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# When do privatizations have popular support? A voting model<sup>☆</sup>

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We consider a general equilibrium model with vertical preferences, where workers and consumers are differentiated respectively by their sensitivity to effort and their intensity of preference for quality. We consider a public monopoly, i.e. which is owned equally by all individuals. The question is under which conditions the firm will be privatized and at which rate/price. The decisions are taken through majority vote in a plurality system. When the firm is controlled by the State, the price is determined through a vote among all the population. Otherwise, the price is the one which maximizes the profit. We prove that, when the maximum disutility of working in the firm is higher than the maximum utility of consuming its output, privatization may emerge as a possible choice of the majority, even if no hypothesis is made on the efficiency of a private management relative to a public one.

*Keywords:* Democracy; General equilibrium; Privatization; Vertical preferences; Majority vote; Public monopoly

## 1. Introduction

The main object of the present paper is to study the relation between democracy and privatization, i.e. whether majority voting may or not lead to the privatization of a public monopoly, when the consumption of the output produced by the monopoly and the employment by the monopoly are important in the economy.

A great wave of privatization began in Great Britain and the USA under the Thatcher and Reagan administrations,<sup>1</sup> lasting until the recent years. Mass privatizations affected many British firms in the eighties and nineties (British Rail, British Aerospace, Rover Group, British Telecom, Sealink ferries, British Petroleum, Rolls-Royce, British Steel Corporation, British Gas among others). Many others took place in Latin America at the same period (water management, transportation, telecommunication enterprises being sold off to the private sector) as part of liberal economic policies. In France, privatizations began under the Chirac government (1986) and continued over all the following administrations

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<sup>1</sup> A few privatizations had occurred earlier: privatization of common lands in Britain, known as the enclosure (1760–1820), in the thirties, privatization by the Nazi government of many state-owned enterprises in several sectors (Railways, mining, steel, ship-lines, banking, local public utilities, shipyard, see [Bel, 2010](#)), privatizations by democratic governments in the fifties and the sixties such as of Volkswagen in 1961 by the West German government.

until now, affecting a great number of important enterprises (Saint-Gobain, Paribas, Société Générale, Havas, Renault, Total to name only a few). In Russia, Eastern and Central Europe, the transition from socialist to market economies was also accompanied in the nineties by massive privatizations of State-Owned Enterprises.

A greater efficiency of private firms was generally invoked to motivate privatization. But efficiency may be intended in different ways, as noticed by [Willner \(2001\)](#) “Attitudes to ownership are however often based on [...] confusion about the meaning of efficiency” (page 723).

A first unambiguous meaning is cost-efficiency. Because the managers of public firms are less easily monitored than the managers of private firms, they are supposed to have less incentives to exert efforts to reduce costs. For instance [Laffont and Tirole \(1991\)](#) write that “the cost of public ownership is a sub-optimal investment by the firm’s managers in those assets that can be redeployed to serve social goals pursued by the public” (page 84).

But there is also a second, more disputable, sense of the word, namely State-Owned Enterprises are often considered inefficient because they pursue other objectives than profit-maximization, leading to levels of output and/or employment which are considered as too high compared to their profit-maximizing levels.<sup>2</sup> The problem with this use of the word is that profit-maximizing output and employment levels are not necessarily welfare-maximizing. In particular, regarding SOEs which are generally

<sup>2</sup> This is in line with the findings of [Duanmu and Pittman \(2019\)](#) suggesting that “SOEs may have ‘multitask’ responsibilities in terms of protecting employment as well as achieving efficiency”.

large firms having substantial market power, profit-maximization is well-known to generally lead to too small output and employment levels, relative to welfare maximizing ones. This is particularly the case for some services such as water provision or health. Regarding the management of SOEs and their possible privatization, labor and/or access to the output are fundamental issues and may legitimately be considered to make the “right” decision. Room has also to be made for democracy and the opinion of majority. The existing theoretical literature heeds attention to labor at the aggregate level, thus from a centralized point of view, while we deal with employment by the monopoly at the individual level, thus from a decentralized point of view. Moreover the literature is scarce on output. The access to such outputs may nonetheless be fundamental, even vital in some cases, for instance water, electricity, transportation etc. A study of the World Bank (Jones et al., 2008) assessed the impact of privatization in 4 sub-saharian countries on workers as well as on consumers and concluded that, if privatization is done properly “it can lead to substantial welfare gains that are reasonably and equitably distributed across stakeholders -consumers, workers, governments, and owners or operators”. However, taking as an example water provision, according to Lobina et al. (2014), “Cities, regions and countries worldwide are increasingly choosing to close the book on water privatization and to “remunicipalize” services by taking back public control over water” because of the failures of the private sector in terms of output provision in quantity and quality. “Where near-universal access to water has been achieved, it has virtually always been through a public commitment” (The Guardian<sup>3</sup>).

These considerations may explain why, even when decided by elected governments, privatizations are often unpopular. In France for instance, according to a YouGov poll,<sup>4</sup> 75% of respondents were ready to sign a petition to stop the privatization of ADP, the Paris airports society, which was decided by the Macron administration. This fear from privatization brings us back to the debate between the nineteenth century economists on the effect of democracy, in particular of universal suffrage, on property rights. The fear of some economists then was that the universal extension of voting would lead to “expropriation of capital” for redistribution purposes (what may take place nowadays in the form of nationalization).<sup>5</sup> Even Ricardo who supported the expansion of suffrage in fact did not support “extending the elective franchise [ ] universally to all the people, but to that part of them which cannot be supposed to have any interest in overturning the rights of property”.<sup>6</sup> In the same spirit, Hayek (1982) proposed to limit drastically the powers of future political majorities in order notably to avoid any infringement of property rights.

Though privatizations are not decided by majority voting, they are easier to implement and more likely to take place if they have a majority support. In the present paper, assuming for heuristic purposes no superior cost efficiency of private relative to public management and considering explicitly employment and output provision, we determine under which circumstances a majority support is more likely to occur for the privatization of a public firm and under which ones it is likely to be opposed by a majority of voters.

<sup>3</sup> Water privatisation: a worldwide failure? <https://www.theguardian.com/global-development/2015/jan/30/water-privatisation-worldwide-failure-lagos-world-bank#top>.

<sup>4</sup> [https://www.huffingtonpost.fr/entry/referendum-sur-adp-75-des-francais-pret-a-signer-une-petition-pour-le-reclamer-sondage-exclusif\\_fr\\_5d1b5d02e4b07f6ca5841d1d](https://www.huffingtonpost.fr/entry/referendum-sur-adp-75-des-francais-pret-a-signer-une-petition-pour-le-reclamer-sondage-exclusif_fr_5d1b5d02e4b07f6ca5841d1d)

<sup>5</sup> Concerning this debate, see for instance Robbins (1978), in particular pp. 201.

<sup>6</sup> in his letter to Trower, 20th December 1818 (Ricardo, Sraffa's ed. 2005).

To this end, we adapt the small general equilibrium model developed in Kahloul et al. (2017) with an initially public monopoly and an exogenous quality of the product, supposing that the firm's output price is determined through majority voting whenever the State retains a majority of shares and is fixed so as to maximize profits when the private investors take over the control.<sup>7</sup> Any sale of shares of the firm is supposed to take place at a price which reflects the post-privatization firm's value<sup>8</sup> and may lead either to a situation when the firm is privately managed (“radical” privatization) or to a situation where the enterprise remains publicly managed. As already noticed, though it is often argued that private firms are more cost efficient because of greater incentives of private owners than politicians to keep costs low by more effectively monitoring and/or motivating appointed managers (see for instance Bishop et al., 1994), we suppose here for heuristic reasons that there is no intrinsic superiority or inferiority of private management over public management: the only difference between the two is that a public firm may choose a price which differs from the profit-maximizing one. The economy encompasses a single firm producing a vertically differentiated product using labour as the unique input and a population of workers/consumers/shareholders characterized by two parameters: preference for product quality and sensitivity to effort. Each individual decides whether to work or not and decides whether to purchase one unit of the product or not, in order to maximize his/her utility given his/her income. When the price is set at some given level, the salary adjusts in order to equalize demand and supply on the labor market.

We study a three-stage game. In the first step, the individuals vote in order to decide the fraction of the shares to be sold to private investors (thus whether to privatize the firm and at which rate).<sup>9</sup> In the second step, the price per share is determined through a vote. Finally, in the third step, the output price is selected.<sup>10</sup> If the private investors hold a majority of shares, the equilibrium price is the profit-maximizing one. When the enterprise is (or remains) controlled by the State, the price is the one which is preferred by a majority of voters. In this latter case, we show that this is the profit-maximizing price iff the public share in the capital of the firm is large enough and/or working in the firm and consuming its output is not very attractive for most agents; but it is otherwise a low price (involving financial losses which have to be covered by the State budget) which is intended to please the, then more important, part of the population which consumes the good and is employed by the enterprise.<sup>11</sup>

Our main results are as follows. When the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, any ownership structure such that the firm maximizes its profit is an equilibrium structure. That of course includes the cases where the State retains less than one half of the shares (private control) but also the cases where it retains an important fraction of the shares but with a majority of voters preferring the profit-maximizing output price. An immediate consequence of this first result is that any cost-efficiency advantage,

<sup>7</sup> Privatization is broadly intended here as the transfer of the ownership of a fraction of the capital of a public sector enterprise to the private sector. We shall speak more specifically of private take-overs following which the State retains only a minority of the shares as “radical privatizations”.

<sup>8</sup> In the case when the firm would make losses after being privatized, we shall consider alternatively that either the State is able to commit to cover the losses of private shareholders or that no (partial) privatization leading to losses is feasible.

<sup>9</sup> We do not consider the distribution of shares to voters (for such an analysis see Biais and Perotti, 2002).

<sup>10</sup> The equilibrium wage rate being a function of the price, as noticed above.

<sup>11</sup> It is important to note that the consumption and work decisions are endogenous to this model, so that the obtained result is far from being directly predictable.

even infinitesimal, of private over public control is then enough to entail a majority support for a radical privatization. When, on the contrary, the maximum disutility of working in the firm is smaller than the maximum utility of consuming its output, then an absolute majority of voters oppose any sale of shares to private investors. Moreover they strictly prefer that the firm remains totally public to a radical privatization, so that a possible small cost-efficiency superiority of private over public management would not be enough to reverse the voting result.

Now, when the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, *at the social optimum* or, equivalently, at the competitive equilibrium, the individuals who choose not to consume the good nor work at the firm are more numerous than individuals who consume the good and work at the firm.<sup>12</sup> In the reverse case, *at the social optimum*, an absolute majority of the population is better off consuming the good or working in the firm.

Though the main part of the paper is about investigating the conditions under which there may or not exist a majority support for the privatization of a SOE, we also analyze here the welfare effects of privatization. Given the quasi-linear individual utility function in our model, welfare is expressed in units of numeraire and is the sum of the utilities of the voters.<sup>13</sup> The conclusions of this analysis are surprisingly similar to the results obtained about majority voting, hinging as well on the comparison between the maximum utility of consuming one unit of the firm's output and the maximum disutility of producing it. When the latter is greater than the former, any ownership structure such that the firm maximizes its profit, including the initial one (all shares belonging to the State), gives the same welfare level, which is greater than an ownership structure leading to a smaller price. When the former is greater than the latter, any privatization reduces welfare.

