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# When Is Product Personalization Profit-Enhancing? A BehaviorBased Discrimination Model 

Didier Laussel, ${ }^{\mathbf{a}}$ Joana Resende ${ }^{\mathbf{b} \text {,* }}$


#### Abstract

This paper investigates duopoly competition when horizontally differentiated firms are able to make personalized product-price offers to returning customers, within a behavior-based discrimination model. In the second period, firms can profile old customers according to their preferences, selling them targeted products at personalized prices. Product-price personalization (PP) allows firms to retain all old customers, eliminating second-period customer poaching. The overall profit effects of PP are shown to be ambigu-ous. In the second period, PP improves the matching between customers' preferences and firms' offers, but firms do not make any revenues in the rival's turf. In the Bertrand out-come, secondperiod profits only increase for both firms if the size of their old turfs are not too different or initial products are not too differentiated. However, the additional second-period profits may be offset by lower first-period profits. PP is likely to increase firms' overall discounted profits when consumers' (firms') discount factor is low (high) and firms' initial products are exogenous and sufficiently different. When the location of initial prod-ucts is endogenous, profits are hurt because of an additional location (strategic) effect aggravating head-to-head competition in the first period. Likewise, when a fraction of active consumers conceals their identity, PP increases second-period profits at the cost of aggressive first-period price competition. Finally, we show that the room for profitable PP enlarges considerably if firms rely on PP as an effective device to sustain tacit collusive out-comes, with firms credibly threatening to respond to first-period price deviations with secondperiod aggressive relocations of their standard products.


Keywords: behavior-based discrimination • price and product targeting • consumer poaching • consumer retention • segmentation • tacit collusion

## 1. Introduction

New-generation digital technologies are enabling firms to combine unprecedented capabilities on customers' recognition with the launch of smart production processes that allow for mass customization at negligible variable costs. An increasing number of firms are thus engaging in strategies of market hypersegmentation through price targeting and product personalization. This trend is emerging in a wide range of industries such as e-commerce (e.g., Alexa makes personalized shopping suggestions to Amazon's users), the hotel industry (e.g., websites such as www.booking.com offer personalized hotel search results based on customers'
previous purchases/online search), media and entertainment (e.g., Netflix or Spotify offer personalized recommendations based on algorithms that account for the individual profile of each consumer), the health sector (e.g., medical devices and even drugs are being personalized according to patients' specific needs), or apparel retailing (e.g., personalized boxing services ${ }^{1}$ are offered by many retailers and online platforms, such as the Trunk Club of Nordstrom). ${ }^{2}$ In all these cases, firms usually start learning about the preferences of customers after some initial interactions (e.g., Spotify's recommendation algorithm starts matching more closely consumers' preferences after following their music choices for

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[^0]some time; the possibility to tailor hotel search results on websites like www.booking.com is more likely after the consumer has done some transactions, or at least some search on the website; likewise, stylists' ability to offer enticing personalized boxes is higher when they already know the clients).

This paper aims to contribute to the ongoing (academic and managerial) discussions on whether product and price personalization strategies may (or not) boost profits for firms. To this end, we develop a two-period Hotelling model where firms are able to make personalized price and product offers to returning customers (throughout the paper, we denote by PP those productprice personalized offers). The model builds on the extant literature on behavior-based price discrimination (BBPD), especially the works by Fudenberg and Tirole (2000) Zhang (2011), or more recently, Choe et al. (2018) (henceforth denoted as CKM 2018). After the seminal works by Fudenberg and Tirole (2000) and Chen (1997), there was a boost in the literature on behavior-based price discrimination. Some examples include Choudhary et al. (2005), Chen and Iyer (2002), Esteves (2009, 2010), Rhee and Thomadsen (2016), or Jing (2016). Other recent works have looked into different forms of price personalization (e.g., Anderson et al. 2015, 2019).

More precisely, we look at competition between two horizontally differentiated firms competing in two periods. In the first period, they first choose the initial specification of their standard product and then compete in uniform prices. Each firm obtains perfect information about the preferences of its own customers while remaining totally unaware of the preferences of the rival's customers (as in CKM 2018): by the end of the first period, firms are able to divide the initial market into two independent customers' segments/turfs (old versus new customers, as in standard BBPD). In the second period, each firm will offer to new customers (the rival's old ones) a (possibly updated) standard product sold at a uniform price. After observing the rival's product and price standard offers, each firm decides on the personalized product-price mix targeted to old customers, using profiling data about their preferences. Personalization costs are assumed to be mostly fixed (e.g., they are associated with the acquisition and maintenance of software, digital platforms, or equipment such as threedimensional printers, whose acquisition and maintenance costs often have a fixed nature).

We investigate the subgame perfect Nash equilibrium (SPNE) of the two-period game, assuming first that firms do not consider the possibility of using threats on future product specification to relax first period price competition. In this baseline model, it is also assumed that there is no leakage between market segments. Both assumptions are relaxed at the end of the paper, where we introduce (i) in Section 7.1, the possibility of consumers' active identity management,
and (ii) in Section 7.2, the possibility of tacit collusive equilibria sustained by punishment threats based on an aggressive specification of the product targeted to the rival's customers in the second period.

Starting with the baseline model, we solve the sequential game by backward induction. When we concentrate on the equilibrium of the second-period subgame, we find that, differently from previous works, there is no room for consumer poaching. ${ }^{3}$ The old provider always makes personalized productprice offers to the returning customers, precluding the rival from making any compelling offer (as the latter lacks the informational advantage and the ability to engage in PP in the rival's turf). This is a particularly interesting finding as it is consistent with practitioners' views that PP may be an important tool to promote consumers' loyalty and increase consumers' retention. ${ }^{4}$

Our second-period results also allow us to highlight some countervailing effects of price-product personalization: on the one hand, PP improves the match between the firms' product (personalized) offers and the ideal product specification of each old customer, leading to higher second-period prices (and higher profits). On the other hand, PP completely eliminates second-period sales in the rival's turf, which hurts profits. Overall, second-period profit effects are positive when (i) firms' initial products are not too differentiated (as in this case, PP allows firms to differentiate their otherwise close products, resulting in higher markups), and (ii) firms' second-period turfs are not too asymmetric. ${ }^{5}$

Basically, PP operates by generating a dissuasive switching cost for old customers, ${ }^{6}$ thus softening competition and increasing profit in the second period, provided firms' turfs and initial products are not too different (which is actually the case in the SPNE). At the same time, it intensifies first-period price competition. Indeed, when we look into the two-period game (without punishment strategies based on product relocation, which will be evoked later), we find that firstperiod price competition is very intense: firms compete head-to-head as an attempt to build a large turf of customers for the coming period. This competitive pressure is reinforced when firms have the possibility to choose the initial location of their products, at the beginning of the game. Actually, maximal differentiation in the first period only occurs when both the firms and the consumers discount heavily future payoffs. The firms' attempt to locate closer to each other is clearly in line with what Zhang (2011, p. 170) calls the "second peril of BehaviorBased personalization." However, herein the effect of consumers' patience on locations, prices, and profits considerably departs from the results of Zhang. This follows directly from the fact that, in our model without equilibrium poaching, greater consumers' patience
raises the price-sensitivity of demand instead of reducing it (as in models with equilibrium poaching).

We assess the profitability of PP strategies in comparison with two relevant benchmarks: price personalization alone (with firms sticking to standard products, as in CKM 2018) and lighter forms of market segmentation (with third-degree price and product discrimination, as in Zhang 2011). We find that PP boosts second-period profits for both firms if the sizes of the firms' turfs are not too different (which is indeed the case in our symmetric SPNE).?

When we look at the two-period game (in which firms have not yet built their consumer base and therefore PP will intensify first-period price competition), the profitability of PP vis-à-vis price personalization alone (as in CKM 2018) critically depends on firms' ability to choose the location of their initial product. If firms can choose the locations of their initial products, PP is always profit detrimental: this results from firms' incentives to choose closer locations to enlarge their turf of second-period old customers, what exacerbates firstperiod price competition.

When the location of firms' initial products is assumed to be exogenous and firms locate sufficiently apart from each other, PP may lead to higher overall profits, provided that firms' discount factor is high enough (so that firms put enough weight on future profits) and/or consumers' discount factor is small enough (so that the first-period demand is not too elastic, alleviating the competitive pressure).

When comparing our two-period equilibrium results to Zhang (2011), we find that overall profits under PP are always lower than the ones in Zhang 2011. ${ }^{8}$ Although the use of accurate information on consumers' preferences allows each firm to fully protect its own turf from consumer poaching (leading to greater second-period profits in the SPNE), PP intensifies first-period price competition.

These results are shown to be robust to the existence of a small fraction of customers engaging in identity management actions. In the extensions section, we build on Chen et al. (2020) to allow for the existence of an infinitesimal proportion of active customers who may hide their identity, either by avoiding being profiled (ex ante active customers) or by pretending to be a new customer after receiving the personalized priceproduct offer (ex post active customers). In line with Chen and Iyer (2002) or, more recently, Chen et al. (2020), the existence of a fraction of active customers always allows firms to sustain higher second-period profits. Firms now know they may be selling their standard good to anonymous old customers, which increases the second-period price targeted to (supposedly) new customers, thus relaxing price competition. However, this intensifies even more first-period competition,
considerably eroding first-period profits. PP may only boost overall profits for both firms when negative firstperiod prices are ruled out and, in addition, firms' discount factor is high enough.

