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# Optimal shock-based maintenance policy for a system in a dynamic environment

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## Abstract

In this article, we consider a single-unit system that operates in a dynamic environment and is subject to shocks. Shocks only affect the system (nonlethal shock) and do not cause sudden failure, and arrive according to a counting process. Both homogeneous and nonhomogeneous Poisson processes are considered for shocks arrival modeling. In order to model the dynamic environment and consider shock effects, a multiplicative failure rate model is proposed. Both corrective maintenance and shock-based preventive maintenance are considered, and two policies are proposed. In the first proposed policy, the system is replaced by a new one upon a failure or based on the predetermined number of shocks, whichever comes first, while the second proposed policy extends the first one by considering an imperfect preventive repair at each inspection time. The inspection times are periodic and the interinspection interval is considered as a decision variable. The proposed policies are optimized according to long-run cost rate criteria. Numerical examples illustrate the applicability and efficiency of the proposed policies.

## KEYWORDS

corrective maintenance, imperfect maintenance, long-run cost rate, shock-based maintenance, tampered failure rate model

## 1 | INTRODUCTION

Nowadays, complex systems are extensively utilized on a daily basis. A serious accident of a complex system causes heavy damages and a social sense of instability. This fact reveals the importance of system maintenance more than ever, however the main point is that the maintenance activities are costly and companies should strive to find new and better ways to control the cost of maintenance activities in order to survive in competitive markets. For this purpose, optimal maintenance policies that minimize costs are of great interest and have been the subject of numerous research studies in the past decades in the reliability literature.

Generally, maintenance actions are classified into two categories: corrective maintenance (CM) where maintenance activities are carried out when the system is failed and preventive maintenance (PM) where maintenance activities are performed when the system is operating. In this article, a bivariate PM model is presented. It is worth noting that any bivariate PM modeling is better or at least not worse than the corresponding univariate one.<sup>1</sup> In this article, a shock-based preventive maintenance is proposed. Since the shock-based preventive maintenance policy is based on monitoring, it

takes advantage of real-time information, such as environmental conditions or their impacts on the system aging. Nowadays, due to the development of sensor technology, it is often possible to gather information on the number of shocks and their magnitudes over time. For example, piezoelectric (PE) accelerometers with integral electronics are vibration sensors, designed for the measurement of dynamic vibration signals at frequencies ranging from very low (near-dc) to 10 kHz.

According to Pham and Wang,<sup>2</sup> maintenance actions can be classified based on their impacts as follows: (a) perfect maintenance which restores the system to as good as new (AGAN) state (b) minimal maintenance which restores the system to its state like before failure (as bad as old or ABAO) (c) imperfect maintenance which restores the system to a level between AGAN and ABAO (d) worse repair which deteriorates the system (e) worst maintenance which undeliberately makes the system either fail or break down. In this article, both perfect and imperfect repairs are considered.

Since external environmental sources influence the system and cause system deterioration and aging, the failure rate process of a system depends on external environment conditions besides its intrinsic characteristics.<sup>3</sup> These conditions are modeled through covariates. To consider the effect of covariates on the failure rate in reliability analysis, two popular models have been considered, including Cox (multiplicative) model<sup>4</sup> and additive model.<sup>5,6</sup> In the additive model, covariates are modeled as additive risks while in the multiplicative model, the covariates are assumed to act multiplicatively on the baseline failure rate. Modeling dynamic environment using additive failure rate process in the maintenance optimization context was first introduced by Cha and Mi<sup>7</sup> and then was extended afterward. For instance, Cha and Finkelstein modeled the effect of external shocks on organisms by an additive mortality rate model based on Strehler and Mildvan idea.<sup>8,9</sup> A bivariate model involving preventive maintenance and random environment effect was considered by Cha et al.<sup>10,11</sup> Cha and Finkelstein<sup>12,13</sup> considered operating systems undergoing random shocks due to environmental effects and presented a maintenance policy considering the environment and quality measures based on the additive failure rate process. Gao<sup>14</sup> presented an optimal sequential PM policy for a repairable system with a monotone increasing failure intensity function. Alberti and Cavalcante<sup>15</sup> used an additive failure rate model to present a two-scale maintenance policy for protection systems subjects to shocks. Moreover, various failure rate models concerning Cox proportional hazard and similar Cox proportional hazard models based on multiplicative failure rate model were established in the literature of maintenance optimization to model dynamic environment effects (see Reference 16,17 and the references therein). Chen et al.<sup>18</sup> planned predictive condition-based maintenance as a time-between-failure prediction model using the Cox proportional hazard model by taking advantage of deep learning and reliability analysis. They presented a model to overcome restrictions such as data sparsity and data censoring. Costa et al.<sup>19</sup> modeled maintenance of train wheel wear by contemplating reliability curves derived from the Cox proportional hazard model. Cha and Finkelstein<sup>20</sup> used stochastic failure rate model to measure the operational quality of k-out-of-n systems.