It is also interesting to understand the factors which are driving the results. Basically the agents have preferences over the ownership structure, namely over the fraction of shares which is held by the State, only in so far as it determines the level of the output price, which, in turn, determines the levels of wage and profits, and the agents' decisions whether or not to consume the product and/or to work at the firm.

One would think that the agents who have no stake in the firm as consumers or workers or both want it to maximize profits and do not care whether this is as a public or a private entity and that, on the contrary, the agents who have a stake in the firm favor an output price lower than the profit-maximizing one, which occurs necessarily at the expense of the agents who have no interest in the enterprise. This retains part of the truth but is too simplistic and accordingly a bit misleading since the agents' decisions to work and consume depend themselves endogenously on the output price which is implied by the ownership structure (and in the case of a public firm by the agents' votes). It may happen for instance that an agent would be better off with a low price, in which case he/she would work and consume, but nevertheless choose not to work nor to consume when the higher profit-maximizing price prevails. In this paper we escape this ambiguity by referring to the social optimum/competitive equilibrium benchmark situation: when a relative majority of agents would in this benchmark case neither consume nor work, any

<sup>12</sup> It should be noticed that the actual patterns of consumption and work (at equilibrium in the game we consider) are in general different from the patterns at the competitive equilibrium. At the profit-maximizing price for instance, we prove that there is **always** a majority of agents who do not consume and do not work.

<sup>13</sup> This definition is rigorous iff there may exist non-distortionary transfers allowing "winners" to possibly compensate "losers" of a contemplated privatization.

ownership structure yielding profit-maximization is preferred by a majority of workers.

**The literature review** There is an abundant literature on privatization. Two main streams exist (Cavaliere, 2006). The first one applies the principal-agent theory to the question of privatization. Comparing State Owned Enterprises (SOE) and regulated private firms, Sappington and Stiglitz (1987) shed light on the role of information asymmetry, risky production and the attitude toward risk of private producers. Shapiro and Willig (1990) assumed that a public framer has to make a choice between operating production through a SOE by delegating administrative power to a "malevolent agent" who pursues a private agenda, or with a regulated private firm facing an asymmetry of information on production. Another group of articles assume benevolent governments. Laffont and Tirole (1991) extended their previous model with incomplete information to compare public and private firms in the framework of incomplete contracts. Schmidt (1996), concerned with the soft budget constraint, compared SOEs with managers weakly motivated by reducing costs and regulated private firms with asymmetry of information.

The second stream of literature deals with privatization from a political economy point of view. Vickers and Yarrow (1988) pointed out the importance of labour in the decision of privatization which will avoid excess employment. Politicians may nevertheless refrain from privatization even if it is Pareto efficient because it will not be supported politically. The strong resistance of workers who will lose their employment will not be outweighed by the support of winners because privatization benefits are widespread. Labor is also at the core of the analysis of Shleifer and Vishny (1994) and Boycko et al. (1996). Because politicians will try to maintain excess employment even in privatized firms, privatization will not necessarily lead to reduced labor costs. Robinson and Torvik (2005) examined the soft budget constraint from another perspective. Politicians want "bad projects" to be supported by voters who will benefit from the redistribution of subsequent resources. Bortolotti and Pinotti (2003) compared the motivation for privatization of "majoritarian" political systems and "consensual-corporatist" democracies and conclude that the former are more likely to privatize. Bi-ais and Perotti (2002) showed the difference in motivation for privatization by right wing and left wing politicians. Right wing politicians are motivated by future support from the constituency of shareholders of newly privatized firms, and left wing by redistribution of revenues accruing from privatization. Borner (2004) distinguished between privatization and restructuring of SOEs on the one hand, and between different types of governments, welfare or voter oriented, on the other hand. While a welfare maximizer will trade-off efficiency gains following privatization with the choice of the socially optimal employment level, a voter-oriented government will weigh the possibility to transfer to citizens the revenues accruing from privatization against the possibility to increase employment level in SOEs, with the objective of being re-elected.

The present paper belongs to the second stream, as we consider a voting model of privatization with no asymmetry of information. The employment level is central in our analysis, but also the level of consumption of the good produced by the monopoly. Willner (2001) is probably the paper which is the closest to ours. He also considered the output and employment levels. In his paper, the public firm's output level is indeed fixed through a Nash bargaining process in which output and profits have complementary weights. This situation is then compared to a private Cournot oligopoly characterized by a given value of the Herfindahl index. Welfare is greater in the public monopoly than under privatization if the output weight in the bargaining process and/or the Herfindahl index are important. The main differences

with the present paper is that (i) we use a general equilibrium model and (ii) we consider that the public firm's price decisions as well as the privatization one are taken through majority voting.

Finally, loosely related to the present paper, there is an abundant literature on privatization in a mixed oligopoly framework, when competition exists initially between public and private firms, generally in partial equilibrium settings (to name only a few, [Lin and Matsumura, 2018](#); [Capuano and De Feo, 2010](#); [Matsumura, 1998](#); [Matsumura and Okumura, 2013](#)).

The paper is organized as follows. Section 2 describes the model and a preliminary result on the consumption and work decisions at the competitive equilibrium. Section 3 provides the outcome of a democratic choice in terms of privatization. Section 4 analyzes the welfare effects of privatization. Section 5 concludes. All proofs are given in [Appendix](#).

## 2. The model

We build on the basic model introduced by [Kahloul et al. \(2017\)](#) and used by [Lahmandi-Ayed and Laussel \(2018\)](#).

The economy encompasses 3 goods: labour as the unique input, a numeraire and an indivisible "differentiated" good as the unique output ("differentiated" in the sense that it may possibly be of different qualities perceived differently by consumers).

There is a population of workers/consumers/shareholders. Each individual is endowed with an indivisible unit of labour and a given quantity  $e$  of a (composite) numeraire good. Denote by  $\lambda$  the share of the individual in the firm. Each agent  $(\alpha, \theta)$  is doubly characterized by:

- a "working parameter"  $\alpha \in [0, \bar{\alpha}]$  which captures the worker's sensitivity to effort,
- a "consumption parameter"  $\theta \in [0, \bar{\theta}]$  which measures the intensity of the consumer's preference for the product's quality.

Individuals are uniformly distributed over  $[0, \bar{\alpha}] \times [0, \bar{\theta}]$  with a density normalized to 1. Each worker/consumer  $(\alpha, \theta)$  chooses sequentially:

- first, whether to remain idle ( $\bar{W}$ ) or to work ( $W$ ) in the differentiated sector, given the salary,
- then to compose his/her consumption bundle, in particular whether to consume ( $C$ ) or not ( $\bar{C}$ ) one unit of the differentiated product, given the price  $p$  of the good.

One firm produces the differentiated product. One unit of the differentiated good requires one unit of labour.

Individuals derive their utility from the consumption of these two goods as follows:

$$V(x, t) = \theta qx + t,$$

where

- $x$  is the consumption of the differentiated product of exogenous quality  $q$ . Quantity  $x$  may take the values 0 or 1.
- $t$  is the consumption of the numeraire good which belongs to  $\mathbb{R}$ .

If the individual chooses to work, he/she receives a salary  $\omega$  and must incur a training cost (or disutility)<sup>14</sup>  $\alpha q$ . If he/she chooses not to work, he/she receives no salary (and does not

<sup>14</sup> This cost may also be interpreted as an opportunity cost, the agents having heterogeneous productivities outside the differentiated sector, i.e. in producing the numeraire.

have to be trained), his/her revenue being limited to the initial endowment in the numeraire and to his/her share in the firm's profit.

Outsiders are supposed to have zero net utility. This may mean either that the monopoly is the only employer and that outsiders are indeed idle. Or it may mean that the outside work option is an unskilled work implying a fixed disutility to effort independent of the type of the worker and firms offering a wage just equal to that disutility.

Denote by  $\nu$  the share of the "State" in the firm. This means that every individual owns the share  $\lambda = \frac{\nu}{\alpha \bar{\alpha}}$ .

Initially the firm is entirely public, i.e. it is fully owned by the State.

The share to be privatized (thus the share  $\nu$  to be kept), the price per share and the price of the output, are determined as follows:

1. In a first stage, the fraction  $\nu$  of the firm which is to remain public is determined through majority voting.
2. Given the value of  $\nu$  selected in the first stage, the price per share  $z$  is determined through a vote. We impose  $z \geq 0$ .
3. Given the choice of  $\nu$  and  $z$ , two cases are distinguished:
  - If  $\nu \geq 1/2$ , the firm is controlled by the State. Then the output price is determined through majority voting.
  - If  $\nu < 1/2$ , the firm is controlled by a private owner. We then suppose that the firm chooses the price so as to maximize its profit.

On the last step of the game, two remarks may be pointed out. First when the firm is controlled by the state ( $\nu \geq 1/2$ ), supposing that the price is chosen through majority voting is an innovative hypothesis (even if it is similar to the hypothesis of the choice of the price through voting of shareholders we adopted with the same model in [Lahmandi-Ayed and Laussel \(2018\)](#)). In the literature, public firms are indeed assumed to maximize a weighted sum of surpluses, which corresponds to a centralized approach. However more and more people are asking to be associated in everyday decision and not only in the election of representative people, pushing towards participatory democracy. Our model may be considered as a first attempt to account for this form of democracy and try to predict its consequences. Second, when the firm is controlled by a private owner, we suppose that it maximizes its profit. This amounts for instance to suppose either that the ownership is very concentrated (the weight of the owners is negligible in the population), which may justify to suppose that consumption and work concerns are negligible for them relative to the firm's profit, or that firms' shares are held by foreign owners. Any other hypothesis would necessitate to know precisely the distribution of the private ownership on a part of the population, would necessitate more complicated computations and would not be less arbitrary.

Under some conditions, when controlled by the State, the firm may incur losses at equilibrium. The firm is thus unsaleable unless the State commits to cover the losses. Covering the losses of a private owner is not always acceptable and politically feasible. We shall however derive our results under the two possible assumptions.

- $\mathcal{H}_1$ : The State cannot cover the possible losses a private shareholder may incur.
- $\mathcal{H}_2$ : The State commits to cover the possible losses a private shareholder may incur.<sup>15</sup>

<sup>15</sup> In this case it sells shares at a zero price but, given the commitment, this is equivalent to a negative price.

Helpfully, we denote by  $\gamma = \frac{\bar{\alpha}}{\bar{\theta}}$  (as in [Lahmandi-Ayed and Laussel, 2018](#)). This is the ratio between the maximum disutility of manufacturing one unit of output and the maximum utility of consuming it. It is going to play a crucial role in the results.

It is important to note that the consumption and work decisions of individuals are endogenous. They depend on the levels of salary and output price which are themselves obtained endogenously. The population of consumers or workers, not known in advance, is part of the outcome at equilibrium. A benchmark situation would thus be helpful for our interpretations. In this respect, we consider the choices of each individual and of the majority in terms of consumption and work in the competitive equilibrium benchmark case.