The scope for profitable PP enlarges considerably when we modify our baseline model to allow for punishment strategies based on aggressive second-period standard product design choices (see Section 7.2) ${ }^{9}$ as in this scenario, firms may sustain higher (tacit-collusive) first-period prices. This possibility follows from firms' ability to credibly threat to respond to downward (first-period) price deviations with aggressive (secondperiod) product positioning in the rival's turf. ${ }^{10}$ When such punishment strategies are allowed, we show the existence of equilibria where two-period profits are greater than the ones obtained with the repetition of the static equilibrium. As far as we know, this is a completely new result in the BBPD literature.

Besides the closely related papers already mentioned, our paper also enriches the wide literature on product customization. An important line of literature in this topic deals with customers' self-selection and second-degree price discrimination. In those works, firms' information about consumers' preferences is both statistical and exogenous (Dewan et al. 2003, Choudhary et al. 2005, Syam et al. 2005). ${ }^{11}$

There are also some other papers that look at product personalization without looking at the process of information collection. Ghose and Huang (2009) analyzed competition between two firms that either choose quality customization and second-degree price discrimination or quality personalization and firstdegree price discrimination. Bernhardt et al. (2007) consider a sequential game in which firms first make investments on information quality and then they see (privately) signal realizations of consumers' preferences. Firms compete in product-specific price offers to consumers. ${ }^{12}$

The rest of the paper is organized as follows. Section 2 presents the main ingredients of the model, and Section 3 looks at second-period competition, including firms' personalized price-product offers targeted to old customers. Section 4 investigates first-period price competition. Section 5 studies endogenous location choices. Section 6 sums up the managerial implications of our baseline model, and Section 7 presents some relevant extensions, including active identity management (Section 7.1) and tacit collusion based on second-period product upgrade (Section 7.2). Finally, Section 8 concludes. All the proofs of nontrivial results are presented in the online appendix.

## 2. Model

Consider a duopoly market where two firms (Firms 1 and 2 ) produce horizontally differentiated goods. In the
beginning of period 1, they choose the specification/ location of their (initially) standard products: Firm 1 locates at $x_{1}=a$ and Firm 2 locates at $x_{2}=1-b$. Without loss of generality, we assume $1-a-b \geq 0$ so that Firm 1 is located at the left of Firm 2. Firms' marginal production cost is assumed to be constant and normalized to zero. Consumers are uniformly distributed on the segment $[0,1]$, and they incur a cost from consuming a product variety that departs from their ideal one (corresponding to the conventional Hotelling "transportation cost"). Following d'Aspremont et al. (1979), we assume that this cost is a quadratic function of the distance between the location of each product and the location of the consumer's ideal variant: a consumer whose ideal variety is $x$ incurs a cost $t\left(x-x_{i}\right)^{2}$ when buying at Firm $i$ (located at $x_{i}$ ). Consumers' reservation price is equal to $U$, which is assumed to be high enough to guarantee full market coverage. ${ }^{13}$

Assumption 1. Let consumers' reservation price be high enough to guarantee full market coverage, with $U \geq 2 t$.

Firms play a noncooperative game with two periods. The timing of the game is as follows: in the first-period, firms first choose noncooperatively and observably the location of their standard products ( $a$ and $b$, respectively) and then compete in linear uniform prices ( $p_{1}$ and $p_{2}$, respectively). Given $p_{1}$ and $p_{2}$, let $z$ denote the location of the first-period marginal customer, who is indifferent between buying at price $p_{1}$ from Firm 1 or buying from Firm 2 at price $p_{2}$, in the first period. Thus, conditional on $p_{1}$ and $p_{2}$, Firm 1 serves a subset $A_{1}$ of customers (comprising the consumers located between zero and $z$ ), whereas Firm 2 serves a subset $A_{2}$ of consumers (comprising those located between $z$ and 1).

In the second period, for given first-period market shares (summarized by the variable $z$ ), each firm serves two segments: the turf of old customers (whose preferences are only disclosed to the firm who sold them the good in the first period) and the turf of the rival's old customers (whose preferences are only known to the rival firm). In the baseline model without information leakage, ${ }^{14}$ the two segments are fully independent, and we assume that firms make sequential decisions on each segment: first they decide on their (uniform) product-price offers targeted to the rival's turf and then they decide on the product-price offers targeted to their own turf (where they may use information on customers' preferences to engage in product-price personalization). ${ }^{15}$ More precisely, the timing of the game in this second-period is the following:
(i) First, the firms decide the (possibly updated) location of their standard products ( $a^{N}$ and $b^{N}$ ) targeted to the rival's old customers;
(ii) Second, they choose the uniform prices $P_{1}\left(A_{2}\right)$ and $P_{2}\left(A_{1}\right)$ of their standard products (directed to the segment of the rival's old customers); and
(iii) Finally, they take decisions targeted to the segment of their own old customers. Each Firm $i$ first decides whether to offer personalized products to each type $x$ - old customer and then selects the corresponding type-specific prices $v_{i}(x)$, targeted to old customers.

This timing framework extends Chen et al. (2020) to a two-period setting with product personalization within firms' old turf of customers. It is also worth noting that we suppose here that firms' decision on whether to offer personalized products to returning customers is not observable and/or not irreversible. This means that, before observing the rival's second-period standard product-price decisions, firms are unable to do any commitment regarding the targeted product-price offers they will make to their second-period returning customers. ${ }^{16}$ Notice that Zhang (2011) also assumes as we do here that product upgrade decisions are taken in period 2 (see p. 174, first line of section 1 and lemma 1 ).

We also follow Zhang (2011) in what concerns the marginal cost of product personalization, which is assumed to be zero. ${ }^{17}$ This assumption fits well contexts where most of personalization costs have a fixed nature (e.g., investments in software, digital production platforms, or three-dimensional printers, whose costs are independent of the exact specification of the products). We also consider that both firms and customers discount future payoffs, possibly at different rates. The firms' common discount factor is denoted by $\delta$, whereas the consumers' discount factor is denoted by $\beta$. To simplify the exposition, throughout the paper, we let both parameters take any value in [01). However, all results remain valid for $\delta \geq 1 .{ }^{18}$ We assume that $\beta<1$. This restriction is necessary to guarantee that first period demand functions are well-behaved decreasing functions of prices.

We shall see that the possibility to engage in PP allows each firm to successfully protect its old customers' turf against any poaching price of the rival. Because only the old provider knows the preferences of its old customers, it is always able to offer a more enticing price-product mix than the rival firm (whatever the poaching price the latter chooses). Then, the firms being indifferent between all possible uniform price and all possible uniform product designs (offered to new customers), there will be multiple second-period equilibria. Refinement arguments are then needed to select one of them and three at least deserve some consideration. First, to allow for meaningful comparisons, in the baseline model, we mainly follow Chen et al. (2020) ${ }^{19}$ by selecting the Bertrand outcome, which would be the unique equilibrium in the case of simultaneous moves. This focus is justified in Chen et al. (2020), lemma 2, by the use of a simple
refinement argument. ${ }^{20}$ Second, we also consider in Section 7.1 equilibrium outcomes resulting from the existence of a very small proportion of customers who are able to conceal their identity (either ex ante or ex post their second-period purchases ${ }^{21}$ ). This will lead to the selection of other, more profitable, second-period equilibria than the Bertrand one. Finally, in Section 7.2, we investigate equilibria with first-period tacit collusion (under the threat of second-period aggressive product relocation). This will allow us to make the point that PP can actually constitute a credible commitment device to sustain more profitable (tacit-collusive) equilibria in the first period.

In what follows, we focus on the Bertrand outcome. The other two alternative equilibria are left to Section 7.

## 3. Second Period: Tailored Price-Product Offerings

Let us recall that in the second period, firms sequentially take the decisions targeted to the rival's turf (uniform product-price mix), followed by the decisions directed to their own turf. First, firms choose the (possibly updated) location of their second-period uniform products ( $a^{N}$ and $b^{N}$ ). Second, they fix the corresponding second-period price $P_{i}\left(A_{j}\right)$. Third, they choose whether to personalize their product offers to old customers, and finally they set their personalized price schedule $v_{i}(x)$.

As we solve the game by backward induction, we start by looking at the decisions of firms targeted to their old turf. To this end, it is worth noting that an old customer of Firm $i(i=1,2)$ cannot be poached by the rival if the transportation cost incurred when buying the rival's standard product exceeds the transportation cost incurred by the consumer when buying from Firm $i$. Without product personalization, an old customer of type $x$ cannot be poached from Firm 1 (respectively, Firm 2) if and only if $x \leq \frac{1+a^{N}-b^{N}}{2}$ (respectively, $x \geq \frac{1+a^{N}-b^{N}}{2}$ ). These two sets have no intersection, except for $x=\frac{1+a^{N}-b^{N}}{2}$. Hence, without product personalization, depending on the value of $z \neq \frac{1+a^{N}-b^{N}}{2}$, either Firm 1 poaches the rival's customers or the reverse occurs (as in CKM 2018).

When a firm offers personalized products to returning customers, poaching becomes unfeasible: when switching to the rival's (standard) product, customers would always incur a positive transportation cost, which creates a competitive disadvantage to the poaching firm as the old provider is able to offer the consumers' ideal variant without incurring additional costs. Let us now investigate Firm $i^{\prime}$ s optimal pricing strategy targeted to the old turf, as a best reply to the rival $j$ 's offer. The latter consists of a standard product sold at a uniform (poaching) price $P_{j}\left(A_{i}\right)$. Without loss of generality, we concentrate on
the case of Firm 1. Mutatis mutandis, a similar analysis applies to Firm 2.