Several models were presented to describe the dynamic environment effects, such as shock effects in the maintenance context. For example, the generalized Polya process was considered in a maintenance context to model systems subject to shocks by Cha and Finkelstein.<sup>21</sup> The random environment was modeled using Poisson process, considering preventive maintenance and double effect of shocks by Cha et al.<sup>22</sup> Optimal maintenance policy for systems in a random environment considering variability of shock rate was presented by Levitin and Finkelstein.<sup>23</sup> This article presents a new multiplicative-based model which considers environmental effects on the failure rate directly and multiplicatively that has not been addressed so far. For more details about the failure rate modeling in the reliability context, one is referred to Finkelstein.<sup>24</sup>

In this article, an optimal shock-based maintenance policy is presented for a single-unit system operating in a dynamic environment. It is assumed that the system is subjected to shocks and shocks influence the system failure rate directly, through a multiplicative failure rate process. Two maintenance policies with periodic inspections considering corrective replacement and shock-based preventive replacement are considered. In the first proposed policy, a system is replaced by a new one (AGAN) depend on failure or on the predetermined number of shocks, whichever comes first while the second proposed policy is a generalized case of the first one by considering an imperfect preventive repair at each inspection time. The inspection times are periodic, and inter-inspection interval is considered as a decision variable. It is assumed that at each inspection time, the number of occurred shocks is observed. The decision variables of proposed policies are optimized according to long-run cost rate criteria.

The main contributions of this article to the current literature lie in the following aspects.

- A new failure model is proposed based on the multiplicative failure rate process, which considers impacts of shocks in the failure rate model multiplicatively. This model is a generalization of two models, the frailty and tampered failure rate (TFR) model.

- The shock-based preventive maintenance and corrective maintenance policies with/without considering imperfect preventive repairs are considered, and the optimal maintenance policy is investigated based on the multiplicative failure rate model.
- A bivariate PM model with two decision parameters: shock number and inspection interval is presented.

The remainder of this article is organized as follows. In Section 2, we describe the model and assumptions. The reliability function is touched upon in Section 3. In Section 4, two maintenance policies are presented, and optimal policies are surveyed. A numerical example as well as a sensitivity analysis are illustrated in Section 5. Eventually, article conclusions and perspectives are given in Section 6.

## 2 | MODEL DESCRIPTION AND ASSUMPTIONS

Consider a single-unit system with the failure rate  $\lambda(t)$  which is a deterministic function in a static environment. We suppose that the system is subject to nonlethal random shocks, which arrive independently based on the counting process  $N(t)$  over the interval  $(0, t]$  and the magnitude of each shock  $W$  is a random variable and follows the distribution function  $F$ . We assume that shocks increase the failure rate. Let  $\{\lambda_t, t > 0\}$  denote the failure rate process of the system in such a dynamic environment. Cha and Mi<sup>7</sup> and Cha and Finkelstein<sup>8</sup> proposed an additive failure rate process based on the Aalen's additive hazard model as follows:

$$\lambda_t = \lambda(t) + \eta N(t), \quad (1)$$

where  $\eta$  is a deterministic jump that occurred on each shock arrival according to the point process  $\{N(t), t \geq 0\}$ . Thus, the damage incurred by the system from a shock is a jump in the corresponding failure rate. Moreover, model (1) has been generalized to the following model by Cha and Finkelstein:<sup>12</sup>

$$\lambda_t = \lambda(t) + \eta \sum_{i=0}^{N(t)} W_i, \quad (2)$$

where the damages incurred by the system from different shocks were considered as non-negative independent and identically distributed random variables  $W_i$  instead of constant jump  $\eta$ .

Qiu and Cui<sup>25</sup> presented a general case of the above model to evaluate the reliability based on a dependent two-stage failure process with competing risks and Qiu et al.<sup>26</sup> modeled the joint impacts of external shocks and preventive repair simultaneously using this model to present a maintenance plan schedule.

In this article, we propose an alternative model, a multiplicative failure rate process, based on the Cox proportional hazard model as follows:

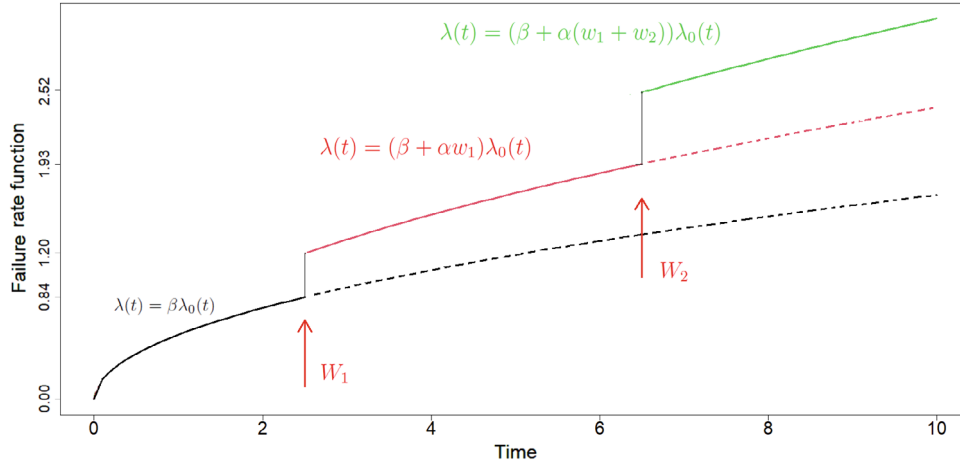
$$\lambda_t = \lambda(t) \left( \alpha \sum_{i=0}^{N(t)} W_i + \beta \right), \quad (3)$$

where  $\alpha$  and  $\beta$  are fixed and  $\{W_i, i \geq 0\}$  represents the magnitude of nonlethal shocks which is a sequence of the non-negative identically and independently distributed random variables with the common cdf  $F$ , pdf  $f$ , the moment generating function  $M$  (if it exists), and  $W_0 = 0$ . Moreover, the number of shocks is a point stochastic process  $\{N(t), t \geq 0\}$ . In this article, both homogeneous and non-homogeneous Poisson processes are considered with rates  $\nu$  and  $\nu(t)$ , respectively. It is assumed that the number of shocks is independent of their magnitudes.<sup>27</sup>

The model (3) could be considered as a generalization of frailty model when the coefficient of baseline failure rate  $(\alpha \sum_{i=0}^{N(t)} W_i + \beta)$  is a stochastic process. The frailty model is often used to study the phenomena of unobservable individual heterogeneity in reliability and biomedical researches. For more details the reader is addressed to Vaupel, Manton and Stallard,<sup>28</sup> Aalen,<sup>29</sup> Yi et al.,<sup>30</sup> Hougaard,<sup>31</sup> Bagdonavicius and Nikulin,<sup>32</sup> and Duchateau and Janssen.<sup>33</sup>

Also, the model (3) could be considered as a deterministic failure rate function given by

$$(\lambda_t | (H_{wt})_{t \in (0, t]}) = \lambda(t) \left( \left( \alpha \sum_{i=0}^{N(t)} W_i | (H_{wt})_{t \in (0, t]} \right) + \beta \right), \quad (4)$$



**FIGURE 1** Failure rate function under two nonlethal shocks with increments  $W_1$  and  $W_2$

where  $(H_{wt})_{t \in (0,t]} := \{N(t) = n, (W_1, \dots, W_n) = (w_1, \dots, w_n)\}$  is the filtration of historical information in  $(0, t]$ , and the failure rate function of the system is an increasing piecewise deterministic process that jumps at random arrival times of shocks. Figure 1 shows an example of failure rate function under the given model in (4).

Furthermore, the model (3) is also a generalization of tampered failure rate (TFR) step-stress model proposed by Bhattacharyya and Soejoeti<sup>34</sup> from the following point of view. Suppose that the shocks arrive at random times  $T_1, \dots, T_{N(t)}$  and the effect of accumulated shocks on the system at time  $T_i, i = 1, \dots, N(t)$  could be considered as a stress. The stresses are constant over interval  $[T_{i-1}, T_i)$  and increase suddenly (jump) at times  $T_i, i = 1, \dots, N(t)$ . In this setup, the model (3) could be expressed as follows:

$$\lambda_s = \begin{cases} \beta \lambda(s) & 0 < s < t_1 \\ (\alpha W_1 + \beta) \lambda(s) & t_1 \leq s < t_2 \\ \vdots & \vdots \\ \left( \alpha \sum_{i=0}^{N(t)} W_i + \beta \right) \lambda(s) & t_{N(t)} \leq s < t \end{cases} \quad (5)$$

which is a generalization of TFR model while  $T_1, \dots, T_{N(t)}$  are random.

Note that when there is no shock, that is,  $W_i = 0; i = 1, \dots, N(t)$ , the proposed model corresponds to the model in static environment and when the magnitude of shocks are the same, that is,  $W_i = w, i = 1, \dots, N(t)$ , it is converted to

$$\lambda_t = \lambda(t)(\beta + \gamma N(t)); \quad \gamma = \alpha w. \quad (6)$$

### 3 | RELIABILITY MODELING

The following theorem will provide the joint probability distribution function of  $\{T > t, N(t)\}$  and the reliability function  $R(t)$  under the non-homogeneous compound Poisson process (NHCPP). It is worth noting that the result for the homogeneous case is analogous.