At such an equilibrium, price must equal marginal cost, so that  $p = \omega$ , and the number of agents who decide to work, i.e.,  $\frac{\bar{\theta}\omega}{q}$  must equal the demand for labor, i.e.,  $\bar{\alpha}(\bar{\theta} - \frac{p}{q})$ . The competitive equilibrium price and wage are then given by  $p^c = \omega^c = q\frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}}$ . All agents of type  $\alpha \leq \frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}}$  work while all agents of type  $\theta \geq \frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}}$  consume.

**Claim.** *At the competitive equilibrium, there are more of agents who neither consume the good nor work at the firm than of agents who consume the good and work at the firm, iff  $\gamma \geq 1$ .*

**Proof.** The number of people who neither consume nor work is simply  $(\bar{\alpha} - \frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}})(\frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}})$ .

The number of agents who consume and work is simply  $\frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}}(\bar{\theta} - \frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}})$ . The difference equals  $(\bar{\alpha} - \bar{\theta})\frac{\bar{\alpha}\bar{\theta}}{\bar{\alpha} + \bar{\theta}}$ . ■

It follows that, when  $\gamma < 1$ , at the competitive equilibrium, there are more individuals working in the firm and consuming its output than individuals neither consuming nor working. Hence, there is an absolute majority of agents working in the firm OR consuming the firm's output.

### 3. Main results

The game is solved by backward induction. We first determine the equilibrium output price of stage 3, then the price per share of the firm, and finally the share of the firm to be kept by the State.

#### 3.1. The output price

When  $\nu < 1/2$ , the firm is controlled by a private owner. The chosen price is  $p(\bar{C}, \bar{W})$  given by Eq. (4) which maximizes the firm's profit.

When  $\nu \geq 1/2$ , the firm is controlled by the State and the price of the output is determined through a vote by all individuals. In this case, we can apply the results obtained by [Lahmandi-Ayed and Laussel \(2018\)](#) replacing  $\mu$  by  $\frac{1}{\nu}$ , except that  $\mu$  has to be in  $[1, 2]$  for  $\nu$  to be in the right interval, i.e.  $[1/2, 1]$ . As a result, the price candidates identified by [Lahmandi-Ayed and Laussel \(2018\)](#) may now be negative. When this is the case, a new analysis is necessary to determine the price winning a relative/absolute majority. Taking as price candidates the maximum between zero and the price candidates of [Lahmandi-Ayed and Laussel \(2018\)](#), we eventually obtain the same result.

The price candidates that may be chosen are now given by:

$$p(C, W) = \max(0, \frac{q\bar{\theta}(\nu(2\bar{\alpha} + \bar{\theta}) - (\bar{\alpha} + \bar{\theta}))}{2\nu(\bar{\alpha} + \bar{\theta})}), \quad (1)$$

$$p(C, \bar{W}) = \max(0, \frac{q\bar{\theta}(\bar{\theta}(\nu - 1) + 2\nu\bar{\alpha})}{2\nu(\bar{\alpha} + \bar{\theta})}), \quad (2)$$

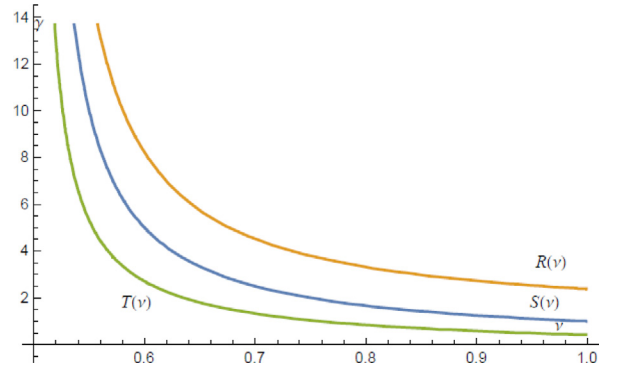


Fig. 1. Absolute/Relative majority vote for the output price in the  $(\nu, \gamma)$ -space.

$$p(\bar{C}, W) = \frac{q\bar{\theta}(\nu(\bar{\theta} + 2\bar{\alpha}) - \bar{\alpha})}{2\nu(\bar{\theta} + \bar{\alpha})}, \quad (3)$$

$$p(\bar{C}, \bar{W}) = \frac{q\bar{\theta}(\bar{\theta} + 2\bar{\alpha})}{2(\bar{\theta} + \bar{\alpha})}. \quad (4)$$

Note that  $p(\bar{C}, \bar{W}) > 0$  always and  $p(\bar{C}, W) > 0$  for  $\nu > 1/2$ .

Our first result on the output price chosen for each couple  $(\nu, \gamma)$ , [Proposition 1](#), corresponds to a rewriting of [Proposition 1](#) of [Lahmandi-Ayed and Laussel \(2018\)](#) replacing  $\mu$  by  $\frac{1}{\nu}$  and using the new expressions of the price candidates. Let

$$R(\nu) = r(\frac{1}{\nu}) = \frac{6\nu^2 + \nu + \sqrt{128\nu^4 + 4\nu^2 + 12\nu + 1}}{8\nu(2\nu - 1)}, \quad (5)$$

$$S(\nu) = s(\frac{1}{\nu}) = \frac{1}{2\nu - 1}, \quad (6)$$

$$T(\nu) = t(\frac{1}{\nu}) = \frac{3(2\nu + 1) - 16\nu^2 + \sqrt{128\nu^4 + 36\nu^2 - 12\nu - 7}}{4(4\nu^2 - 1)}. \quad (7)$$

Note that all the functions defined above are decreasing, as depicted in [Fig. 1](#), and helpfully that  $S(\nu = 1) = 1$ .

**Proposition 1.** *When  $\nu < 1/2$ ,  $p(\bar{C}, \bar{W})$  is chosen.*

*For  $\nu \geq 1/2$ , the winning price in a plurality vote is either  $p(\bar{C}, \bar{W})$  or  $p(C, W)$ . More precisely, we have the following.*

1. *The profit-maximizing firm's price  $p(\bar{C}, \bar{W})$  is the ideal price of an absolute majority if and only if*

$$\gamma \geq R(\nu). \quad (8)$$

*In this case,  $p(\bar{C}, \bar{W})$  is the Condorcet winner.*

2. *The profit-maximizing firm's price  $p(C, W)$  is the ideal price of only a relative majority if and only if*

$$S(\nu) \leq \gamma < R(\nu). \quad (9)$$

3. *Price  $p(C, W)$  is preferred by only a relative majority if and only if*

$$T(\nu) < \gamma \leq S(\nu). \quad (10)$$

4. *Price  $p(C, W)$  is preferred by an absolute majority if and only if*

$$\gamma \leq T(\nu). \quad (11)$$

*In this case,  $p(C, W)$  is the Condorcet winner.*

To establish [Proposition 1](#) in [Lahmandi-Ayed and Laussel \(2018\)](#), we proceeded as follows. Depending on the output price

<sup>16</sup> This is detailed further in [Appendix](#).

and  $\theta$ , an individual chooses to consume or not the differentiated good. This determines the demand of the output as function of the output price, and at the same time the demand of labour. Depending on the wage and  $\alpha$ , an individual chooses to work or not, which determines the labour supply as function of the wage. Balancing supply and demand on the labour market allows to express the wage as function of the output price, so that:

$$\omega = \bar{\alpha}(q - \frac{p}{\theta}). \quad (12)$$

The firm's profit is then the following function of its price<sup>17</sup>:

$$\pi = \bar{\alpha}[p(1 + \frac{\bar{\alpha}}{\theta}) - \bar{\alpha}q][\bar{\theta} - \frac{p}{q}]. \quad (13)$$

As  $p$  varies, the individual goes through different regimes in terms of consumption and working, resulting in different expressions of the corresponding indirect utility. We then identify  $p(C, W)$ ,  $p(C, \bar{W})$ ,  $p(\bar{C}, W)$  and  $p(\bar{C}, \bar{W})$  to be the prices candidates to be local interior maxima of these utilities, each one corresponding to the best price in each state of consumption-working. We then determine the voters for each candidate price. Finally, we count the number of voters for each candidate price, thus obtaining the winner in a plurality system. Some hints are provided in the appendix (Some details from [Kahloul et al., 2017](#)). Further details may be found in [Kahloul et al. \(2017\)](#) and [Lahmandi-Ayed and Laussel \(2018\)](#). The novelty in the present paper is that the candidates identified by [Lahmandi-Ayed and Laussel \(2018\)](#) may now be negative, in which case they are replaced by zero. Therefore some of the computations undertaken by [Lahmandi-Ayed and Laussel \(2018\)](#) are no longer valid. Indeed when a price candidate is null, the corresponding utility has a new expression, which necessitates new calculations for the comparison of this new utility and the other ones and modifies the areas corresponding to each candidate.

### 3.2. The price per share

The price per share  $z$  of the firm depends on what is expected to occur at stage 3, i.e. in terms of the output price, thus the firm's profit, which in turn depend on  $\gamma$  and  $\nu$  as stated in [Proposition 1](#). It may also depend on whether we consider  $\mathcal{H}_1$  or  $\mathcal{H}_2$ , as the expected firm's profit may be negative. The formal result is provided in [Proposition 2](#).

**Proposition 2** (The Price per Share). *At equilibrium of step 2, the price per share of the firm,  $z$ , depends on  $\nu$  and  $\gamma$  as follows:*

1. If  $\nu < 1/2$ , then  $z = \frac{q\bar{\theta}^2}{4(\bar{\alpha}+\bar{\theta})}$  and the output price is given by  $p(\bar{C}, \bar{W})$ .
2. If  $\nu \geq 1/2$ , two cases are distinguished.
  - If  $\gamma \geq S(\nu)$ , then  $z = \frac{q\bar{\theta}^2}{4(\bar{\alpha}+\bar{\theta})}$  and the output price is given by  $p(\bar{C}, \bar{W})$ .
  - If  $\gamma < S(\nu)$ , then, under  $\mathcal{H}_1$ , the firm is unsaleable and  $z$  is not defined. Under  $\mathcal{H}_2$ ,  $z = 0$ .

When the agents expect firm's profit to be negative, either the firm is unsaleable (under  $\mathcal{H}_1$ ) or the price per share is null (under  $\mathcal{H}_2$ ). In the latter case, the private purchaser is completely indifferent between buying or not. If we suppose that the State gives  $\epsilon$  to urge the purchaser to accept, the transaction will occur, whatever low  $\epsilon$  is. The above results are thus obtained when  $\epsilon$  converges to zero.

### 3.3. The share to be kept by the state

We are going to establish the results under each possible assumption concerning the possibility for the State to cover the losses a private stakeholder may incur after buying her shares. It turns out that the results are independent of this assumption. But the proofs are not the same under  $\mathcal{H}_1$  or  $\mathcal{H}_2$ . In other words, we obtain the same result under each one of the two possible hypotheses but for different reasons.