Given $P_{2}\left(A_{1}\right)$, the price that Firm 1 targets to an old customer of type $x$ cannot exceed the price for which this customer is indifferent between buying the two products. Likewise, such targeted price cannot exceed the price that makes this customer indifferent between buying and not buying. Accordingly, (i) if Firm 1 offers this customer a standard product, the price $v_{1}(x)$ must be such that $v_{1}(x)=\min \left\{P_{2}\left(A_{1}\right)+t\left(1-b^{N}-x\right)^{2}-t\left(x-a^{N}\right)^{2}, U-\right.$ $\left.t\left(x-a^{N}\right)^{2}\right\}$; (ii) if, instead, Firm 1 offers a product that matches the customer's ideal specification, then $v_{1}(x)$ $=\min \left\{P_{2}\left(A_{1}\right)+t\left(1-b^{N}-x\right)^{2}, U\right\}$.

Comparing the markup that Firm 1 makes on a given old-consumer $x$ when it offers a personalized product versus a standard one, we can conclude upon observation that, when we reach this stage of the game, firms always profit from PP. Hence, firms' decision to offer personalized products to returning customers constitutes a dominant strategy: for any product specification $\left(1-b^{N}\right)$ chosen by Firm 2 in the second period, it is always optimal for Firm 1 to offer each old-customer $x$ a product that exactly matches tastes. It is also clear that each firm makes a strictly positive margin on its old customers, whatever the rival's poaching price $P_{2}\left(A_{1}\right) \geq 0$.

Lemma 1. Given the new customers' prices $P_{1}\left(A_{2}\right)$ and $P_{2}\left(A_{1}\right)$, the second-period equilibrium is such that it is a dominant strategy for Firm $i(i=1,2)$ to protect its turf by offering to all its old customers a customized product at a personalized price

$$
\begin{equation*}
v_{i}(x)=\min \left\{P_{j}\left(A_{i}\right)+t\left(x-x_{j}\right)^{2}, U\right\} \tag{1}
\end{equation*}
$$

with $x_{1}=a^{N}, x_{2}=1-b^{N}$. This will dissuade all of them to switch to its rival.

The "no poaching" result stated in Lemma 1 departs from previous literature (namely the twosided poaching results in Fudenberg and Tirole (2000) and Zhang (2011) or the one-sided poaching result in CKM (2018)). ${ }^{22}$ The rationale lies on the fact that PP results in a switching cost to old customers, which softens firms' personalized pricing schedule (leading to a positive effect on second-period profits). ${ }^{23}$

Lemma 1 takes as given the second-period uniform price-product mix that each firm targets to the rival's turf. We now move forward by investigating the uniform price and the standard product that each firm offers to the rival's customers. In this respect, it is worth noting that Lemma 1 implies that the poaching firm (i.e., Firm $j$ ) makes a zero profit on the rival $i$ 's turf whatever the price $P_{j}\left(A_{i}\right)$ it may choose. Hence, the game will have multiple equilibria. Among all these equilibria, in the baseline model we will start by looking at the more competitive one, that is, the Bertrand one. Although there is some
rationale to justify this focus, ${ }^{24}$ this is also for the sake of meaningful comparison with CKM (2018).

Proposition 1. At the second-period Bertrand equilibrium, the poaching price is $P_{j}\left(A_{i}\right)=0, i, j=1,2, i \neq j$. Given Assumption 1,
(i) Firm i charges each returning customer $x$ a personalized price:

$$
v_{i}(x)=t\left(x-x_{j}\right)^{2}, i, j=1,2, i \neq j
$$

(ii) firms' second-period equilibrium profits are

$$
\begin{align*}
& \pi_{1}(z)=\frac{t}{3}\left(\left(1-b^{N}\right)^{3}-\left(1-b^{N}-z\right)^{3}\right) \\
& \pi_{2}(z)=t\left(\frac{1}{3}\left(1-z^{3}\right)+a^{N}(1-z)\left(a^{N}-z-1\right)\right) \tag{2}
\end{align*}
$$

Proposition 1 implies that, at the Bertrand equilibrium described previously, the second-period profits of Firm $i, \pi_{i}(z)$, only depend on the size of Firm $i^{\prime}$ s initial customer base and on the updated specification of $j$ 's product. This result is to be expected: even if a Firm tailored its standard product to become more suitable to the preferences of new customers, the rival would always be able to offer them a better bargain by appropriately adjusting its product/price mix. However, this brings a new source of equilibrium multiplicity: in the second period, Firm $i$ is indifferent between all the possible specifications $x_{i} \in[0,1]$ of its standard product. In the following sections, among all possible equilibria, we select a natural one where each firm chooses to keep the specification of its product unchanged: indeed, a small cost of relocating, even if infinitesimal, would be enough to dissuade completely the two firms from making any product update. In Section 7.2, we shall show the existence of alternative equilibria, where first-period tacit collusion may be sustained by firms' threaten to update the second-period location of the standard product offered to new customers (to punish rivals for deviations from a first-period price (tacit) agreement). ${ }^{25}$ Before turning our attention to these other equilibria, we investigate profit effects under the absence of these product relocation threats.

In Corollary 1, we assess the profitability of secondperiod PP vis-à-vis price personalization alone (as in CKM 2018) and third-degree product and price discrimination (as in Zhang 2011). The profit comparison with CKM (2018) is illustrated in Online Appendix A.2.1 and the profit comparison with Zhang (2011) in Online Appendix A.2.2. Because the analysis is relatively similar in both cases, Figure 1 reproduces, for $a^{N}=b^{N}=l$, the area (in gray) of values of $(l, z) \in[0,1 / 2]$ $\times[0,1]$, where Firm 1's ability to personalize its product leads to higher second-period profits herein than in CKM (2018).

Figure 1. Effect of PP on $F_{1}{ }^{\prime} \mathrm{s}$ Profits


Corollary 1. Under symmetric locations $\left(a^{N}=b^{N}=l\right)$, product personalization boosts second-period equilibrium profits for both firms iff their first-period market shares are not too different.

Figure 1 highlights the countervailing effects of PP. Whether Firm 1 is the small or the big firm, PP allows firms to match the old consumers' ideal specification and charge them higher prices. This is the positive profit effect of PP. When Firm 1 is the small firm ( $z \leq 1 / 2$ ), PP also eliminates firms' revenues from sales to new customers (which hurts profits). When Firm 1 is the big firm $(z \geq 1 / 2)$, PP elicits a tougher rival's poaching price (which also hurts profits). In both cases, the positive effect is always present, whereas the negative ones vanish when $z \rightarrow 1 / 2$. Figure 1 indeed shows that if initial locations are very similar, the first effect tends to dominate the second one as PP allows firms to differentiate their (otherwise close) products, collecting the corresponding brand premium. When firms are initially located far from each other, PP only increases second-period profits if firstperiod market shares are similar enough. ${ }^{26}$

The comparison between our second-period profits and the corresponding counterparts in Zhang (2011) leads to similar conclusions (summarized in Online Appendix A2.2). In this case, it is also worth adding that our second-period equilibrium profits are increasing in the size of firms' own (first-period) market share, whereas Zhang (2011) obtains that second-period profits are increasing with the asymmetry of the size of firms' first-period market shares. ${ }^{27}$

Thus far, we concentrated our attention in the secondperiod subgame, taking $z$ as given. This analysis is useful to get some insights on competition patterns when PP is implemented in markets where firms already have an installed customer base (i.e., mature markets). Now, we proceed by investigating first-period outcomes that allow us to understand how PP affects competition when firms are still building their own consumer installed base (so that $z$ is endogenous). As we solve the game by backward induction, we start with first-period price competition and then we look at optimal initial product location.

## 4. First-Period Price Competition

Let us first investigate how the position of the marginal first-period customer $z$ (who is indifferent between buying at Firm 1 or buying at Firm 2 in the first period) is affected by $p_{1}$ and $p_{2}$. We consider consumers have rational expectations about the offers they will receive in period 2. In particular, they anticipate that poaching will be ruled out, and consequently, the indifferent consumer in period 1 (located at $z$ ) knows that the choice is between staying with Firm 1 for two periods or staying with Firm 2 for two periods (always getting a personalized product-price offer in the second period). Firm 1's first-period demand is thus obtained by solving the indifference condition, which writes generally as

$$
\begin{align*}
p_{1}+t(a-z)^{2}+\beta t\left(1-b^{N}-z\right)^{2}= & p_{2}+t(1-b-z)^{2} \\
& +\beta t\left(a^{N}-z\right)^{2} . \tag{3}
\end{align*}
$$

Recall also that, for the time being, we are focusing on the second-period Bertrand outcome, and we are also selecting an equilibrium without second-period relocation, that is, such that $a^{N}=a$ and $b^{N}=b$. (This assumption is relaxed in Section 7.2.) Then, we obtain the following:

$$
\begin{equation*}
z\left(p_{1}, p_{2}, \beta\right)=\frac{1+a-b}{2}+\frac{p_{2}-p_{1}}{2 t(1-a-b)(1-\beta)}, \tag{4}
\end{equation*}
$$

whereas the first-period demand to Firm 2 is simply $1-z\left(p_{1}, p_{2}, \beta\right)$.

