

$$\Delta(\xi) = \int_0^\xi \frac{d\xi}{|\mu(\xi)|}$$

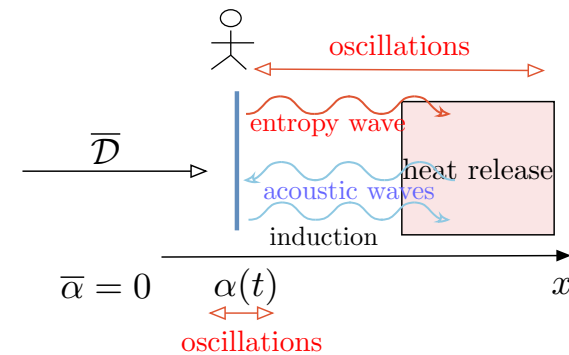
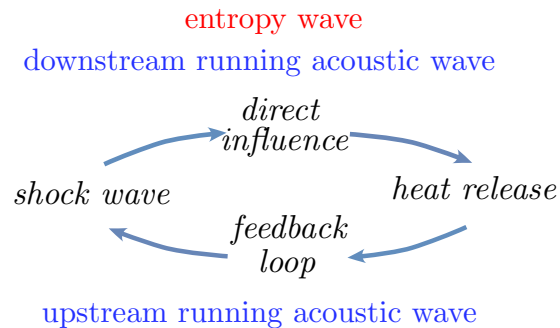
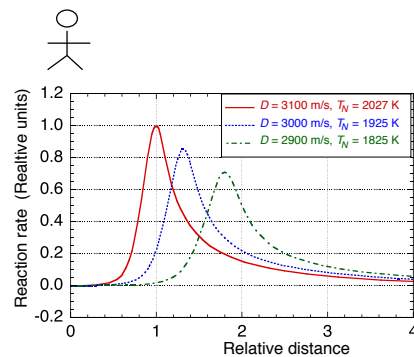
$$\dot{a}_\tau(\tau) = \int_0^\infty \left[\frac{1}{4\sqrt{f}} \Omega'_N(\xi) + G(\xi) \right] \dot{a}_\tau(\tau - \Delta(\xi)) d\xi$$

Instability due to thermal sensitivity

Stabilizing term due to residual compressible effects

GENERAL CONCLUSION

Galloping detonations are due to a **phase shift** in the loop between the lead shock and the heat release, controlled by the **entropy** wave and the **upstream running acoustic** wave



Strongly overdriven detonation in the Newtonian limit (P.C. & L. He 1996)

quasi-isobaric flow

dominant mechanism: **entropy wave** \longrightarrow

CJ (or near CJ) conditions close to the instability threshold (P.C. & F.A. Williams 2002)

transonic flow

dominant mechanism: **acoustic wave** \longleftarrow

Comparison with DNS