**Proposition 3.** *Whether under  $\mathcal{H}_1$  or under  $\mathcal{H}_2$ , at equilibrium,  $\nu$  and the output price depend on  $\gamma$  as follows:*

- When  $\gamma > 1$ , there exists a unique  $\tilde{\nu} = \frac{\gamma+1}{2\gamma} \in ]1/2, 1[$  satisfying  $\gamma = S(\tilde{\nu})$ . At equilibrium, a continuum of values of  $\nu$  may emerge. That is:  $\nu \in ]0, 1/2[ \cup ]\tilde{\nu}, 1[$ ; and the output price is in all cases  $p(\bar{C}, \bar{W})$ .
- When  $\gamma \leq 1$ ,  $\nu = 1$  and the output price is  $p(C, W)$ .

In the first case (high  $\gamma$ ), the status quo and a privatization giving control to a private owner are equivalent and may both emerge at equilibrium. Thus a privatization may emerge at equilibrium even when we do not suppose any superior efficiency of a private management relative to a public one. However, if we introduce a difference in efficiency between the two types of management, whatever small this difference is, the indeterminacy will disappear and radical privatization will be the only outcome at equilibrium.

Although the result is the same, independently of whether the State covers or not the losses a private purchaser may make after transaction, the proof depends on this hypothesis. The proof thus encompasses two parts, supposing first  $\mathcal{H}_1$ , then  $\mathcal{H}_2$ . Under each one of the two hypotheses, the analysis depends on the output price chosen at the last step, which according to [Proposition 1](#), depends on  $\gamma$  and  $\nu$ . More precisely, it depends on whether  $\gamma$  is greater or smaller than  $S(\nu)$ . Since  $S$  is decreasing with 1 as the minimal value, we have to compare  $\gamma$  to 1.

For  $\gamma < 1$ , when the State controls the firm ( $\nu > 1/2$ ), the price  $p(C, W)$  is chosen at the last stage of the game, yielding a negative profit. Under  $\mathcal{H}_1$ , the firm is unsaleable and only  $\nu = 1$  may emerge at equilibrium among the values of  $\nu$  giving control to the State. Therefore  $\nu = 1$  has to be compared to the values  $\nu < 1/2$  transferring the control of the firm to private owners and yielding the price  $p(\bar{C}, \bar{W})$  at the last stage of the game. Under  $\mathcal{H}_2$ , the State covers the losses, which means that each individual has to bear his/her share of the losses.

When  $\gamma \geq 1$ ,  $\gamma$  may be greater or smaller than  $S(\nu)$ , depending on whether  $\nu$  is greater or smaller than  $\tilde{\nu}$ . We have to study the indirect utilities for  $\nu \in ]0, 1/2[ \cup ]\tilde{\nu}, 1[$ , yielding  $p(\bar{C}, \bar{W})$  at the last stage and a positive profit; and for  $\nu \in ]1/2, \tilde{\nu}[$  yielding  $p(C, W)$  and a negative profit, in which case we have to distinguish  $\mathcal{H}_1$  and  $\mathcal{H}_2$  as the indirect utilities are different under each of these assumptions. The local maxima in each case are compared to determine the majority voting outcome in terms of  $\nu$ .

As a conclusion, the model predicts that democratic support for privatization is likely to occur when most of the population are neither consumers of the good nor employees of the firm (at the competitive equilibrium). When it comes to water facilities or health, we can hardly imagine the existence of non-consumers. On the one hand, this may explain the observation made by [Lobina et al. \(2014\)](#) about the cities and countries increasingly choosing to take back public control over water provision and management after an episode of privatization. On the other hand, when privatization is adopted for these services, we can conjecture that it would be against the will of the majority. This

<sup>17</sup> For details on the derivation of the results see [Kahloul et al. \(2017\)](#).

explains some observations of the opposition to privatizations all over the world. It allows even to conjecture that a significant part of the real-world privatizations have certainly taken place against popular support.<sup>18</sup>

#### 4. Privatization and social welfare

It is interesting to know whether or not the majority choice coincides with the welfare enhancing one. By welfare we intend the sum of the individuals' utilities measured in units of numeraire, as is usual in models with semi-linear utility functions.<sup>19</sup> Notice that this definition supposes the possibility of non-distortionary transfers between "gainers" and "losers" of any contemplated change. In their absence, the procedure of "treating a dollar as a dollar, no matter to whom it accrues" has been criticized (see, e.g., [Hendren, 2014](#); [Saez and Stantcheva, 2013](#)).

In this section, we suppose that  $\nu$ , the firm's share to remain public, is decided in a centralized way, hence modifying only the first step of the game and keeping the same following steps (steps 2 and 3). We consider  $\nu = 1$  as the initial situation.

For a given  $\nu$ , we are going to express the social welfare as a function of the output price  $p$ . Notice that, from the labor market equilibrium condition, the equilibrium wage

$$\omega = \gamma(q\bar{\theta} - p).$$

The social welfare following the privatization is the sum of 5 terms<sup>20</sup>:

1. the aggregate net utility from consuming the good:  $\theta q - p$  for each consuming individual.
2. the aggregate net utility from working:  $\omega - \alpha q$  for each working individual.
3. the profit (equal to the sum of all dividends).
4. + the money from the sale of the fraction  $(1 - \nu)$  of the firm.
5. - the profits which will not be distributed as dividends after transaction.

The sum of the three first terms equals the aggregate utility from consuming the good ( $\theta q$ ) minus the aggregate disutility from working ( $\alpha q$ ).

The sum of the two last terms has been proved to equal to zero.

As for the private investors, whether they belong to the considered population or not, amounts to the same, as their net surplus is equal to zero. Indeed the money they spend to acquire the share  $(1 - \nu)$  of the firm corresponds exactly to what they earn as dividends after transaction.

The social welfare is thus simply the aggregate utility from consuming the good,

$$\bar{\alpha} \int_{\frac{p}{q}}^{\bar{\theta}} \theta q d\theta,$$

minus the aggregate disutility from working

$$\bar{\theta} \int_0^{\frac{w}{q}} \alpha q d\alpha.$$

<sup>18</sup> Thanks are due to an anonymous referee for this remark.

<sup>19</sup> We deal here with a general equilibrium model so that profits are distributed and appear in the individuals' utilities each in proportion of his/her share. When private investors buy shares we proved that they do that at a price which leaves them with no net profit.

<sup>20</sup> We decompose the social welfare in a way different from the one used to decompose the individual utility in the proof of [Proposition 2](#). The difference is only for exposition needs. They amount to the same.

Subtracting and substituting  $\omega$  for its value, we obtain the social welfare as a function of the output price as follows<sup>21</sup>:

$$W = \frac{1}{2q} [\bar{\theta} (q\bar{\theta} - p) \gamma (p + q\bar{\theta} + p\gamma - q\bar{\theta}\gamma)].$$

We are now ready to provide our result on the centralized choice of  $\nu$ .

**Proposition 4.** *Suppose that  $\nu$  is chosen so as to maximize the total welfare while keeping unchanged the two following steps of the game.*

- When  $\gamma < 1$ , the unique share that maximizes the welfare is  $\nu^* = 1$ .
- When  $\gamma \geq 1$ , all the values of  $\nu^* \in [0, 1/2[ \cup [\bar{\nu}, 1]$  are equivalent and maximize the total welfare.

The optimal choice of  $\nu$  in terms of global welfare coincides with the outcome of majority voting. In other words, the centralized and decentralized choices amount to the same in this setting.

For low values of  $\gamma$  ( $\gamma < 1$ ), privatization is desirable neither from a majority of voters' point of view nor in terms of global welfare. In both cases, the firm remains entirely under the control of the State, the output price is below its profit-maximizing level and the equilibrium profit is negative. For high values of  $\gamma$  ( $\gamma \geq 1$ ), the welfare-maximizing choice, as well as the outcome of majority voting, involves profit maximization, what may follow either from a limited sale of shares to private stockholders, the State retaining a large enough majority of shares, or from a radical privatization.

To obtain this result, we compare two types of privatization: privatization such that the firm remains under the control of the state (with  $p(C, W)$  as the output price) and radical privatization transferring the control to the private investors (with  $p(\bar{C}, \bar{W})$ ). The analysis is not the same depending on whether the State can or cannot commit to cover or not the private investors' losses after privatization. Supposing  $\mathcal{H}_1$  excludes the values of  $\nu < 1$  yielding a negative profit. The difference between high and low values of  $\gamma$  is that when  $\gamma < 1$ , for all  $\nu \in [1/2, 1]$ , the output price is always given by  $p(C, W)$  yielding a negative profit, whereas when  $\gamma \geq 1$ , the output price and the resulting profit depend on the position of  $\nu$  relative to  $\bar{\nu}$ .

#### 5. Conclusion

In this paper we have tried to determine whether there will be a majority support for the privatization of a State monopoly and, incidentally, whether it can increase social welfare. Supposing that there is no superior nor inferior cost-efficiency of private management, we have focused on the output and employment effects of privatization. Under public control the output price and hence the output and employment levels are assumed to be the ones which please a majority of voters. The equilibrium price turns out to be either the profit-maximizing one, which is greater than the welfare-maximizing one, or the price which maximizes the utility of the voters who work in the firm and consume its product, a price smaller than the welfare-maximizing one. Under private control, the profit-maximizing price always follows. In this framework, the only possible relevant effect of privatization is to allow a switch from one price to the other (and from one

<sup>21</sup> Notice that  $W$  is a concave function of  $p$  which takes its maximum at  $p^{opt} = q\bar{\theta} \frac{\gamma}{1+\gamma}$ . It is interesting, though not surprising, to remark that  $p(C, W) < p^{opt} < p(\bar{C}, \bar{W})$ .



wage to the other). What we found is that privatization is neither strictly better than the status quo for a majority of voters nor strictly welfare increasing. However when the maximum disutility of working in the firm is greater than the maximum utility of consuming its output, so that a minority of voters have a direct stake in the firm as consumers/workers, an infinitesimal cost-efficiency superiority of private over public control is enough to entail a majority support for a radical privatization which, at the same time, is welfare enhancing. In the reverse case, the only equilibrium has the State retaining all shares of the firm, what is also the best outcome from the social welfare point of view, a result which only a substantial cost-efficiency advantage of private control is likely to reverse.

The results on the welfare analysis have been obtained under the hypothesis that there is no shadow price of public funds, which is questionable, in particular in the developing world. A natural extension would be to relax this assumption by considering a positive shadow price.<sup>22</sup>

We assumed that, under private control, the previously State Owned Enterprise is not regulated.<sup>23</sup> An obvious extension would be to analyze what would be the equilibrium regulatory policy for the private firm decided through majority voting and then to see whether, in these circumstances, a privatization inducing a switch from a SOE to a private firm regulated in this way would find a majority support.