When consumers become more patient ( $\beta$ increases), they weigh more what happens in the second period, and they anticipate that, by then, they will be charged a personalized price equal to the transportation they would incur when buying the rival firm's standard product (because second-period personalized prices are perfectly negatively correlated to the traveling costs to the rival product). The perfect negative correlation implies that, for $\beta=1$, the overall costs to consumers (the sum of the traveling cost in period 1 and the second-period personalized price) would be the same
for all consumers regardless of the first-period choice, which would only depend on the price differential, entailing a demand discontinuity. ${ }^{28}$ This is why we suppose here that $\beta<1$. The fact that more patient consumers are more price sensitive distinguishes this model without second-period consumer poaching from the conventional BBPD results (Villas-Boas 1999, Fudenberg and Tirole 2000, Zhang 2011). In those models, more forward-looking consumers tend to be less price sensitive. ${ }^{29}$

Let us now look at the first-period equilibrium prices. For given locations $a$ and $b$, the two firms compete in prices, rationally anticipating second-period payoffs in Equations (2), with $z$ given by (4).

Accordingly, firms' overall discounted profits are $\Pi_{1}$ $=p_{1} z+\delta \pi_{1}(z)$ and $\Pi_{2}=p_{2}(1-z)+\delta \pi_{2}(z)$, and the firstperiod Nash equilibrium in prices can then be obtained as follows:

$$
\begin{aligned}
& p_{1}^{*}=\arg \max _{p_{1}} p_{1} z\left(p_{1}, p_{2}^{*}, \beta\right)+\delta \pi_{1}\left(z\left(p_{1}, p_{2}^{*}, \beta\right)\right), \\
& p_{2}^{*}=\arg \max _{p_{2}} p_{2}\left(1-z\left(p_{1}^{*}, p_{2}, \beta\right)\right)+\delta \pi_{2}\left(z\left(p_{1}^{*}, p_{2}, \beta\right)\right) .
\end{aligned}
$$

The solution to these conditions allows us to obtain $\left(p_{1}^{*}(a, b), p_{2}^{*}(a, b)\right)$. The resulting expressions are rather long, and they are presented in Online Appendix A4. To get some insight, let us focus on the cases where firms are located symmetrically with respect to the center (later on, we will prove that this is actually the case in this SPNE). For $a=b=l \leq 1 / 2$, we get the following:

$$
\begin{equation*}
p_{i}^{*}(l, l)=\frac{t}{4}(1-2 l)(4(1-\beta)+(2 l-1) \delta), i=1,2, \tag{5}
\end{equation*}
$$

leading to $z^{*}=\frac{1}{2}$ (which according to Corollary 1 , assures profit-enhancing PP in the second period). In addition, prices $p_{i}^{*}(l, l)$ in (5) are clearly decreasing with both discount factors: (i) the higher $\beta$, the more price elastic are the first-period demand functions, with the usual negative impact on equilibrium prices; and (ii) when $\delta$ increases, firms attach more value to the size of their captive customer base (for the second period), resulting in tougher first-period price competition.
For prices (5), overall profits are as follows:

$$
\begin{align*}
\Pi_{i}^{*}(l, l)= & {\left[\frac{1}{2}\left(\frac{t}{4}(1-2 l)(4(1-\beta)+(2 l-1) \delta)\right)\right] } \\
& +\delta\left[\frac{t}{3}\left((1-l)^{3}-\left(\frac{1}{2}-l\right)^{3}\right)\right] . \tag{6}
\end{align*}
$$

We had conveniently written $\Pi_{i}^{*}(l, l)$ as the sum of the first-period profits and the (discounted) equilibrium second-period profits, corresponding to the first and the second bracketed terms (respectively) in (6). The profits in (6) show that, for fixed symmetric locations, the
negative effect of $\beta$ on equilibrium overall profits comes uniquely from its negative effect on first-period profits (an increase in $\beta$ leads to more elastic demands). Differently, variations of the firms' discount factor $\delta$ generate two opposite effects. The effect on first-period profits is negative because a greater $\delta$ value means that firms compete head-to-head in the first period to enlarge their captive market for the second period. However, a higher $\delta$ also means that firms attach more value to secondperiod profits, increasing overall discounted profits. For fixed symmetric locations, the last effect dominates so that $\Pi_{i}^{*}(l, l)$ increase with $\delta$.

## 5. Endogenous Standard Product Specification

We now search for firms' equilibrium initial locations, ${ }^{30}$ when those are selected simultaneously and noncooperatively at the beginning of the game.

Proposition 2. There is a unique symmetric equilibrium of the location game in which the firms share the market evenly $\left(z^{*}=\frac{1}{2}\right)$, with
$a^{*}=b^{*}=\max \left\{\frac{1}{2}-\frac{1-\beta}{3}\left(\frac{1}{4(1-\beta)+\delta}+\frac{2}{1-\beta+\delta}\right), 0\right\}$.

Corollary 2. As $\beta$ or $\delta$ increase, firms optimally choose closer locations.

From Proposition 2 and Corollary 2, it follows that maximal differentiation will only arise at equilibrium when both firms and consumers are sufficiently impatient: Given (7), maximal differentiation arises when ${ }^{31}$ $\delta \leq \frac{\sqrt{17}-3}{2}(1-\beta)$. Online Appendix A7 provides an illustration of the subdomain $(\beta, \delta)$ that leads to maximal differentiation.

Corollary 2 shows that products become less differentiated when both firms and customers are sufficiently patient. The effect of $\delta$ on equilibrium locations is clearly in line with what Zhang (2011) calls the "second peril of behavior-based personalization" (see proposition 2, p. 177). Obviously when the duopolists value more the second-period profits, they have a greater incentive to locate closer to their rival in the first period to guarantee a wider captive market for the future. However, the effect of the consumers' discount factor goes in the opposite direction of Zhang (2011): here, an increase in $\beta$ leads the duopolists to locate closer to each other. Taking the rival's location as given, each firm is then eager to move toward the center in order to expand its first-period demand (this incentive is stronger when $\beta$ goes up, as demand becomes more elastic). From (7), we get $z^{*}=\frac{1}{2}$ and the
following equilibrium first-period prices:

$$
\begin{align*}
p_{i}^{*}= & t\left(1-\beta-\frac{\delta}{4}\right), i=1,2, \text { if } \delta \leq \frac{\sqrt{17}-3}{2}(1-\beta) \\
p_{i}^{*} & =t \frac{(1-\beta)^{2}(3-3 \beta+\delta)\left(8(1-\beta)^{2}+7(1-\beta) \delta+\delta^{2}\right)}{(4-4 \beta+\delta)^{2}(1-\beta+\delta)^{2}} \\
& \text { if } \delta \geq \frac{\sqrt{17}-3}{2}(1-\beta) \tag{8}
\end{align*}
$$

which are decreasing in both $\beta$ and $\delta .{ }^{32}$ The twoperiod equilibrium profits write

$$
\begin{align*}
& \Pi_{i}^{*}=\frac{t}{6}(3(1-\beta)+\delta), i=1,2, \text { if } \delta \leq \frac{\sqrt{17}-3}{2}(1-\beta) \\
& \Pi_{i}^{*}=t \frac{\left(18(1-\beta)^{2}+7(1-\beta) \delta+\delta^{2}\right)}{24(1-\beta+\delta)} \text { if } \delta \geq \frac{\sqrt{17}-3}{2}(1-\beta) \tag{9}
\end{align*}
$$

Corollary 3. The total (two-period) profits always decrease with $\beta$. They are first decreasing and then increasing with $\delta$.

To understand the comparative statics with respect to $\delta$, recall that we have previously shown that overall profits under exogenous locations always increase with $\delta$. When we endogenize initial locations, there is an additional layer of complexity, because a variation in $\delta$ also affects firms' location decision: when maximum differentiation is not obtained, more patient firms locate closer to the center. The degree of differentiation between the initial products is mitigated, entailing an additional negative profit effect. Summing up all the effects, the impact of $\delta$ on profits ends up being nonmonotonic. ${ }^{33}$ For small $\delta$ values, equilibrium profits increase with $\delta$ (indeed, as long as we remain in the maximal differentiation area, there is no

Figure 2. Variation of $\Pi^{*}$ with $\delta$

location additional effect). For intermediate values of $\delta$, equilibrium profits decrease with $\delta$ : managers now react to an increase in $\delta$ by choosing to locate closer to the center (this intensifies first-period price competition, overweighing the fact that firms put more weight in second-period profits, when $\delta$ goes up within this subdomain). Finally, for sufficiently large $\delta$ values, equilibrium profits increase again with $\delta$ (because the positive direct effect of $\delta$ on second-period profit becomes dominant for sufficiently high $\delta$ ). Figure 2 depicts in gray the subdomain $(\beta, \delta)$, where an increase in $\delta$ increases profits at the SPNE.

## 6. Managerial Implications

Let us first investigate if managers who are already collecting information on customers preferences (to engage in price personalization) should enlarge the scope of their targeting strategies to include personalized product offers. Then, we analyze if firms profit from collecting hypertargeted information on customers' preferences (or if instead they are better off under lighter forms of market segmentation).