**Theorem 1.** Suppose nonlethal shocks and their magnitudes until time  $t$ ,  $\{W(t), t \geq 0\}$  where  $W(t) = W_1 + W_2 + \dots + W_{N(t)}$ , occur according to the NHCPP with an intensity function  $v(t)$  such that  $v(t) \geq 0$  for  $t \geq 0$  and let  $\lambda(t)$ , a deterministic function, indicates the baseline failure rate that models the normal environment for the system, where  $\lambda(t) \geq 0$ . Define  $\Lambda(t) = \int_0^t \lambda(s)ds$  and  $V(t) = E(N(t)) = \int_0^t v(s)ds$  as the cumulative baseline failure rate function and expected number of nonlethal

shocks in  $(0, t]$ , respectively. Then the joint probability distribution function of  $\{T > t, N(t)\}$  is obtained as follows:

$$P(T > t, N(t) = n) = \exp\{-\beta\Lambda(t) - V(t)\} \frac{\left(\int_0^t v(s)M_W(\alpha(\Lambda(s) - \Lambda(t)))ds\right)^n}{n!} \quad (7)$$

and the corresponding reliability function is

$$R(t) = \exp\{-\beta\Lambda(t) - V(t) + \int_0^t v(s)M_W(\alpha(\Lambda(s) - \Lambda(t)))ds\}. \quad (8)$$

*Proof of Theorem 1.* Consider  $0 \leq T_1 \leq T_2 \leq \dots$  and  $W_1, W_2, \dots$  respectively as the sequential arrival times and magnitudes of shocks in the NHCPP  $\{W_1, \dots, W_{N(t)}, N(t)\}$  with intensity  $v(t)$ . Note that the full history of  $\{N(u), 0 \leq u \leq t\}$  can be specified as  $\{T_1, T_2, \dots, T_{N(t)}, W_1, W_2, \dots, W_{N(t)}, N(t)\}$  where arrival times and magnitude of shocks are independent. Hence, the conditional distribution of  $T > t$  given  $T_1, \dots, T_{N(t)}, W_1, \dots, W_{N(t)}, N(t)$  based on (3) is

$$\begin{aligned} P(T > t | (T_1, \dots, T_{N(t)}, W_1, \dots, W_{N(t)}, N(t)) = (t_1, \dots, t_n, w_1, \dots, w_n, n)) \\ = e^{-\int_0^{t_1} \beta \lambda(s) ds} e^{-\int_{t_1}^{t_2} (\alpha w_1 + \beta) \lambda(s) ds} \dots e^{-\int_{t_n}^t (\alpha \sum_{i=0}^n w_i + \beta) \lambda(s) ds} \\ = e^{-\beta \int_0^t \lambda(s) ds} e^{-\alpha \sum_{i=1}^n w_i \int_{t_i}^t \lambda(s) ds}. \end{aligned} \quad (9)$$

The joint probability distribution of  $\{T > t, N(t)\}$  is calculated by integrating out  $T_1, \dots, T_n$  of  $P(T > t, t_1, \dots, t_n, N(t) = n)$ , see appendix A.

$$\begin{aligned} P(T > t, N(t) = n) &= e^{-\beta \int_0^t \lambda(s) ds - \int_0^t v(s) ds} \int_0^t \int_0^{t_1} \dots \int_0^{t_{n-1}} \prod_{i=1}^n v(t_i) M_W(\alpha(\Lambda(t_i) - \Lambda(t))) dt_1 dt_2 \dots dt_n \\ &= e^{-\beta \int_0^t \lambda(s) ds - \int_0^t v(s) ds} \frac{\left(\int_0^t v(s) M_W(\alpha(\Lambda(s) - \Lambda(t))) ds\right)^n}{n!}, \end{aligned} \quad (10)$$

where the latest equality is concluded from the following property, (Cha and Finkelstein<sup>8</sup>):

$$\int_0^t \dots \int_0^{t_3} \int_0^{t_2} f(t_i) dt_1 dt_2 \dots dt_n = \frac{\left(\int_0^t f(x) dx\right)^n}{n!}.$$

Eventually, the reliability function is calculated using NHPP properties as follows:

$$\begin{aligned} R(t) &= e^{-\beta \int_0^t \lambda(s) ds - \int_0^t v(s) ds} \sum_{n=0}^{\infty} \frac{\left(\int_0^t v(s) M_W(\alpha(\Lambda(s) - \Lambda(t))) ds\right)^n}{n!} \\ &= e^{-\beta \Lambda(t) - V(t) + \int_0^t v(s) M_W(\alpha(\Lambda(s) - \Lambda(t))) ds}. \end{aligned} \quad (11)$$

## 4 | MAINTENANCE

In this section, a shock-based preventive maintenance action is proposed where the system is maintained at inspection times after failure or after reaching to a prefixed number of shocks, whichever comes first. This maintenance action is considered based on two different policies. In the first policy, it is assumed that inspections do not impact the failure rate, while in the second policy inspections are considered as imperfect preventive repairs. Suppose that inspections are scheduled at periodic times  $(k+1)\tau$ ;  $k = 0, 1, 2, \dots$  and interinspection interval,  $\tau$ , is unknown. The time interval  $(k\tau, (k+1)\tau]$  is called the  $(k+1)$ th period. Maintenance actions are performed based on the following assumptions.