In the whole analysis, we have supposed a uniform distribution of agents on  $[0, \bar{\theta}] \times [0, \bar{\alpha}]$ , which makes extreme values of  $\alpha$  and  $\theta$  as likely as intermediate ones. This assumption is certainly not innocuous. An interesting extension of our work would be to consider other distributions, in particular distributions with most individuals with characteristics “around the middle” as in triangular or normal distributions. Other simplifying hypotheses which may be relaxed are first the zero net utility of the outside work option which is a simple way to suppose that working in the monopoly is perceived as the best option for everybody; second to ignore the accusation that State Owned Enterprises require less efforts from employees than private ones (Bradburd, 1995); third to assume a linear utility in the numeraire-good, which may understate income effects. Indeed price distortions not only cause a welfare loss but moreover increase inequality (Rodríguez-Castelán et al., 2021). Finally it would be interesting to check the robustness of the results if we take  $[\underline{\theta}, \bar{\theta}]$  instead of  $[0, \bar{\theta}]$  (with  $\underline{\theta} > 0$ ) to account for the fact that the output is a vital commodity. This should reinforce the results on privatization as it strengthens the importance of consumption and the ensuing effect. However the analysis is likely to become very difficult and the model untractable in this case.<sup>24</sup> We conjecture from the developments which were necessary in this paper to account for the fact that the demand for the good cannot exceed the number of agents,<sup>25</sup> that similar arguments can be used to prove more generally the robustness of our results when the good is a vital commodity possibly consumed by all agents.<sup>26</sup>

<sup>22</sup> Since this model is a general equilibrium one, it would be natural to endogenize the shadow price of public funds. This would require to identify a specific (distortionary) tax system.

<sup>23</sup> On the contrary, Laffont and Tirole (1991) compare a public enterprise and a private regulated firm.

<sup>24</sup> The necessary analysis we conducted when zero was a possible corner solution was already not simple and obliged us at some stages to rely on numerical calculations.

<sup>25</sup> What is equivalent when  $\underline{\theta} = 0$  to non negative prices. This complication was absent from Lahmandi-Ayed and Laussel (2018).

<sup>26</sup> Thanks are due to two anonymous referees for suggesting these interesting extensions.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix. Some details from Kahloul et al. (2017)

*Consumption choice.* An individual  $\theta$  owning some revenue  $R$  has to choose between consuming one unit ( $x = 1$ ) and consuming zero units ( $x = 0$ ) of the differentiated good, so as to maximize his/her utility.

If  $x = 0$ , the whole revenue is spent on the purchase of the numeraire:  $t = R$ ; and the resulting indirect utility is:  $U(0) = R$ . If  $x = 1$ , the quantity of the numeraire, implied by the budget constraint, equals:  $t = R - p$ ; and the resulting utility is:  $U(1) = \theta q + R - p$ .

Individual  $\theta$  chooses  $x = 1$  if and only if  $U(1) > U(0)$ , i.e.  $\theta q > p$ .

*Work decision.* From the analysis above, a non-consumer of the differentiated good has the indirect utility  $U = R$ . His/her revenue will be equal to  $R = (\text{non-work revenues}) + \omega - \alpha q$ , if s/he chooses to work; and will be equal to  $R = (\text{non-work revenues})$  if s/he chooses not to work.

(Non-work revenues) are composed by the initial endowment and dividends if the individual is a shareholder.

Individual  $\alpha$  chooses the option that maximizes his/her indirect utility, thus here, the one maximizing his/her revenue. Hence s/he works if  $\omega > \alpha q$ .

A consumer of the differentiated good has the indirect utility  $U = \theta q + R - p$ . Again, the work decision maximizing the utility is the one maximizing the revenue, which results in the same conclusion.

Thanks to the linearity of the utility function, the consumption and work decisions are independent. The consumption decision depends only on  $\theta$  and the work decision only on  $\alpha$ .

### Proof of Proposition 1

In the present paper,  $\nu$  plays the role of  $\frac{1}{\mu}$  in Lahmandi-Ayed and Laussel (2018), except that  $\mu$  has to be in  $[1, 2]$  for  $\nu$  to be in the right interval, i.e.  $[1/2, 1]$ . The analysis conducted by Lahmandi-Ayed and Laussel (2018) holds under the condition that all the candidates be positive. Otherwise, one (or more) candidate is (are) null and new computations are necessary.

We have the following,

- $p(C, W) > 0$  iff  $\nu > \frac{1+\gamma}{2\gamma+1}$ .
- $p(C, \bar{W}) > 0$  iff  $\nu > \frac{1}{2\gamma+1}$ .
- $p(\bar{C}, W) > 0$  iff  $\nu > \frac{\gamma}{2\gamma+1}$ .
- $\frac{\gamma}{2\gamma+1} < 1/2$  (hence  $p(\bar{C}, W) > 0$  for all  $\nu \geq 1/2$ .) and  $\frac{1}{2\gamma+1} < \frac{1+\gamma}{2\gamma+1}$ .

Therefore, for all  $\nu \geq 1/2$ ,

- for  $\nu > \frac{1+\gamma}{2\gamma+1}$  all four prices are strictly positive;
- for  $\nu \in (\frac{1}{2\gamma+1}, \frac{1+\gamma}{2\gamma+1}]$ ,  $p(C, W) = 0$  and the three other prices are strictly positive;
- for  $1/2 \leq \nu \leq \frac{1}{2\gamma+1}$ ,  $p(C, W) = p(C, \bar{W}) = 0$  and the two other prices are strictly positive.

We are going to show that there is a majority for  $p(C, W)$  provided that  $\gamma \leq S(\nu)$  (or, equivalently,  $\nu \leq \frac{1-\gamma}{2\gamma}$ ). We have to distinguish three cases stemming from the conditions implied by the positivity of the different price candidates.

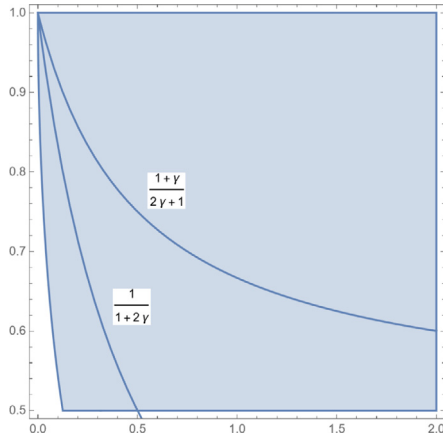


Fig. 2. Area where  $\alpha_2' \geq \alpha_1$  (case (ii)).

(i) For  $\nu > \frac{1+\gamma}{2\gamma+1}$  all the price candidates are positive, the analysis conducted by [Lahmandi-Ayed and Laussel \(2018\)](#) is valid and shows that there is a relative majority for  $p(C, W)$  iff  $\gamma \leq S(\nu)$ .

(ii) For  $\nu \in (\frac{1}{2\gamma+1}, \frac{1+\gamma}{2\gamma+1}]$ , we have  $p(C, W) = 0$ . In the expression of the utilities associated to each price candidate, only  $U(C, W)$  changes and becomes:

$$U(C, W) = q(\theta + (1 - \nu)\bar{\alpha} - \alpha).$$

First,  $U(C, W) > U(C, \bar{W})$  iff

$$\alpha < \alpha_2' = \bar{\theta} \frac{-1 + (2 + 4\gamma(2 + \gamma)\nu - (\nu + 2\gamma\nu)^2)}{4\nu(1 + \gamma)}, \quad (14)$$

$\alpha_2'$  is the equivalent of  $\alpha_2$  (obtained in [Lahmandi-Ayed and Laussel \(2018\)](#) to separate the vote zones between  $p(C, W)$  and  $(C, \bar{W})$ ) when  $p(C, W) = 0$ .

Second,  $U(C, W) > U(\bar{C}, W)$  iff

$$\theta > \theta_1' = \bar{\theta} \frac{(\nu + \gamma(2\nu - 1))^2}{4\nu(1 + \gamma)}. \quad (15)$$

$\theta_1'$  is the equivalent of  $\theta_1$  (obtained in [Lahmandi-Ayed and Laussel \(2018\)](#) to separate the vote zones between  $p(C, W)$  and  $(\bar{C}, W)$ ) when  $p(C, W) = 0$ .

The condition  $U(C, W) > U(\bar{C}, \bar{W})$  on the other hand yields

$$\theta - \alpha > -\bar{\alpha}(1 - \nu) + \bar{\theta} \frac{\nu}{4(1 + \gamma)} \quad (16)$$

The values of  $\alpha_1$  and  $\theta_2$  (obtained by [Lahmandi-Ayed and Laussel \(2018\)](#) to separate the vote zones between respectively  $p(\bar{C}, \bar{W})$  and  $p(\bar{C}, W)$  on the one hand, and between  $p(\bar{C}, \bar{W})$  and  $p(C, W)$  on the other hand) remain unchanged as they stem from comparisons not involving  $p(C, W)$ .

In the relevant area, i.e. for  $\nu \in (\frac{1}{2\gamma+1}, \frac{1+\gamma}{2\gamma+1}]$ , we check numerically that  $\alpha_2' \geq \alpha_1$ . (Fig. 2 pictures in blue the area for which this inequality does hold. It includes the whole relevant area.)

Now notice that  $\theta_2 - \theta_1' = \alpha_2' - \alpha_1$ . Thus  $\theta_1' < \theta_2$ .

Neither  $p(\bar{C}, W)$  nor  $p(C, W)$  may win a relative majority. Indeed, we show numerically that  $p(C, W)$  collects more voices than each one of them. Fig. 3 depicts in part (a), the area where  $p(C, W)$  collects more voices than  $p(C, \bar{W})$ , and in part (b), the area where  $p(C, W)$  collects more voices than  $p(\bar{C}, W)$ . Each one of these areas covers the whole area we are considering. Hence in a plurality system only  $p(C, W)$  or  $p(\bar{C}, \bar{W})$  may win a relative or absolute majority.

The number of individuals voting for  $p(C, W)$  is given by

$$A(p(C, W)) = \alpha_2'(\bar{\theta} - \theta_1) - \frac{(\alpha_2' - \alpha_1)(\theta_2 - \theta_1')}{2}.$$

The number of individuals voting for  $p(\bar{C}, \bar{W})$  is given by:

$$A(p(\bar{C}, \bar{W})) = (\bar{\alpha} - \alpha_1)\theta_2 - \frac{(\alpha_2' - \alpha_1)(\theta_2 - \theta_1')}{2}.$$

Hence there is a relative majority for  $p(C, W)$  iff  $\alpha_2'(\bar{\theta} - \theta_1) - \theta_2(\bar{\alpha} - \alpha_1) \geq 0$ .

As pictured in Fig. 4 with Mathematica this holds true (blue area) over the whole area considered.

(iii) Finally for  $1/2 \leq \nu < \frac{1}{1+2\gamma}$ , we have  $p(C, W) = p(C, \bar{W}) = 0$ . This means that  $p(C, \bar{W})$  is no longer a local maximum. Only  $p(C, W) = 0$ ,  $p(\bar{C}, W)$  and  $p(\bar{C}, \bar{W})$  are local maxima.