### 6.1. Product Personalization Pays Off?

We have already seen that PP generates countervailing profit effects. On the one hand, it increases secondperiod profits, as firms are able to improve the match between their (type-specific) product offer and the customers' individual tastes, leading to higher consumerspecific markups. On the other hand, PP exacerbates first-period price competition, hurting profits. For the sake of simplicity, let us start with the case of fixed exogenous locations at the extremes, so that (6) implies

$$
\begin{equation*}
\Pi_{i}^{*}(0,0)=\frac{1}{6} t(3(1-\beta)+\delta), \quad i=1,2 . \tag{10}
\end{equation*}
$$

To isolate the profit effects of PP alone, we need to compare our equilibrium profits (10) to the counterparts derived in CKM (2018), who have identified the existence of two asymmetric mirror Nash equilibria in pure strategies. ${ }^{34}$ In this respect, it is important to clarify that whenever we refer to PP, we are always referring to a setting where both prices and product design are adjusted to the individual tastes of each old customer. Differently, the results of CKM (2018) refer to a setting with uniform products and personalized prices. Although some effects of personalization strategies may already emerge under price personalization alone (e.g., the fact that overall profits may be lower with price personalization alone than in BBPD or the fact that price personalization may increase secondperiod profits with respect to BBPD), we also present some new insights arising only when firms can personalize both prices and products. Those insights include our finding that customer poaching in the

Figure 3. Effect of PP on Industry Profits

second period becomes unfeasible under PP or the possibility of using aggressive product placement strategies in the second period (in the rival's turf) to sustain higher first-period prices (see Section 7.2).
The comparison of our results with CKM (2018) shows that there is a domain of values of $\beta$ and $\delta$, where PP boosts overall two-period profits for both firms. This is illustrated in the small white area of Figure 3. Although this subdomain is a relatively small area, one should notice that it includes for instance the point $(\beta, \delta)=(0,1)$, which may have some empirical appeal, addressing the case of naïve customers and forward-looking firms.

If we now look at firms' individual equilibrium profits, we find that PP leads to lower equilibrium profits for both firms when $\beta$ is high enough or $\delta$ low enough. On the contrary, for high-enough $\delta$ and low $\beta$, both firms gain with PP. In this case, first-period demand is less elastic (as $\beta$ is low), which relaxes firstperiod price competition and mitigates the corresponding negative profit effects. Indeed, in Online Appendix A5, we compare first-period equilibrium prices in our model for fixed locations (see Equation (6)) with the corresponding counterparts in CKM (2018). We get that PP always leads to tougher firstperiod pricing policies when $\beta$ is high enough or $\delta$ is sufficiently small. When $\beta$ is small and $\delta$ is large, the first-period pricing effects of PP are ambiguous as the large (small) firm sets lower (higher) prices in CKM (2018) than herein. Thus, there is a small range of intermediate $(\delta, \beta)$ values where only the small firm benefits from the ability to tailor products to its returning customers.

The previous comparison has been made for exogenous locations. What happens under endogenous location choices? To get some clues, we compare our results to the ones obtained in section 4 of CKM (2018), which focuses on the special but empirically relevant case $\beta=0$ and $\delta=1$. The authors obtain asymmetric mirror location equilibria with $a^{*}=\frac{2 \sqrt{56029}-347}{621} \simeq 0.2$ and $b^{*}=0$ on the one hand, and $a^{*}=0$ and $b^{*}=\frac{2 \sqrt{56029}-347}{621}$ on the other hand. Differently, we obtain symmetric locations with $a^{*}=b^{*}=0.1$. Again, this symmetry exacerbates first-period price competition. With endogenous location, PP may still raise the small firm's profits (and industry profits as well), but it always lowers the big firm's profit (by entailing initial symmetric locations, PP eliminates the large firm's initial location advantage in CKM (2018)). ${ }^{35}$

### 6.2. Hypersegmentation Pays Off?

We now compare our results to Zhang (2011), who addresses group pricing-product choices when firms are only able to separate old from new customers. In this case, firms compete separately in uniform prices over two smaller market segments, without any informational advantage in the old turf. Herein, the old provider ends up being able to keep a captive market as it can make personalized product-price offers to its old customers (whereas its rival can offer only a standard product at a uniform price to this segment).

In both cases, we get symmetric equilibrium outcomes, with $z^{*}=\frac{1}{2}$. However, in the case of Zhang (2011), an equilibrium exists iff $\delta$ is small enough and/or the $\beta$ sufficiently high. ${ }^{36}$ The result of the comparison is simple: Whenever an equilibrium exists in both models, firms are hurt by using more accurate information to engage in personalization strategies. This is clearly in line with the results in the literature for the case of exogenous locations (CKM 2018), and it boils down the old tradeoff between competition for the market and competition in the market. As PP gives firms the prospect of extracting larger profits from future returning customers, firms aggressively compete for the market in the first period by engaging in aggressive first-period prices and locating closer to the center. Our comparison with Zhang (2011) shows that if firms had the possibility to coordinate their personalization strategies, they would choose to limit their customization by offering the same product to all repeated customers or at least a fewer number of products to some loyal customers (even if they had the ability to customize products for all old customers). However, this does not arise as a dominant strategy for the firms: if Firm $i$ takes as given that Firm $j$ only has a small number of varieties (e.g., one variety targeted to new customers and another one targeted to old customers as in Zhang (2011)), Firm $i$ actually profits from individual product personalization, leading to a
prisoner dilemma akin to the one identified in Thisse and Vives (1988).

## 7. Extensions

### 7.1. Untrackable Customers

In the previous sections, we focused on the most competitive equilibrium, which we called the Bertrand equilibrium. Nonetheless, as mentioned earlier, the indifference of each firm with respect to its (secondperiod) standard product's prices and locations yields a multiplicity of second-period equilibria. In this section, we are going to show that the existence of a very small (exogenous) fraction of untrackable customers may allow firms to select different second-period equilibria, yielding higher second-period profits.

From a technical point of view, the existence of active consumers (who can hide their identity, either ex ante or ex post, to get offers targeted to new customers) breaks the poaching firm's indifference. Accordingly, we can now uniquely determine the equilibrium poaching price(s). In this equilibrium, firms take into consideration that they may now be selling their standard products to (anonymous) consumers with a strong preference for their brand, which refrains them from engaging in aggressive poaching prices. This, in turn, alleviates the competitive pressure over returning customers, leading to higher second-period profits. This positive profit effect is akin to the one obtained in Chen et al. (2020) or Chen and Iyer (2002). However, when we look at the SPNE in our two-period game, it turns out that PP does not necessarily increases firms' total discounted profits. The following sections provide an in-depth analysis of the profit effects associated with consumers' identity management. We first look at ex ante identity management (where customers avoid being profiled by their old provider already in the first period) and then we address ex post identity management (where firms are able to track active customers but they may hide their identity when doing their second-period purchases).
7.1.1. Ex Ante Untrackable Customers. Suppose that there is an exogenous very small fraction $\mu$ of active consumers who avoid being profiled when they buy in the first period (this is what Chen et al. (2020) call ex ante identity management). These customers are perceived as new customers by both firms, which means that in the second period they will have to choose between firms' standard offers. For the sake of simplicity, we assume that the fraction of $\mu$ customers is uniformly distributed in the Hotelling line. We also consider that the two firms are initially located at the extremes of the Hotelling segment. Moreover, the timing of the game is similar to the baseline model, except
that we take first-period locations as given, assuming firms are located at the extremes of the Hotelling line.

The game is solved by backward induction. In the last stage, firms' best reply to the rival's poaching price remains the same as in the baseline model. However, product-price decisions targeted to the rival's turf are now affected by the existence of a group of active customers. On the one hand, firms know that they remain unable to sell their standard good to the rival's old (profiled) customers, who receive compelling PP offers from the old supplier. On the other hand, firms know that a fraction of new customers consists of their own old unprofiled customers, who are anonymously buying the standard product at a uniform price. We denote those uniform prices by $p_{1}^{N}$ and $p_{2}^{N}$ (where the superscript $N$ means that such uniform prices are the ones targeted to new customers, which will also be directed to unprofiled old customers). Similarly, the location of firms' second-period standard product is denoted by $a^{N}$ and $b^{N}$, which could differ from the initial location parameters $a$ and $b$ (although in equilibrium this will not be the case, because of our assumption that firms are initially located at the extremes).

An important difference with our baseline model (where nobody buys the standard product) is that here firms have a market for their (second-period) standard product: old anonymous consumers.

Lemma 2. When a fraction $\mu$ of active consumers avoids being profiled (ex ante identity management), in the sec-ond-period:
(i) firms' standard products are optimally located at the extremes (with $a^{N *}=b^{N *}=0$ ), being sold at the standard Hotelling equilibrium prices, $p_{1}^{N *}=p_{2}^{N *}=t$.
(ii) Each Firm $i$ offers a personalized product to each $x$-old profiled customer, selling it at a type-specific price $m_{i}(x)=$ $p_{j}^{N *}+t\left(x_{j}-x\right)^{2}, x_{1}=0, x_{2}=1 .{ }^{37}$
(iii) In the limit, when $\mu \rightarrow 0$, firms' second-period equilibrium profits are equal to

$$
\begin{equation*}
\pi_{1}(z)=t\left(z+\frac{1-(1-z)^{3}}{3}\right), \pi_{2}(z)=t\left((1-z)+\frac{\left(1-z^{3}\right)}{3}\right) \tag{11}
\end{equation*}
$$

The first part of Lemma 2 is the standard outcome of the Hotelling static game with quadratic travel costs and endogenous locations: a firm only cares about the effect of its (new) standard product price on unprofiled old customers because it anticipates its inability to sell to the rival's profiled old clients. In addition, it is no longer true that firms' second-period profits do not depend on firms' second-period location: $a^{N}$ and $b^{N}$ affect the profits obtained out of unidentified customers, with the usual maximal differentiation result
arising. The second part of Lemma 2 follows directly from Lemma 1 in Section 3. The third part of Lemma 2 shows that, when $z^{*}=\frac{1}{2}$ (which will be the case at the SPNE), second-period profits are, because of PP, unambiguously greater than at the static equilibrium. They are equal to $\frac{19 t}{24}$, above the conventional Hotelling profits $\frac{t}{2}$. This runs opposite to what Zhang (2011, p. 170) called "the first peril of behavior-based discrimination." Because we are looking at the secondperiod subgame, this result is particularly interesting to understand why firms may cope with consumers attempts to remain unprofiled (namely in mature markets, where firms already have their own customer installed base). However, in emergent markets (where firms are still building their customer base), the situation may be different: Proposition 3 shows that the profit advantage vanishes in the SPNE of the twoperiod game: For $\mu \rightarrow 0$, the individual firms' overall profits under ex ante identity management ( $\Pi_{i}^{I M *}$ ) coincide with the corresponding counterpart in the baseline model (for exogenous locations at the extremes of the Hotelling line), so that all overall profit comparisons in the previous sections remain valid under the present specification with ex ante identity management.
Proposition 3. When $\mu \rightarrow 0, p_{i}^{*} \rightarrow t\left(1-\beta-\frac{5}{4} \delta\right)$ and $\Pi_{i}^{I M *} \rightarrow \frac{t}{6}(3(1-\beta)+\delta)=\Pi_{i}^{*}(0,0)$.