- Inspection is performed at the end of each period

- Time needed for inspection and maintenance actions is negligible
- The system failure is not self-announced
- At each inspection time, the number of received shocks is observed

In both policies, the system is replaced by a new identical one (AGAN) if a failure is detected or the number of receiving shocks exceeds preventive threshold  $n^*$ . For the first case, the maintenance task is called perfect corrective maintenance (PCM) and for the second, perfect preventive maintenance (PPM). The time interval between the installation (set-up) and its replacement is called cycle. Let  $P_p(k\tau)$  and  $P_c(k\tau)$  denote perfect preventive and perfect corrective maintenance probabilities for policy I,  $P_p^q(k\tau)$  and  $P_c^q(k\tau)$  denotes perfect preventive and perfect corrective maintenance probabilities for policy II, and  $P_r(k\tau)$  denote replacement probability at the end of the  $k$ th period.

#### 4.1 | Maintenance policy I: Perfect preventive maintenance

According to the shock-based maintenance policy, at the end of  $(k+1)$ th period one decides on PCM or PPM based on the observed information at  $(k+1)\tau$ . At inspection time  $(k+1)\tau$  the system is maintained as

- Perfect preventive replacement if

$$(T > (k+1)\tau \quad \& \quad N(k\tau) \leq n^* \quad \& \quad N((k+1)\tau) > n^*).$$

- Corrective replacement if

$$(k\tau < T < (k+1)\tau).$$

##### 4.1.1 | Maintenance probability calculations

The perfect preventive replacement is carried out at the end of the  $(k+1)$ th period when number of shocks exceed a critical value, say  $n^*$ , that is,

$$(T > (k+1)\tau \quad \& \quad N(k\tau) \leq n^* \quad \& \quad N((k+1)\tau) > n^*).$$

**Proposition 1.** The probability of PPM at the end of the  $(k+1)$ th period is given by

$$\begin{aligned} P_p((k+1)\tau) &= P(T > (k+1)\tau \quad \& \quad N(k\tau) \leq n^* \quad \& \quad N((k+1)\tau) > n^*) \\ &= \sum_{m^*=n^*+1}^{\infty} \sum_{n=0}^{n^*} P(T > (k+1)\tau, N(k\tau + \tau) = m^*, N(k\tau) = n | T > k\tau) P(T > k\tau) \\ &= \sum_{m^*=n^*+1}^{\infty} \sum_{n=0}^{n^*} P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m^* - n, N(k\tau) = n | T > k\tau) P(T > k\tau), \end{aligned} \quad (12)$$

where  $\Delta N(k\tau + \tau) = N(k\tau + \tau) - N(k\tau) = m^* - n = m$  and  $P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, N(k\tau) = n | T > k\tau)$  is derived as follows:

Using (3) the reliability function at  $(k+1)\tau$  given received information at  $k$ th inspection is

$$\begin{aligned} P(T > (k+1)\tau | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w, \Delta N(k\tau + \tau) = m, t_{n+1}, \dots, t_{n+m}, w_{n+1}, \dots, w_{n+m}) \\ &= e^{-\int_{k\tau}^{t_{n+1}} (\alpha \sum_{i=0}^n w_i + \beta) \lambda(s) ds - \int_{t_{n+1}}^{t_{n+2}} (\alpha \sum_{i=0}^{n+1} w_i + \beta) \lambda(s) ds - \dots - \int_{t_{n+m}}^{k\tau + \tau} (\alpha \sum_{i=0}^{n+m} w_i + \beta) \lambda(s) ds} \\ &= e^{-\int_{k\tau}^{k\tau + \tau} (\alpha w + \beta) \lambda(s) ds - \sum_{i=n+1}^{n+m} \int_{t_i}^{k\tau + \tau} (\alpha w_i) \lambda(s) ds} \end{aligned}$$

$$\begin{aligned}
&= e^{-(\alpha w + \beta)\{\Lambda(k\tau + \tau) - \Lambda(k\tau)\} - \sum_{i=n+1}^{n+m} (\alpha w_i)\{\Lambda(k\tau + \tau) - \Lambda(t_i)\}} \\
&= e^{-(\alpha w + \beta)\{\Lambda(k\tau + \tau) - \Lambda(k\tau)\}} \prod_{i=n+1}^{n+m} e^{-\alpha w_i(\Lambda(k\tau + \tau) - \Lambda(t_i))},
\end{aligned} \tag{13}$$

where the joint probability distribution of  $\{\Delta N(k\tau + \tau), T_{n+1}, \dots, T_{n+\Delta N(k\tau + \tau)}, W_{n+1}, \dots, W_{n+\Delta N(k\tau + \tau)}\}$  is given by