$\theta_1'$ ,  $\alpha_1$  and Eq. (16) giving the frontier between  $p(C, W)$  and  $p(\bar{C}, \bar{W})$  remain identical as in case (ii). The area where the best price would be  $p(C, \bar{W})$  vanishes since when  $p = 0$ , everybody consumes **and works**. To obtain the new figure (Fig. 5) giving the areas corresponding to individuals' choices between the three remaining alternatives, we make the zone corresponding to  $p(C, \bar{W})$  disappear from Figure 2 of [Lahmandi-Ayed and Laussel \(2018\)](#). Then only remain the critical value of  $\alpha$  separating  $p(\bar{C}, W)$  and  $p(\bar{C}, \bar{W})$  and the critical value of  $\theta$  separating  $p(\bar{C}, W)$  and  $p(C, W)$ .

Eq. (16) intersects  $\alpha = \bar{\alpha}$  at  $\hat{\theta} = \bar{\theta} \frac{(1+2\gamma)^2\nu}{4(1+\gamma)}$ . Since in this area  $\nu < \frac{1}{1+2\gamma}$ ,  $\hat{\theta} < \bar{\theta} \frac{1+2\gamma}{4(1+\gamma)} < \bar{\theta}$ .

We show numerically that  $p(\bar{C}, W)$  is never chosen by a relative majority, as  $p(C, W)$  always collects more voices. Fig. 6 depicts the area where  $p(C, W)$  collects more voices than  $p(\bar{C}, W)$ , which covers all the area of interest here. Hence, again only  $p(C, W)$  or  $p(\bar{C}, \bar{W})$  may win a relative (or absolute) majority.

By Fig. 5 the number of agents who favor  $p(C, W)$  equals  $A(p(C, W)) = (\bar{\theta} - \hat{\theta})\bar{\alpha} + (\hat{\theta} - \theta_1)\alpha_1 + \frac{1}{2}(\hat{\theta} - \theta_1)(\bar{\alpha} - \alpha_1)$  and the number of agents who favor  $p(\bar{C}, \bar{W})$  equals  $A(p(\bar{C}, \bar{W})) = \theta_1(\bar{\alpha} - \alpha_1) + \frac{1}{2}(\hat{\theta} - \theta_1)(\bar{\alpha} - \alpha_1)$ .

Accordingly,

$$A(p(C, W)) - A(p(\bar{C}, \bar{W})) = (\bar{\theta} - \hat{\theta})\bar{\alpha} + (\hat{\theta} - \theta_1)\alpha_1 - \theta_1(\bar{\alpha} - \alpha_1).$$

In Fig. 7 we have pictured in blue the area in the  $(\gamma, \nu)$ -space where this difference is positive. As can be seen it includes all the area where  $\nu < \frac{1}{1+2\gamma}$ .

**Proof of Proposition 2** Whatever the type of the individual, its indirect utility to be considered at this step is the sum of four terms:

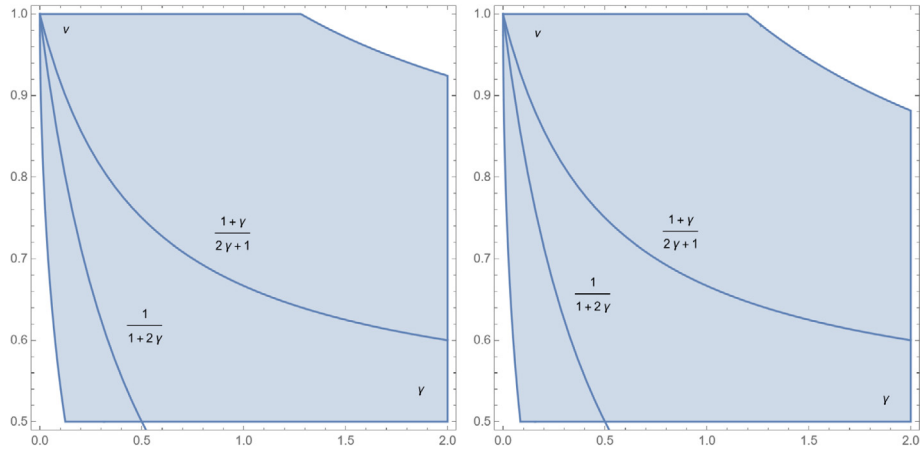
- 1 a consumption term,
- 2 a working term,
- 3 a term corresponding to the share in the firm's profit,
- 4 a term corresponding to the share of the individual in the revenues from selling a part of the firm, equal to  $z(1 - \nu)$ , which is obviously increasing with  $z$ , for all  $\nu < 1$ .

The three first terms do not depend on the price per share  $z$ . The indirect utility is thus obviously increasing with  $z$ , for all  $\nu < 1$ . Therefore, all individuals prefer the highest value possible of  $z$ .

Suppose  $\nu < 1/2$ . Then the firm will be under the control of a private owner who will choose price  $p(\bar{C}, \bar{W})$ , yielding the firm's profit

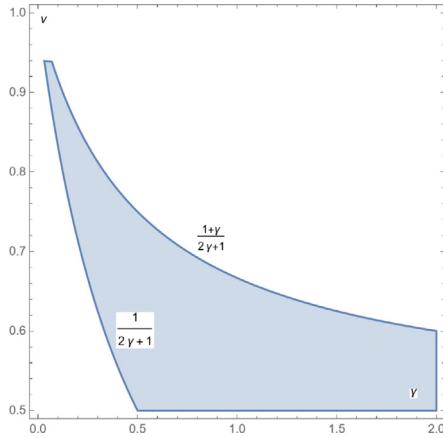
$$\bar{\pi} = \frac{q\bar{\alpha}\bar{\theta}^3}{4(\bar{\alpha} + \bar{\theta})},$$

which is positive.



(a)  $p(C, W)$  collects more votes than  $p(C, \bar{W})$  (b)  $p(C, W)$  collects more votes than  $p(\bar{C}, W)$

**Fig. 3.** Figure showing why  $p(C, \bar{W})$  and  $p(\bar{C}, W)$  never win a relative majority (Case (ii)).



**Fig. 4.** Figure showing that there is a relative majority for  $p(C, W)$  for  $v \in (\frac{1}{2\gamma+1}, \frac{1+\gamma}{2\gamma+1}]$ .

A potential private owner will not accept to buy a part of the firm if the price per share exceeds the dividend per share. Hence the maximal value possible of  $z$  is simply given by  $\frac{\pi}{\alpha\bar{\theta}}$ .

Suppose now that  $v \geq 1/2$ . We have to distinguish two cases.

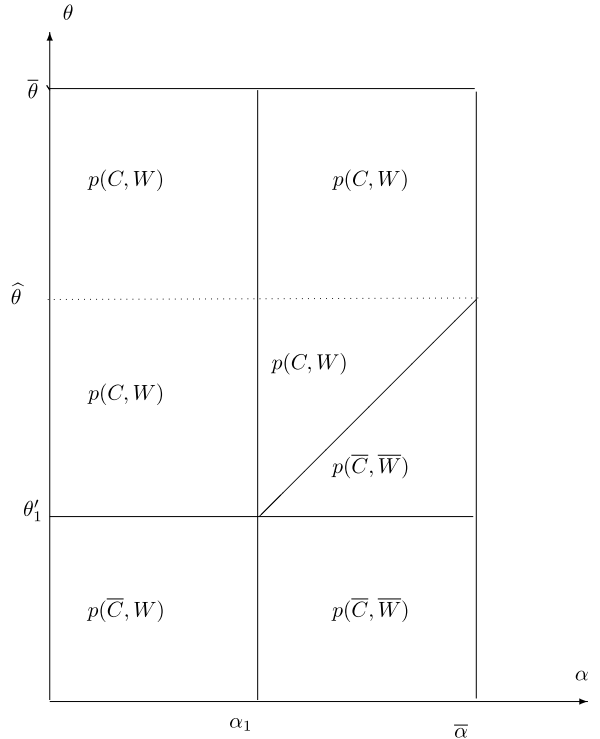
- If  $\gamma \geq S(v)$ , it is price  $p(\bar{C}, \bar{W})$  which is voted by a majority, yielding the same profit as the case  $v < 1/2$ . We continue the reasoning similarly.
- If  $\gamma < S(v)$ , it is price  $p(C, W)$  which is voted, yielding the profit

$$\underline{\pi} = \frac{q\alpha\bar{\theta}(v^2\bar{\theta}^2 - (\bar{\alpha} + \bar{\theta})^2)}{4v^2(\bar{\alpha} + \bar{\theta})}, \text{ when } v \geq \frac{1+\gamma}{1+2\gamma}, \quad (17)$$

$$\underline{\pi} = -q\alpha^2\bar{\theta}, \text{ when } v \leq \frac{1+\gamma}{1+2\gamma}. \quad (18)$$

which is negative for all  $v$ . This means that the firm is unsaleable unless the State commits to cover the losses after transaction. Therefore,

- under  $\mathcal{H}_1$ , there is no transaction, thus the price per share is not defined.
- under  $\mathcal{H}_2$ , the price per share is null.



**Fig. 5.** The best price from the viewpoint of each individual for  $1/2 \leq v < \frac{1}{1+2\gamma}$ .

### Proof of Proposition 3

#### Part 1: Under $\mathcal{H}_1$

Suppose first that  $\gamma > 1 = S(1)$ .

Given the decreasingness of  $S$ , when  $v < \tilde{v}$ ,  $\gamma < S(v)$ , the firm's output price is  $p(C, W)$ , which yields a negative profit. The firm is thus unsaleable under  $\mathcal{H}_1$ . Thus such a value of  $v$  cannot emerge at equilibrium. At equilibrium, we have necessarily  $v \geq \tilde{v}$  (thus  $\gamma > S(v)$ ) or  $v < 1/2$ , and an indifference of all the individuals between all these values of  $v$ .

Suppose now that  $\gamma \leq 1 = S(1)$ .

Since  $S$  is a decreasing function with  $v$ , then  $\gamma < S(v)$  for all  $v \in [1/2, 1]$ .

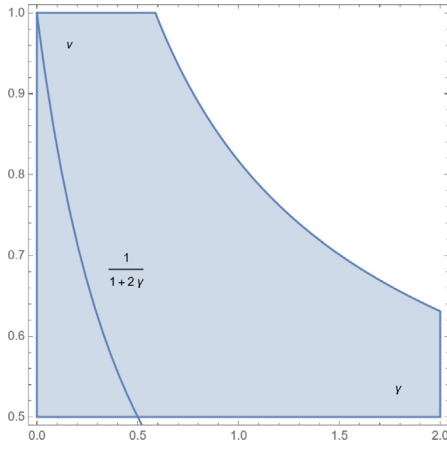


Fig. 6. Area where  $p(C, W)$  collects more voices than  $p(\bar{C}, \bar{W})$  (case (iii)).

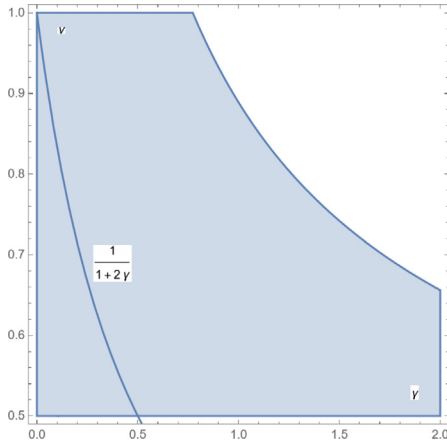


Fig. 7. Figure showing that there is a relative majority for  $p(C, W)$  for  $v < \frac{1}{1+2\gamma}$ .