Proposition 3 shows that the extra second-periods profits are exactly compensated by the reduction in first-period profits. Equilibrium first-period prices are indeed $p_{1}^{*}=p_{2}^{*}=t\left(1-\beta-\frac{5}{4} \delta\right)$, smaller than the static equilibrium price $t$ and also strictly smaller than the ones, $p^{*}(0,0)$, in our baseline model. From (5), $p^{*}(0,0)-p_{1}^{*}=t \delta$.
7.1.2. Ex Post Untrackable Customers. Suppose now a very small fraction $\mu$ of consumers are able, if in their interest, to hide their identity from their old supplier at the time they return to buy. Differently from the previous case, these customers receive the old supplier's personalized offers. However, if those are not enticing enough, they may prefer to create a new account (e.g., registering a different email address) to get more favorable conditions than the initial personalized offer. For the sake of simplicity, we assume again that firms are initially located at the extremes of the Hotelling line (in this case, all the results would go through if we allow for endogenous initial locations). The timing of the game also remains unchanged with respect to the baseline model. The novelty lies here in old customers' ability to pretend to be a new customer (in the last stage).

Lemma 3. In the second-period equilibrium with ex post identity management, whatever the exogenous fraction of untrackable customers $\mu>0$, there is an equilibrium where
(i) Each Firm i chooses a standard product price $P_{i} \geq U$;
(ii) Each firm sells a personalized product to its old customers, with $v_{i}(x)=U, \forall x \in A_{i}$; and
(iii) The second-period equilibrium profits equal $\pi_{1}(z)=$ $U z$ and $\pi_{2}(z)=U(1-z)$.

The limit equilibrium when $\mu \rightarrow 0$ is the one described in Lemma 3. Comparing with the baseline model, ex post identity management allows firms to sustain higher second-period profits. In this case, they actually get the highest possible second-period profits: all customers are paying the reservation price for a product that exactly matches their tastes (replicating the perfect discrimination outcome). Consumers' secondperiod surpluses are zero and, in the first period, they choose their provider only as a function of first-period prices and locations. This means that first-period demands are then identical to ones in the usual Hotelling model, that is, $z=\frac{1}{2}+\frac{p_{2}-p_{1}}{2 t}$. First-period equilibrium prices $p_{1}$ and $p_{2}$ are then selected to maximize firms' accumulated profits, given by $\Pi_{1}=\left(p_{1}+\delta U\right) z$ and $\Pi_{2}=\left(p_{2}+\delta U\right)(1-z)$, so that $p_{i}^{*}=t-\delta U$.

Proposition 4. When consumers are able to engage in ex post identity management, firms' two-period profits are equal to $\Pi_{i}^{*}=t / 2$.

Although PP allows firms to charge all customers with their reservation prices (in the second period), fiercer first-period price competition wipes out the second-period gains, resulting in smaller profits than in the baseline model whenever $\beta>\delta / 3$. It is also worth noting that the equilibrium profits in Proposition 4 are obtained when first-period negative prices are allowed (if they were not allowed, we would get $\Pi_{i}^{*}=\delta \frac{U}{2}$, only when $\left.t<\delta U\right)$. Actually, under Assumption 1 and $\delta \leq 1$, negative prices always occur in equilibrium (if allowed). This means that nonnegativity price constraints may partially prevent the overall profit negative effect (by limiting profit losses in the first period).

### 7.2. First-Period Tacit Collusion Equilibria Under Second-Period Relocation Threats

We close Section 7 by investigating the possible consequences of the fact that in the second period firms are indifferent between all the possible relocations of their standard product offer to the rival's turf. This indifference implies that relocation is potentially a credible threat to avoid fierce first-period competition. The most severe possible punishment would be the selection of a standard product's specification which minimizes the rival's second period profits. Given Equations (2),
this corresponds to relocating just in the middle of the rival's turf, that is, $b^{N}=1-\frac{z}{2}$ and $a^{N}=\frac{1+z}{2}$, resulting into second-period profits $\pi_{1}(z)=\frac{t}{12} z^{3}$ and $\pi_{2}(z)=\frac{t}{12}(1-z)^{3}$, respectively.

Consider for the sake of simplicity, the case of exogenous initial locations at the extremes. We are searching for a tacit collusive SPNE such that both firms select identical first-period prices $p_{1}^{C}=p_{2}^{C}=p^{C}$. If both firms are observed to stick initially to these prices or to higher ones (we assume that a firm does not want to punish a deviation which raises its own profits), then each firm commits not to upgrade its standard poaching product (so that the second-period equilibrium is the one described in Proposition 1 with $a^{N}=$ $b^{N}=0$ ). If one or both of them deviate toward a price lower than $p^{C}$, then they both choose to relocate their standard product so that $b^{N}=\frac{2-z}{2}$ and $a^{N}=\frac{1+z}{2}$. The remaining of the game corresponds to the Bertrand equilibrium with profits $\pi_{1}(z)=\frac{t}{12} z^{3}$ and $\pi_{2}(z)=\frac{t}{12}(1$ $-z)^{3}$. When the (forward-looking) consumers observe a downward deviation from the price $p^{C}$ by one or both firms, they rationally expect the standard products to be upgraded in period 2 in the way described previously, and they also anticipate the corresponding equilibrium personalized prices $v_{1}(x)=t\left(x-\frac{z}{2}\right)^{2}$ and $v_{2}(x)=t\left(x-\frac{1+z}{2}\right)^{2}$. Then, the first-period demand function, conditional on an observed downward deviation, becomes

$$
\begin{equation*}
z^{\text {Dev }}\left(p_{1}, p_{2}, \beta\right)=\frac{1}{2}+\frac{2\left(p_{2}-p_{1}\right)}{t(4+\beta)} \tag{12}
\end{equation*}
$$

Hence, the two-period profits of Firm $i(i=1,2)$ at the candidate collusive equilibrium are simply

$$
\Pi^{C}\left(p^{C}\right)=\frac{1}{2} p^{C}+\delta \frac{7 t}{24}
$$

The profits $\Pi^{C}\left(p^{C}\right)$ at the candidate collusive equilibrium are greater than the profits $\frac{t}{2}(1+\delta)$ from the repetition of the static equilibrium iff $p^{C} \geq t\left(1+\delta \frac{5}{12}\right)$.

To investigate firms' ability to enforce this tacit collusive equilibria, suppose that one firm (say Firm 1 wlog) contemplates to deviate from the collusive equilibrium. The profit it can obtain from such a deviation is

$$
\begin{equation*}
\Pi^{D e v}\left(p^{C}\right)=\max _{p_{1}} p_{1} z^{D e v}\left(p_{1}, p^{C}, \beta\right)+\delta \frac{t}{12} z^{D e v}\left(p_{1}, p^{C}, \beta\right)^{3} \tag{13}
\end{equation*}
$$

Any couple $\left(p^{C}, p^{C}\right)$ of first-period prices corresponds to a SPNE provided that $\Pi^{\text {Dev }}\left(p^{C}\right) \leq \Pi^{C}\left(p^{C}\right)$.

Proposition 5. For all $(\beta, \delta) \in[0,1) \times[0,1]$, there exist collusive SPNE which yield two-period profits $\Pi^{C}\left(p^{C}\right)$ at least as great as the profits from the repetition of the static equilibrium, with $p^{C} \geq t\left(1+\delta \frac{5}{12}\right)$. They are sustained by the credible threats of second-period relocation strategies $b^{N}=\frac{1-z}{2}$ and $a^{N}=\frac{1+z}{2}$, upon the observation of a firstperiod downward deviation from $\left(p^{C}, p^{C}\right)$.

From Proposition 5, we have the remarkable result that PP to old customers may lead to greater profits than the simple repetition of the static equilibrium (and accordingly to greater profits than in conventional BBPD models without product personalization). The following example shows that relocation threats may sustain equilibria that are a lot more profitable than the repetition of the static equilibrium.