$$\begin{aligned}
&f_{\Delta N(k\tau + \tau), T_{n+1}, \dots, T_{n+\Delta N(k\tau + \tau)}, W_{n+1}, \dots, W_{n+\Delta N(k\tau + \tau)}}(m, w_{n+1}, \dots, w_{n+m}, t_{n+1}, \dots, t_{n+m}) \\
&= e^{-\int_{k\tau}^{t_{n+1}} v(s)ds} v(t_{n+1}) f(w_{n+1}) e^{-\int_{t_{n+1}}^{t_{n+2}} v(s)ds} v(t_{n+2}) f(w_{n+2}) \dots e^{-\int_{t_{n+m}}^{k\tau + \tau} v(s)ds} \\
&= e^{-\int_{k\tau}^{k\tau + \tau} v(s)ds} \prod_{i=n+1}^{n+m} f(w_i) v(t_i),
\end{aligned} \tag{14}$$

combining (13) and (14) results in

$$\begin{aligned}
&P\left(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, t_{n+1}, \dots, t_{n+m}, w_{n+1}, \dots, w_{n+m} | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w\right) \\
&= e^{-(\alpha w + \beta)\{\Lambda(k\tau + \tau) - \Lambda(k\tau)\} - \int_{k\tau}^{k\tau + \tau} v(s)ds} \prod_{i=n+1}^{n+m} e^{-(\alpha w_i)\{\Lambda(k\tau + \tau) - \Lambda(t_i)\}} f(w_i) v(t_i),
\end{aligned} \tag{15}$$

then integrating out of  $W_{n+1}, \dots, W_{n+m}$  and  $T_{n+1}, \dots, T_{n+m}$  ends in

$$\begin{aligned}
&P\left(T > (k+1)\tau, \Delta N(k\tau + \tau) = m | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w\right) \\
&= e^{-(\alpha w + \beta)\{\Lambda(k\tau + \tau) - \Lambda(k\tau)\} - \int_{k\tau}^{k\tau + \tau} v(s)ds} \frac{\left(\int_{k\tau}^{k\tau + \tau} v(x) M_{\alpha W}(\Lambda(x) - \Lambda(k\tau + \tau)) dx\right)^m}{m!}
\end{aligned} \tag{16}$$

and based on (16) and the joint distribution function of  $(N(k\tau), \sum_{i=0}^{N(k\tau)} W_i)$  we obtain

$$f_{(N(k\tau), \sum_{i=0}^{N(k\tau)} W_i)}(n, w) = f^{n*}(w) \frac{e^{-\int_0^{k\tau} v(s)ds} \left(\int_0^{k\tau} v(s)ds\right)^n}{n!}, \tag{17}$$

where  $f^{n*}$  is the  $n$  convolution of  $f$ , then the conditional probability distribution function is derived as:

$$\begin{aligned}
&P\left(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, N(k\tau) = n, \sum_{i=0}^n W_i = w | T > k\tau\right) \\
&= \frac{\left(\int_{k\tau}^{k\tau + \tau} v(x) M_{\alpha W}(\Lambda(x) - \Lambda(k\tau + \tau)) dx\right)^m}{m!} e^{-(\alpha w + \beta)\{\Lambda(k\tau + \tau) - \Lambda(k\tau)\} - \int_0^{k\tau + \tau} v(s)ds} \times \\
&\quad \frac{f^{n*}(w) \left(\int_0^{k\tau} v(s)ds\right)^n}{n!}
\end{aligned} \tag{18}$$

and finally with integration out  $w$ , the conditional distribution function of  $(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, N(k\tau) = n)$  given  $T > k\tau$  is obtained as follows:

$$\begin{aligned}
&P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, N(k\tau) = n | T > k\tau) \\
&= \frac{\left(\int_{k\tau}^{k\tau + \tau} v(x) M_{\alpha W}(\Lambda(x) - \Lambda(k\tau + \tau)) dx\right)^m}{m!} \frac{e^{-\int_0^{k\tau + \tau} v(s)ds} \left(\int_0^{k\tau} v(s)ds\right)^n}{n!} \\
&\quad \times M_{\alpha \sum_{i=0}^n W_i + \beta}(\Lambda(k\tau) - \Lambda(k\tau + \tau)).
\end{aligned} \tag{19}$$



**Proposition 2.** The probability of PCM at time  $(k+1)\tau$  is calculated as follows:

$$\begin{aligned} P_c((k+1)\tau) &= P(k\tau < T < (k+1)\tau) = P(T < (k+1)\tau | T > k\tau)P(T > k\tau) \\ &= P(T > k\tau) - P(T > (k+1)\tau), \end{aligned} \quad (20)$$

where the last equality is simply calculated using reliability function (11) and in the second equality, the conditional probability  $P(T < (k+1)\tau | T > k\tau)$  is straightforward. Hence, the PCM probability is given by

$$P(T > (k+1)\tau | T > k\tau) = e^{\int_{k\tau}^{k\tau+\tau} v(x)M_{\alpha W}(\Lambda(x)-\Lambda(k\tau+\tau))dx - V(k\tau+\tau) + \beta(\Lambda(k\tau)-\Lambda(k\tau+\tau)) + (M_{\alpha W}(\Lambda(k\tau)-\Lambda(k\tau+\tau))V(k\tau))}. \quad (21)$$