For  $v \in [1/2, 1]$ , the output price chosen by the majority is  $p(C, W)$  and the firm is unsaleable. Under  $\mathcal{H}_1$ , among the segment  $[1/2, 1]$ , only  $v = 1$  is eligible at equilibrium.

For  $v < 1/2$ , the private owner has the control and the output price is  $p(\bar{C}, \bar{W})$ .

At price  $p(\bar{C}, \bar{W})$ , the indirect utilities of the individuals when we integrate the revenues of the firm's sale, are given by:

$$U(C, W)(p(\bar{C}, \bar{W})) = (\theta - \alpha)q - \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (19)$$

$$U(\bar{C}, W)(p(\bar{C}, \bar{W})) = -\alpha q + \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (20)$$

$$U(C, \bar{W})(p(\bar{C}, \bar{W})) = \theta q - \frac{q\bar{\theta}(2\bar{\alpha} + \bar{\theta})}{4(\bar{\alpha} + \bar{\theta})}, \quad (21)$$

$$U(\bar{C}, \bar{W})(p(\bar{C}, \bar{W})) = \frac{q\bar{\theta}^2}{4(\bar{\alpha} + \bar{\theta})}. \quad (22)$$

The used notation is self-explaining. For instance,  $U(C, W)(p(\bar{C}, \bar{W}))$  is the utility at price  $p(\bar{C}, \bar{W})$  of an individual choosing to consume the differentiated good and to work. All these utilities do not depend on  $v$ . All the values  $v < 1/2$  are eligible to be chosen.

For  $v \geq 1/2$ ,  $p(C, W)$  is voted by a majority at the last step, but we have to distinguish two sub-cases, as  $p(C, W)$  may have two possible expressions.

For  $v \geq \frac{1+\gamma}{2\gamma+1}$ , at price  $p(C, W)$  (which is positive), the utilities are given by:

$$U_{\mathcal{H}_1}(C, W)(p(C, W)) = (\theta - \alpha)q + \frac{q(\bar{\alpha} + (1 - v)\bar{\theta})^2}{4v(\bar{\alpha} + \bar{\theta})} \quad (23)$$

$$U_{\mathcal{H}_1}(C, \bar{W})(p(C, W)) = \theta q - \frac{q\bar{\theta}(v(2\bar{\alpha} + \bar{\theta}) - (\bar{\theta} + \bar{\alpha}))}{2v(\bar{\alpha} + \bar{\theta})} + \frac{q((v - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(v + 1) + \bar{\alpha})}{4v(\bar{\alpha} + \bar{\theta})} \quad (24)$$

$$U_{\mathcal{H}_1}(\bar{C}, W)(p(C, W)) = -\alpha q + \frac{q\bar{\alpha}(\bar{\alpha} + (v + 1)\bar{\theta})}{2v(\bar{\alpha} + \bar{\theta})} + \frac{q((v - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(v + 1) + \bar{\alpha})}{4v(\bar{\alpha} + \bar{\theta})} \quad (25)$$

$$U_{\mathcal{H}_1}(\bar{C}, \bar{W})(p(C, W)) = \frac{q((v - 1)\bar{\theta} - \bar{\alpha})(\bar{\theta}(v + 1) + \bar{\alpha})}{4v(\bar{\alpha} + \bar{\theta})} \quad (26)$$

When  $1/2 \leq v \leq \frac{1+\gamma}{2\gamma+1}$ , we have  $p(C, W) = 0$  and  $\omega = q\bar{\alpha}$ . Every agent consumes and works and the utility of each agent reduces to the consumption and working terms, so that her utility equals:

$$U_{\mathcal{H}_1}(C, W)(p(C, W)) = (\theta - \alpha)q + q\bar{\alpha}. \quad (27)$$

Since the firm is unsaleable while keeping public control (i.e. with  $v \geq 1/2$ ), we have to compare the number of individuals choosing  $v = 1$  and those choosing some  $v < 1/2$ . Notice that at  $v = 1 > \frac{1+\gamma}{2\gamma+1}$ , Eqs. (23) to (26) are the relevant ones. This comparison stems from the comparison for each individual between the two options.

To do so, we have first to determine the decisions of each individual in terms of consumption and work at each option, i.e. for each price. The determination of the consumption and work decisions at each price ( $p(C, W)$  or  $p(\bar{C}, \bar{W})$ ) amounts to position the considered price relative to  $\theta q$  and  $\bar{\theta}q(1 - \frac{v}{\bar{\alpha}})$ . Doing this the consumption and work decisions of individuals are depicted in Figs. 8 and 9, denoting by:

$$\hat{\alpha} = \frac{\bar{\alpha}\bar{\theta}}{2(\bar{\theta} + \bar{\alpha})}, \quad (28)$$

$$\hat{\theta} = \frac{\bar{\theta}(\bar{\theta} + 2\bar{\alpha})}{2(\bar{\theta} + \bar{\alpha})}, \quad (29)$$

$$\tilde{\alpha}(v) = \frac{\bar{\alpha}(\bar{\theta}(v + 1) + \bar{\alpha})}{2v(\bar{\theta} + \bar{\alpha})}, \quad (30)$$

$$\tilde{\theta}(v) = \frac{\bar{\theta}(\bar{\theta}(v - 1) + (2v - 1)\bar{\alpha})}{2v(\bar{\theta} + \bar{\alpha})}. \quad (31)$$

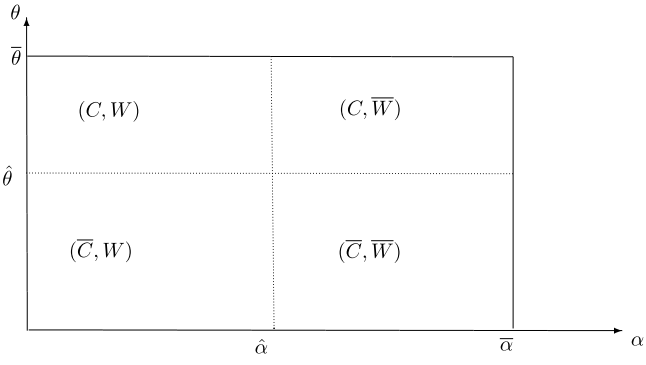
As the comparison must be done between  $v = 1$  and  $v < 1/2$ , Figs. 8 and 9 have to be superimposed making  $v = 1$  in Fig. 9. Denoting by  $\tilde{\alpha} = \tilde{\alpha}(\mu = 1)$  and  $\tilde{\theta} = \tilde{\theta}(v = 1)$  and noting that  $\hat{\alpha} < \tilde{\alpha} < \bar{\alpha}$  and  $\hat{\theta} < \tilde{\theta} < \bar{\theta}$ , Fig. 10 shows the 9 zones of interest for the comparison of the utilities.

We then compare on each zone the utility of the individual at  $p(\bar{C}, \bar{W})$  and at  $p(C, W)$  making  $v = 1$ . Simple calculations lead to Fig. 11, where

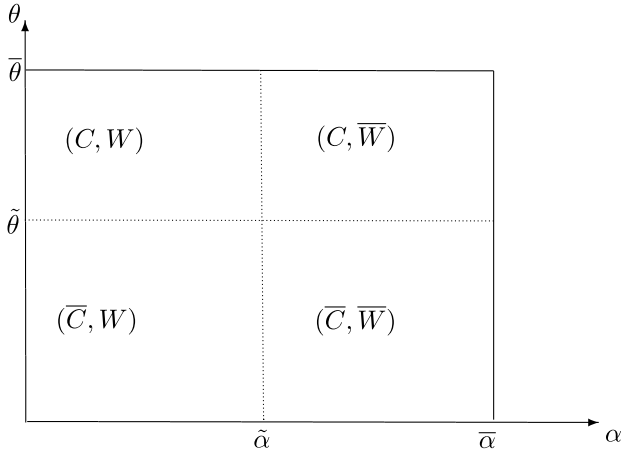
$$\theta_1 = \frac{2\bar{\alpha}\bar{\theta} + \bar{\theta}^2 - \bar{\alpha}^2}{4(\bar{\alpha} + \bar{\theta})} \quad (32)$$

$$\theta_2 = \frac{4\bar{\alpha}\bar{\theta} + \bar{\theta}^2 + \bar{\alpha}^2}{4(\bar{\alpha} + \bar{\theta})} \quad (33)$$

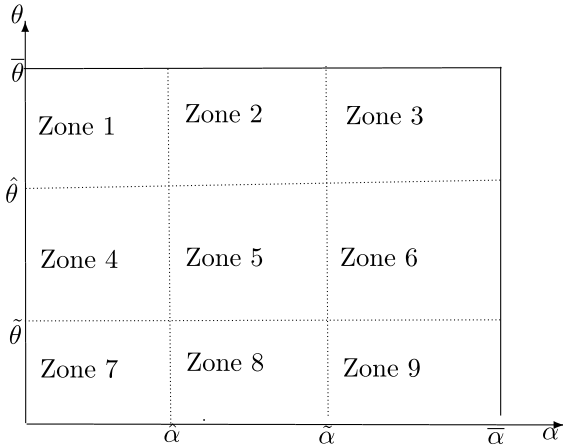
noting that for  $\gamma < 1$ , we have  $\tilde{\theta} < \theta_1 < \theta_2 < \hat{\theta}$ .



**Fig. 8.** The choice of each individual in terms of consumption and work at  $p(C, W)$ .



**Fig. 9.** The choice of each individual in terms of consumption and work at  $p(C, W)$ .

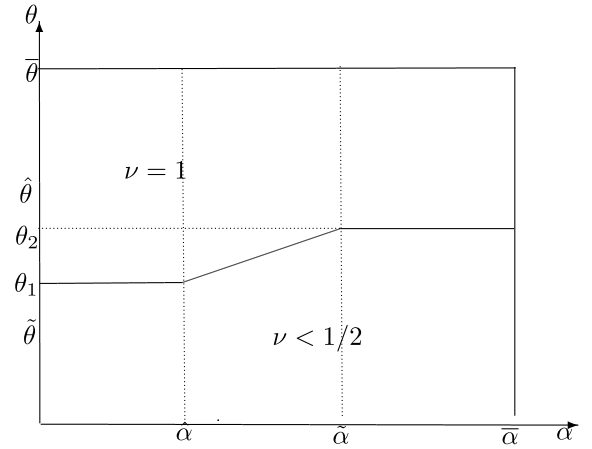


**Fig. 10.** Superimposing Figs. 8 and 9.

Now we calculate the area corresponding to the choice  $\nu = 1$  and the one corresponding to the choice  $\nu < 1/2$  and compare them. The comparison shows that for  $\gamma < 1$ ,  $\nu = 1$  always wins a majority.