Example 1. Let $t=1, \beta=0.5$ and $\delta=0.9$. From Equation (14) in the online appendix, the optimal deviation yields profits equal to

$$
10 p^{C}-\frac{555}{4}+\left(\frac{285}{4}-\frac{10}{3} p^{C}\right) \sqrt{\frac{19}{5}-\frac{8}{45} p^{C}}
$$

For $p^{C}$ to correspond to a SPNE, it must be that the deviation profits are not greater than the candidate equilibrium profits $\frac{1}{2} p^{C}+\frac{6.3}{24}$. The difference between the latter and the former is pictured in Figure 4. It is positive for all positive $p^{C} \leq \bar{p} \simeq 2.5$. This corresponds to the more profitable equilibrium that can be sustained by the relocation threats. Profits at this equilibrium are close to 1.5 , which is substantially greater than the profits corresponding to the repetition of the static equilibrium (equal to 0.95 ).

This example provides an insightful illustration of how PP may lead to more profitable equilibria (both
in comparison with the repetition of standard static competition and in comparison with conventional BBPD). The intuition behind this result is the following: contrary to models with price personalization only, is that PP opens the door to second-period punishment strategies through product relocations in the rival's turf (which considerably soften first-period competition). Interestingly, this happens when the second-period equilibrium is the most competitive, that is, the Bertrand one. In the context of our model, this result would no longer hold if we allowed for the existence of a (very small) fraction of active customers. In the latter case, either (i) firms are no more indifferent with respect to the specification of their second-period standard product (in the case of ex ante identity management), or (ii) their rival's profits do not depend on their relocation choice (in the case of the ex post identity management).

### 7.3. Earlier PP Investment Decisions

In our baseline model, we supposed that a firm's decision to invest or not in PP takes place in the second period of the game, after it has observed its rival's decisions directed to the turf of new customers; that is, each firm chooses to engage or not in product personalization when it already knows the second-period location and price of the rival's standard product. Under this assumption, PP has only the direct positive effects of (i) retaining all old customers and (ii) raising the margin that each firm makes on them so that it is always a dominant strategy. We discuss here the two alternative scenarios where PP investment decisions are taken earlier, either upfront at the very beginning of period 1 or at the beginning of period 2. In those scenarios, if such decisions were irreversible, firms would either personalize their product to all old customers (because this is a

Figure 4. Collusive Profits-Deviation Profits (w.r.t. $p^{C}$ )

dominant strategy when the time comes to implement it), or they would not personalize products at all (which would be the case if the firm had not invested in the relevant technology in due time). In both cases, these supposedly irreversible and observable investment decisions would have strategic effects besides the direct one we address in this paper.

Consider first the case when decisions to engage in product personalization are made in the second period but before firms knowing the rival's standard product-price offer (targeted to new customers). Here we have that a firm with a small installed customer base will indeed set a zero poaching price if it knows that the big firm is going to personalize and a positive one in the reverse case. Accordingly, not investing in PP may be a way for a big firm to soften secondperiod price competition (by eliciting a higher standard product price from its rival). If its market share is great enough, this strategic effect may outweigh the direct effect of PP. A small firm on the contrary would always select PP. When the two firms have identical installed customer bases, the strategic effect vanishes. We accordingly conjecture that, at least for some parameters, we would continue to get the same symmetric first-period equilibrium under this alternative timing structure (in that case, PP continues to have only a positive direct effect and it is a dominant strategy).

Consider now the alternative scenario of a PP investment decision being made upfront at the beginning of the first period. In this case, another strategic negative effect adds to the one mentioned previously. This effect could possibly take place through consumers' expectations impacting the first-period demand function. More precisely, first-period price elasticity is usually smaller when PP is absent than when it is present. Accordingly, not investing in PP, as an upfront observable and irreversible commitment, could soften first-period price competition. ${ }^{38}$ Despite this interesting theoretical result, it is important to bear in mind that often this initial commitment of not investing in PP is most likely not irreversible because firms may have the opportunity to delay their investments to implement mass customization supply chain ${ }^{39}$ (and our results reveal that it would actually be in their interest to do so when the time comes, provided it is technically feasible).

In summary, we argue that our results may be robust to alternative settings in which PP decisions are taken at the beginning of the second period (before knowing the rival's second-period standard productprice offer). Differently, when the decision to invest in PP needs to be taken upfront in the first period, firms may prefer not to invest on PP. However, they would need to find credible commitment devices to convince the rival and the customers that they would not engage in PP later in the game.

## 8. Conclusion

This paper investigates optimal pricing and product strategies when firms are able to make personalized (product and price) offers to their loyal customers. In our baseline model, firms are able to trace the preferences of all returning customers, remaining uninformed about the preferences of the rival's customers. In line with recent business practices allowed by new generation digital technologies, we find that firms will always make use of such privileged information on old customers to sell them a personalized product at a targeted price. Our results highlight that PP may allow firms to build captive markets for the secondperiod, increasing the effectiveness of their customer retention strategies.

Indeed, we find that in equilibrium there is no consumer poaching, so that all customers end up getting a product that perfectly matches their ideal specification (different consumers get different surpluses depending on how far they are from the rival brand, which affects the degree of price pressure that the latter may exert on the old supplier). This result is in line with recent practitioners' view that PP is an effective strategy to promote consumers' retention and increase brand loyalty.

When looking at the SPNE of the two-period game corresponding to the Bertrand equilibrium, we show that, absent any product relocation strategy, there exists a symmetric PSNE where firms compete aggressively in first-period prices in an attempt to build a wider captive market for the second period. The competitive pressure is exacerbated if firms are given the possibility to choose their initial product location. In that case, maximal differentiation only arises when agents put low weight in future profits. If firms value enough their future payoffs, they both choose to locate closer to the center, intensifying first-period price competition. The same result arises if consumers attach a high-value to future payoffs (as first-period demands become more elastic).

When we extend the baseline model to introduce the possibility of using aggressive second-period product relocation strategies to sustain higher profit equilibria, we find that first-period price competition becomes much softer (as firms fear the rival's retaliation in the subsequent period). The introduction of relocation punishment strategies even allow firms to get greater overall profits than the ones obtained in the repetition of the static equilibrium (which are, themselves, greater than the ones obtained in the conventional BBPD setting).

A natural question arising from this study is whether managers should or not engage in PP strategies. Our investigation reveals that the answer to this question depends on the specific industry features. When firms make use of product relocation threats to relax first-period price competition, we find that there
is a wide scope for profitable PP. When this tacit collusive possibility is ruled out (as in the baseline model), the following results are obtained: first, PP profitability heavily depends on whether we are looking to a setup where firms already have an inherited customer base (i.e., a mature market) or a setting where firms do not have yet an installed consumer base (e.g., emergent markets). In the first case, we conclude that PP will boost (second-period) profits for both firms, provided that the size of firms' installed customer base is not too different. When we look at emergent markets (absent product relocation punishment strategies), PP may be profit detrimental. This boils down to the usual tradeoff between competition for the market versus competition in the market: the fact that PP allows both firms to keep all their old customers captive (getting higher second-period profits) increases first-period price competition. If the specification of firms' initial product is exogenous, PP boosts profits for both firms if consumers are naive enough and firms are sufficiently patient. When firms are able to endogenously choose their initial location, first-period competition erodes all possible future profit gains. Hence, when managers are not implementing product relocation threatening strategies because of legal, market, or credibility factors, they should engage in PP when (i) they already have an existing base of old customers and firms' initial market sizes are not too asymmetric, or (ii) they are still building their customer base but the product specification is stabilized, consumers discount heavily future payoffs, and firms value them highly enough.

Finally, we also investigated how consumers' identity concealing actions (to hide their preferences from the old supplier, either before or after doing their second-period purchases) may affect the profit effects of PP. We find that consumers' identity management always relaxes competitive pressure in the second period, increasing the corresponding profits. However, it also intensifies first-period price competition.

The results obtained in this paper open the door to future research avenues on profit-enhancing personalization strategies. Considering that our analysis suggests firms would be better off if they could coordinate to lighter forms of market segmentation instead of individual product-price offers, it is worthwhile to investigate the optimal amount of information each firm would choose to collect about its customers. This would then open the door to relevant investigations regarding firms' optimal degree of product customization (as firms can only make personalized offers to consumers whose preferences are disclosed). Finally, an important message of this paper is that costless PP completely eliminates poaching. This result remains valid if we consider a model with linear personalization and transportation costs, as long as the ratio between personalization and transportation
costs is not too large. Accordingly, further (empirical) research is needed to identify markets where this ratio is indeed small.