## 4.2 | Maintenance policy II: Imperfect preventive maintenance

In this subsection, it is assumed that at inspections some actions are performed with the same cost as the previous section and reduce the effect of the dynamic environment by factor  $q$ ; ( $0 \leq q \leq 1$ ) such that the coefficient  $(\alpha \sum_{i=0}^{N(t)} W_i + \beta)$  of the baseline failure rate is converted to,  $(q\alpha \sum_{i=0}^{N(t)} W_i + \beta)$  which is considered as periodically imperfect preventive maintenance. Thus, at an inspection time,  $(k+1)\tau$ ,  $k \in \{\{0\} \cup \mathbb{N}\}$  the system is maintained as

- Perfect preventive replacement if

$$(T > (k+1)\tau \quad \& \quad N(k\tau) \leq n^* \quad \& \quad N((k+1)\tau) > n^*).$$

- Corrective replacement if

$$(k\tau < T < (k+1)\tau).$$

- An imperfect maintenance is carried out.

### 4.2.1 | Maintenance probability calculations

**Proposition 3.** The conditional probability of  $(T > (k+1)\tau)$  given observed information at  $k\tau$  is calculated as follows:

$$\begin{aligned} P(T > (k+1)\tau | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w, \Delta N(k\tau + \tau) = m, t_{n+1}, \dots, t_{n+m}, w_{n+1}, \dots, w_{n+m}) \\ &= e^{-\int_{k\tau}^{t_{n+1}} (q\alpha \sum_{i=0}^n w_i + \beta) \lambda(s) ds} e^{-\int_{t_{n+1}}^{t_{n+2}} (q\alpha \sum_{i=0}^n w_i + \alpha w_{n+1} + \beta) \lambda(s) ds} \times \dots \times \\ &\quad e^{-\int_{t_{n+m}}^{k\tau+\tau} (q\alpha \sum_{i=0}^n w_i + \alpha \sum_{i=n+1}^{n+m} w_i + \beta) \lambda(s) ds} \\ &= e^{-(q\alpha w + \beta) \{ \Lambda(k\tau+\tau) - \Lambda(k\tau) \} - \alpha \sum_{i=n+1}^{n+m} w_i \{ \Lambda(k\tau+\tau) - \Lambda(t_i) \}} \end{aligned} \quad (22)$$

and combining (22) with the joint probability function of  $\{\Delta N(k\tau + \tau) = m, T_{n+1}, \dots, T_{n+m}, w_{n+1}, \dots, w_{n+m}\}$  results in

$$\begin{aligned} P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, t_{n+1}, \dots, t_{n+m}, w_{n+1}, \dots, w_{n+m} | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w) \\ &= e^{-(q\alpha w + \beta) \{ \Lambda(k\tau+\tau) - \Lambda(k\tau) \} - \int_{k\tau}^{k\tau+\tau} v(s) ds} \prod_{i=n+1}^{n+m} e^{-\alpha w_i \{ \Lambda(k\tau+\tau) - \Lambda(t_i) \}} f(w_i) v(t_i), \end{aligned} \quad (23)$$

then integrating out  $w_{n+1}, \dots, w_{n+m}$  and  $T_{n+1}, \dots, T_{n+m}$  respectively, concludes that

$$\begin{aligned} P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m, t_{n+1}, \dots, t_{n+m} | T > k\tau, N(k\tau) = n, \sum_{i=0}^n W_i = w) \\ &= e^{-(q\alpha w + \beta) \{ \Lambda(k\tau+\tau) - \Lambda(k\tau) \} - \int_{k\tau}^{k\tau+\tau} v(s) ds} \frac{\left( \int_{k\tau}^{k\tau+\tau} M_W(-\alpha \{ \Lambda(k\tau + \tau) - \Lambda(x) \}) v(x) dx \right)^m}{m!} \end{aligned} \quad (24)$$

and eventually

$$\begin{aligned}
P(T > (k+1)\tau, N(k\tau) = n, \Delta N(k\tau + \tau) = m | T > k\tau) \\
= \frac{\left( \int_{k\tau}^{k\tau+\tau} M_W(-\alpha \{ \Lambda(k\tau + \tau) - \Lambda(x) \}) v(x) dx \right)^m}{m!} e^{-\beta \{ \Lambda(k\tau+\tau) - \Lambda(k\tau) \} - \int_0^{k\tau+\tau} v(s) ds} \\
M_{\sum_{i=0}^n W_i}(-q\alpha \{ \Lambda(k\tau + \tau) - \Lambda(k\tau) \}) \frac{\left( \int_0^{k\tau} v(s) ds \right)^n}{n!}
\end{aligned} \quad (25)$$

and substituting Equation (25) in the below probability function gives the probability of PPM under policy II.