### Part 2: under $\mathcal{H}_2$ .

If the output price  $p(C, W)$  wins, we just have to add to the expressions of the utilities given in Equations from (23) to (27) the share of each individual in the losses, equal to  $(1-\nu)\frac{\pi}{\alpha\bar{\theta}}$ . Recall that the expression of  $\pi$  is given by Eqs. (17) and (18).



**Fig. 11.** Votes in terms of  $\nu$  for  $\gamma \leq 1$  under  $\mathcal{H}_1$ .

We prove easily that whenever  $\nu \geq \frac{1+\gamma}{2\gamma+1}$ , all the obtained utilities  $U_{\mathcal{H}_2} = U_{\mathcal{H}_1} + (1-\nu)\frac{\pi}{\alpha\bar{\theta}}$  at  $p(C, W)$  (calculated using Eqs. (23) to (26)) are increasing in  $\nu$ . For  $\nu \in [1/2, \frac{1+\gamma}{2\gamma+1}]$ ,  $U_{\mathcal{H}_2} = (\theta - \alpha)q + \nu q\bar{\alpha}$ , which is also increasing in  $\nu$ . Hence whatever her type, every agent's utility at  $p(C, W)$  is continuous and increasing in  $\nu$ , for  $\nu \in [1/2, 1]$ .

For  $\nu \leq 1$ , for all  $\nu \geq 1/2$ ,  $p(C, W)$  is chosen by a majority. Since  $U_{\mathcal{H}_2}$  is always increasing in  $\nu$ , among  $\nu \geq 1/2$ , only  $\nu = 1$  is eligible. We have thus only to compare the options  $\nu = 1$  using  $U_{\mathcal{H}_2}$  and the option  $\nu < 1/2$ . But at  $\nu = 1$ ,  $U_{\mathcal{H}_1}$  and  $U_{\mathcal{H}_2}$  are equal. The comparison has already been made under  $\mathcal{H}_1$  and we concluded that  $\nu = 1$  wins a majority.

For  $\gamma > 1$ , noticing that  $\frac{\gamma+1}{2\gamma} > \frac{\gamma+1}{2\gamma+1}$ , there is a change for the output price at  $\tilde{\nu}$  satisfying  $\gamma = S(\tilde{\nu})$ , thus given by:

$$\tilde{\nu} = \frac{\gamma + 1}{2\gamma}.$$

For  $1/2 \leq \nu \leq \tilde{\nu}$ , the output price  $p(C, W)$  is chosen; and for  $\nu > \tilde{\nu}$ , the output price  $p(\bar{C}, \bar{W})$  is chosen. For  $\nu < 1/2$ , the output price  $p(\bar{C}, \bar{W})$  is also chosen and the situation amounts for everybody to  $\nu > \tilde{\nu}$ . Given the increasingness of the utilities  $U_{\mathcal{H}_2}$  over  $[1/2, \tilde{\nu}]$ , the comparison to be made relevantly is between  $\nu = \tilde{\nu}$  and  $\nu = 1$  (equivalent to any other  $\nu > \tilde{\nu}$  or  $\nu < 1/2$  but requiring no transaction). As done previously, to determine the choice of each individual in terms of consumption and work, we have to superimpose Figs. 8 and 9, but this time making  $\nu = \tilde{\nu}$  in Fig. 9. We then obtain the equivalent of Fig. 10 with:

$$\tilde{\alpha}(\tilde{\nu}) = \frac{\bar{\theta}\gamma(1+2\gamma)}{2(\gamma+1)},$$

and

$$\tilde{\theta} = \frac{\bar{\theta}}{2(\gamma+1)}.$$

As for the utilities, we have to determine  $U_{\mathcal{H}_2}(p(C, W))$  at  $\nu = \tilde{\nu}$ . They are given by the following.

$$U_{\mathcal{H}_2}(C, W)(p(C, W)) = (\theta - \alpha)q + \frac{\bar{\theta}q\gamma(2\gamma+1)}{2(\gamma+1)} - \frac{q\bar{\theta}}{2(\gamma+1)} + \frac{q\bar{\theta}(1-4\gamma^2)}{4(\gamma+1)}, \quad (34)$$

$$U_{\mathcal{H}_2}(C, \bar{W})(p(C, W)) = \theta q - \frac{q\bar{\theta}}{2(\gamma+1)} + \frac{q\bar{\theta}(1-4\gamma^2)}{4(\gamma+1)}, \quad (35)$$

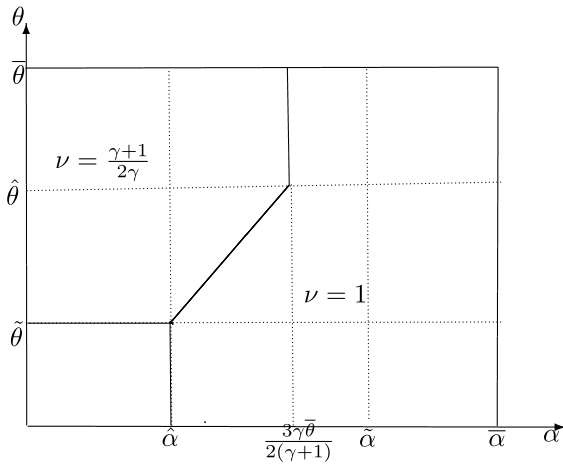


Fig. 12. Votes in terms of  $\nu$  for  $\gamma > 1$  under  $\mathcal{H}_2$ .

$$U_{\mathcal{H}_2}(\bar{C}, W)(p(C, W)) = -\alpha q + \frac{\bar{\theta} q \gamma (2\gamma + 1)}{2(\gamma + 1)} + \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}, \quad (36)$$

$$U_{\mathcal{H}_2}(\bar{C}, \bar{W})(p(C, W)) = \frac{q\bar{\theta}(1 - 4\gamma^2)}{4(\gamma + 1)}. \quad (37)$$

The comparison on each zone among the 9 zones, between the utilities leads to Fig. 12.

The area corresponding to  $\nu = 1$  is equal to  $\frac{\gamma^2 \bar{\theta}^2}{\gamma + 1}$ , while the area corresponding to  $\nu = \tilde{\nu}$  is equal to  $\frac{\bar{\theta}^2 \gamma (4\gamma + 3)}{4(\gamma + 1)^2}$ . The comparison between the two expressions shows that the area corresponding to  $\nu = 1$  is larger than the second one for all  $\gamma > 1$ .

#### Proof of Proposition 4

**First case**  $\gamma < 1$ . We have  $p = p(C, W)$  at  $\nu = 1$ . Two types of privatization have to be considered.

The first one is such that the State retains the control of the firm, thus  $\nu \in [1/2, 1]$ , which implies price  $p(C, W)$  at step 3 and a negative profit. Under  $\mathcal{H}_1$ , the firm is unsaleable and the only eligible  $\nu \in [1/2, 1]$  is  $\nu = 1$ .

Under  $\mathcal{H}_2$ , one has simply to replace  $p$  in  $W$  by  $p(C, W)$  (with an arbitrary  $\nu \in [1/2, 1]$ .) We obtain

$$W(\nu) = \frac{q\bar{\theta}^3(3\nu - 1 - \gamma)\gamma(1 + \gamma + \nu)}{8\nu^2(1 + \gamma)} \text{ when } \nu \geq \frac{1 + \gamma}{2\gamma + 1}, \quad (38)$$

$$W(\nu) = \frac{1}{2}q\bar{\theta}^3\gamma(1 - \gamma) \text{ when } 1/2 \leq \nu \leq \frac{1 + \gamma}{2\gamma + 1} \quad (39)$$

This is a continuous function which is constant then increasing of  $\nu$ . Thus it takes its maximum value at  $\nu = 1$ . Any partial privatization where the State retains the control of the firm decreases social welfare as defined here.

To conclude partially, whether under  $\mathcal{H}_1$  or  $\mathcal{H}_2$ ,  $\nu = 1$  is the local maximum or the only eligible value among  $\nu \in [1/2, 1]$ .

The second type of privatization is a radical one transferring the control of the firm to private investors who maximize profits. Therefore we have to compare the welfare at  $\nu = 1$  with the value of welfare for  $\nu < 1/2$ .

Evaluating  $W$  at  $p = p(C, W)$  and  $\nu = 1$ , we obtain (here  $p(C, W) > 0$ , and no discussion is needed):

$$\frac{q\bar{\theta}^3(4 - \gamma^2)}{8(1 + \gamma)}. \quad (40)$$

A radical privatization would lead to price  $p = p(\bar{C}, \bar{W})$ , thus to social welfare:

$$\frac{3q\bar{\theta}^3\gamma}{8(1 + \gamma)}. \quad (41)$$

Subtracting (40) from (41), one obtains

$$\frac{3q\bar{\theta}^3\gamma}{8(1 + \gamma)} - \frac{q\bar{\theta}^3(4 - \gamma^2)}{8(1 + \gamma)} = q\bar{\theta}^3 \frac{(\gamma - 1)(\gamma + 4)}{8(\gamma + 1)}$$

so that we can conclude that a radical privatization would reduce social welfare.

**Second case**  $\gamma > 1$ . In the initial situation, the firm maximizes profits and  $p = p(\bar{C}, \bar{W})$ . Accordingly  $W$  equals (41). Any radical privatization or a partial privatization toward some  $\nu > \frac{1 + \gamma}{2\gamma}$  obviously yields to the same level of social welfare since the firm remains a profit-maximizer. Thus the situation would be equivalent in terms of welfare.

Let us now consider a partial privatization toward  $\nu \in [1/2, \frac{1 + \gamma}{2\gamma}]$ .

Under  $\mathcal{H}_1$  this privatization is not possible as the firm's profit is negative making the firm unsaleable. Thus the possible values of  $\nu$  are only the ones belonging to  $\in [0, 1/2] \cup [\tilde{\nu}, 1]$  and the proof ends here.

Under  $\mathcal{H}_2$ , a partial privatization toward  $\nu \in [1/2, \frac{1 + \gamma}{2\gamma}]$  leads to a social welfare given by (38) and (39) which has been shown to be increasing in  $\nu$ . So we have to compare the levels of  $W$  given by (38) and (41) for  $\nu = \frac{1 + \gamma}{2\gamma}$  (as  $\frac{1 + \gamma}{2\gamma} > \frac{1 + \gamma}{2\gamma + 1}$ ). The latter is:

$$W\left(\frac{1 + \gamma}{2\gamma}\right) = q\bar{\theta}^3 \frac{(3 - 2\gamma)(2\gamma + 1)}{8(\gamma + 1)}.$$

The difference equals:

$$q\bar{\theta}^3 \frac{(3 + \gamma - 4\gamma^2)}{8(1 + \gamma)} = q\bar{\theta}^3 \frac{(-4\gamma - 3)(\gamma - 1)}{8(\gamma + 1)} < 0.$$

Thus such a partial privatization will decrease the total welfare.

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<sup>27</sup> Indeed the derivative of this function w.r.t.  $\nu$  is of the same sign as  $\gamma + 1 - \nu$ , which is positive as  $\nu \leq 1$ .

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