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## Endnotes

${ }^{1}$ More details available on https://www.trunkclub.com/. The Nordstrom case is presented here only for illustrative purposes as many other retailers also offer this type of service.
${ }^{2}$ Personalization has also become quite trendy in high-end markets such as luxury fashion or the automotive sector (e.g., Tesla's personalized dashboard).
${ }^{3}$ This is in sharp contrast with Fudenberg and Tirole (2000) and Zhang (2011), who get two-sided equilibrium poaching, but also with CKM (2018), who obtain one-way poaching. With positive linear personalization (and transportation) costs, the no poaching result remains valid provided firms' first-period market shares are not too different. The proof is available from the authors upon request.
${ }^{4}$ For example, Lindecrantz et al. (2020) argues that "Personalization at scale $\ldots$ often delivers a 1 to 2 percent lift in total sales for grocery companies and an even higher lift for other retailers, typically by driving up loyalty and share-of-wallet among already-loyal customers." Information retrieved online: https://www.mckinsey.com/industries/ retail/our-insights/personalizing-the-customer-experience-drivingdifferent iation-in-retail. Along the same lines, many other opinion articles and field experts point toward the enormous potential of customization to increase brand loyalty and customer satisfaction (e.g., https://freshdesk.com/general/product-customization-for-customer-satisfaction-blog/ or https://www.forbes.com/sites/forbescommunications council/2020/02/13/to-build-brand-loyalty-in-the-digital-age-personalize-customer-experiences/?sh=1564883b13ec.). Although our model certainly is a very stylized representation of complex real-world PP strtategies, it allows us to identify one theoretically plausible justification behind the argument that product personalization may be quite effective in promoting consumers' loyalty.
${ }^{5}$ If a firm's old turf is too narrow, the elimination of poaching results in large profit losses because of the absence of sales in the rival's large turf. If a firm's old turf is too large, PP elicits a tougher rival's poaching price, which also hurts profits.
${ }^{6}$ On switching costs, see, for example, the seminal paper by Klemperer (1987).
${ }^{7}$ This result holds both in relation to the standard product setting with personalized prices of CKM (2018) and in relation to the thirddegree product/price discrimination setting of Zhang (2011).
${ }^{8}$ This result is in sharp constrast with the one obtained by Laussel et al. (2020a) in an infinite-horizon monopoly. Although in their case, the monopolist is hurt by using coarse information to implement price BBD, herein, duopolists are actually better off when they use coarse than full information on customers' preferences.
${ }^{9}$ We gratefully thank the anonymous associate editor for drawing our attention to these equilibria.
${ }^{10}$ This possibility follows from the fact that, in our baseline model, the firms' own second-period profits do not depend on the (re)location of their own product. This makes firms indifferent between all possible locations. However, firms' location choice affects their rival's second-period profits, which means that firms can credibly threaten to punish their rival if they observe that it has deviated from a first-period tacitly agreed equilibrium price.
${ }^{11}$ The famous Mussa and Rosen (1978) model is already about price and quality customization. Laussel et al. (2020b) extend their analysis to a dynamic setting.
${ }^{12}$ In this framework, both symmetric and asymmetric equilibria may arise, with the two firms investing in product personalization in symmetric equilibria and only one of them doing it in the case of asymmetric equilibria.
${ }^{13}$ Assumption 1 ensures that at the static Hotelling equilibrium (with uniform products), even the farthest consumer from a firm would prefer to buy the more distant product rather than not to buy.
${ }^{14}$ The no leakage assumption implies that each old customer only gets information on the deal, remaining uninformed about the offers targeted to new customers and the offers targeted to other old customers. In Section 7, we shall consider the case where a small fraction of old consumers engage in identity management to get informed about offers targeted to new customers.
${ }^{15}$ The sequentiality of decisions between the two market segments (with uniform price to new customers preceding personalized prices) is rather usual in the literature (Thisse and Vives 1988, Choe et al. 2018, Chen et al. 2020). It is generally considered that standard product prices are chosen less frequently and at a higher management level than personalized offers (Garella et al. 2021).
${ }^{16}$ As we shall comment in Section 7, this implies that, when they decide whether to personalize their products to their old customers, the firms can account only for the direct effects of PP and not for the strategic ones.
${ }^{17}$ An alternative modeling framework with linear transportation costs and linear personalization costs is available from the authors upon request. By setting nil marginal personalization costs, we are ruling out asymmetric equilibria, whose economic properties are very close to the ones obtained in CKM (2018).
${ }^{18}$ As argued by Rhee and Thomadsen (2016), the assumption of discount factors above one may be useful to address cases in which the two-period dynamics represent a short first period of time and a subsequent long period of time (whose outcomes would be condensed in the second-period payoff).
${ }^{19}$ See the proof of lemma 1, page 5682 , lines 5 and 6.
${ }^{20}$ They consider the limit of a perturbed game where the two firms compete on uniform prices over a small common contestable segment belonging to the poached firm's turf. More precisely, they focus on the limit case arising when the length of the contestable segment tends toward zero.
${ }^{21}$ Ex ante identity management arises when consumers are able to hide their identity when buying for the first time: the firm is unable to recognized them when they return (and therefore they do not receive any personalized offer, opting between the two standard products). Ex post identity management takes place when consumers are able to hide their identity when doing their second purchase: they receive the personalized offer and they can choose between it and the two standard offers. As noticed by Chen et al. (2020, p. 4022): "When some consumers are active, a firm's poaching price is uniquely determined in all cases."
${ }^{22}$ To understand why PP eliminates poaching herein but not in Zhang (2011), note that, in this case, firms compete separately in
uniform prices over two smaller market segments, without any informational advantage on the old turf (both firms know who are each other's new and old customers). On the contrary, herein firms collect accurate information about the preferences of old consumers (whereas the rival only clusters them all as a group of potential new customers). Hence, the old provider is able to make PP offers, whereas its rival can offer only a standard product at a uniform price.
${ }^{23}$ We are grateful to an anonymous referee to suggesting us this link with the switching cost literature.
${ }^{24}$ This corresponds to the limit equilibrium of a game where firms simultaneously choose the price of the standard good for a fraction of consumers and the personalized product-price offers for a fraction $1-q$ of consumers (the limit equilibrium is obtained by taking $q \rightarrow 0$ ).
${ }^{25}$ We thank the associate editor for drawing our attention to the possibility of this type of equilibrium in a dynamic two-period model.
${ }^{26}$ Although first-period market shares are endogenous to the model (when the full game is taken into consideration), it is still interesting to study how first-period market shares affect secondperiod equilibrium profits, allowing us to shed some light on competitive patterns in mature markets (which are better described by our second-period subgame because firms already have an installed customer base). Herein we obtain, $\frac{\partial \pi_{1}(z)}{\partial z}=t(b+z-1)^{2}>0$ and $\frac{\partial \pi_{2}(z)}{\partial z}=-t(a-z)^{2}<0$.
${ }^{27}$ In our notation, they would be equal to $\frac{3}{4} t\left(z^{3}+(1-z)^{3}\right)$, so that individual second-period profits reach their maximum when one of the firms monopolizes the market in the first period.
${ }^{28}$ We are very grateful to an anonymous reviewer for drawing our attention to this point and for highlighting the similarities between the first-period demand system in this paper and the one in Chen (1997).
${ }^{29}$ To understand the intuition for this smaller sensitivity in these models, see Zhang (2011), p. 176, last section.
${ }^{30}$ The location game actually has three equilibrium candidates. However, two of them do not satisfy the SOC (either for Firm 1 or for Firm 2), leaving us with a unique equilibrium for the location game.
${ }^{31}$ Of course when the firms are not constrained to locate within the Hotelling line, we always obtain $a^{*}=b^{*}=\frac{1}{2}-\frac{1-\beta}{3}\left(\frac{1}{4(1-\beta)+\delta}+\frac{2}{1-\beta+\delta}\right)$, with firms locating outside the line when $\delta$ and $\beta$ are low.
${ }^{32}$ This can be checked easily using the RegionPlot function of Mathematica over the area $(\beta, \delta) \in[0,1]^{2}$.
${ }^{33}$ Equilibrium profits decrease with $\delta$ if $\delta \in\left[\frac{\sqrt{17}-3}{2}(1-\beta),(2 \sqrt{3}-1)\right.$ $(1-\beta)]$ and they increase with $\delta$ if $\delta \geq(2 \sqrt{3}-1)(1-\beta)$ or $\delta \leq$ $\frac{\sqrt{17}-3}{2}(1-\beta)$ (last condition defines the maximal differentiation area). ${ }^{34}$ In CKM (2018), the equilibrium profits of the small and the big firm are, respectively, $\Pi_{S}^{*}=t \frac{36(2-\beta)^{3}+24 \delta(2-\beta)^{2}+(2-\beta) \delta^{2}+\delta^{3}}{4(12-6 \beta+\delta)^{2}}$ and $\Pi_{B}^{*}=$ $t \frac{36(2-\beta)^{3}+36 \delta(2-\beta)^{2}+3(2-\beta) \delta^{2}-2 \delta^{3}}{4(12-6 \beta+\delta)^{2}}$, where $S$ stands for the small firm and $B$ for the big one.
${ }^{35}$ Equilibrium profits are here equal to $\frac{13}{24} t(\simeq 0.54 t)$. They must be compared with the corresponding counterparts in CKM (2018), where profits are $\simeq 0.59892 t$ and $0.41153 t$, for the big and the small firm, respectively, (to check this, introduce equilibrium locations in the profit equations in their proof of lemma 8).
${ }^{36}$ We compare our equilibrium profits to the counterparts in Zhang (2011) as they appear from the main text and the online appendix: $\frac{1}{2}\left(\frac{t\left(24+18 \delta+25 \beta+\sqrt{324 \delta^{2}-180 \delta(8+5 \beta)+(24+25 \beta)^{2}}\right.}{32}\right)+\delta \frac{3 t}{16}$. This expression shows
profits when firms are not constrained to locate inside the Hotelling line. For the sake of comparison, we also consider profits in our model in the same case, corresponding to the second line of (9). It is worth adding that Zhang's equilibrium exists iff $\delta \leq \frac{1}{810}(1789+$ $1125 \beta-29 \sqrt{2881+2250 \beta}$. Our first-period equilibrium prices for fixed locations in (5) are always lower than prices in (A-3) of Zhang's online appendix. Moreover, comparing the values of $l^{*}$ from (7) with (A-4) of Zhang's online appendix, we find firms choose closer product specifications herin than in the setting of Zhang.
${ }^{37}$ From Assumption 1, $m_{i}(x)<U$.
${ }^{38}$ This would necessarily require from the consumers a deep understanding of the second period equilibrium effects.
${ }^{39}$ The second period, corresponding to a mature market, may actually be quite long, and it is not unrealistic to think that the firms may always at some point invest in PP if they have not done it previously.

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