$$\begin{aligned}
P_p^q((k+1)\tau) &= P(T > (k+1)\tau \quad \& \quad N(k\tau) \leq n^* \quad \& \quad N((k+1)\tau) > n^*) \\
&= \sum_{m^*=n^*+1}^{\infty} \sum_{n=0}^{n^*} P(T > (k+1)\tau, \Delta N(k\tau + \tau) = m^* - n, N(k\tau) = n | T > k\tau) P(T > k\tau).
\end{aligned} \quad (26)$$

**Proposition 4.** The probability of PCM based on the implementation of imperfect maintenance is obtained as

$$\begin{aligned}
P_c^q((k+1)\tau) &= P(k\tau < T < (k+1)\tau) = P(T < (k+1)\tau | T > k\tau) P(T > k\tau) \\
&= P(T > k\tau) - P(T > (k+1)\tau),
\end{aligned} \quad (27)$$

where

$$\begin{aligned}
P(T > (k+1)\tau | T > k\tau) &= e^{-\beta \{ \Lambda(k\tau+\tau) - \Lambda(k\tau) \} - \int_0^{k\tau+\tau} v(s) ds} \sum_{m=0}^{\infty} \frac{\left( \int_{k\tau}^{k\tau+\tau} M_W(-\alpha \{ \Lambda(k\tau + \tau) - \Lambda(x) \}) v(x) dx \right)^m}{m!} \times \\
&\sum_{n=0}^{\infty} M_{\sum_{i=0}^n W_i}(-q\alpha \{ \Lambda(k\tau + \tau) - \Lambda(k\tau) \}) \frac{\left( \int_0^{k\tau} v(s) ds \right)^n}{n!} \\
&= e^{\int_{k\tau}^{k\tau+\tau} v(x) M_{qW}(\Lambda(x) - \Lambda(k\tau+\tau)) dx - V(k\tau+\tau) + \beta(\Lambda(k\tau) - \Lambda(k\tau+\tau)) + (M_{qW}(\Lambda(k\tau) - \Lambda(k\tau+\tau)) V(k\tau))},
\end{aligned} \quad (28)$$

### 4.3 | Long-run cost rate function

Let  $c_{ins}$ ,  $c_p$  and  $c_c > c_p > c_{ins}$  be costs of inspection, PPM and PCM respectively in two policies. It is planned that the system is inspected at regular times  $(k+1)\tau$ ;  $k = 0, 1, 2, \dots$  in both policies, and its replacement depends on the conditions. The system is preventively replaced with cost  $c_p$ , if the number of arrival nonlethal shocks exceeds the critical threshold  $n^*$ , and it is correctively replaced with cost  $c_c$ , if it has been failed in  $(k\tau, (k+1)\tau]$ .

We are going to select  $\tau$  and  $n^*$  such that the long-run cost rate function takes its minimum value. The optimal values of  $\tau$  and  $n^*$  lead to real needed inspection times and the most useful functionality of the system, respectively.

Denote the total maintenance cost until time  $t$  by  $C(t)$ . Based on the renewal reward theorem, the expected long-run maintenance cost rate is

$$C(T) = \lim_{t \rightarrow \infty} \frac{C(t)}{t} = \frac{E(C_r)}{E(T_r)}, \quad (29)$$

where  $E(T_r)$  is the mean length of the renewal cycle and  $E(C_r)$  is expected maintenance cost spent during a renewal cycle and are obtained as

$$E(T_r) = \sum_{k=0}^{\infty} (k+1)\tau P_r((k+1)\tau),$$

where, for policy I:

$$\begin{aligned}
P_r((k+1)\tau) &= P(k\tau < T < (k+1)\tau) + P(T > (k+1)\tau, N(k\tau + \tau) > n^*, N(k\tau) \leq n^*) \\
&= P_c((k+1)\tau) + P_p((k+1)\tau)
\end{aligned}$$

and

$$E(C_r) = \sum_{k=0}^{\infty} ((k+1)c_{ins} + c_p)P_p((k+1)\tau) + ((k+1)c_{ins} + c_c)P_c((k+1)\tau),$$

and for policy II:

$$\begin{aligned} P_r((k+1)\tau) &= P(k\tau < T < (k+1)\tau) + P(T > (k+1)\tau, N(k\tau + \tau) > n^*, N(k\tau) \leq n^*) \\ &= P_c^q((k+1)\tau) + P_p^q((k+1)\tau) \end{aligned}$$

and

$$E(C_r) = \sum_{k=0}^{\infty} ((k+1)c_{ins} + c_p)P_p^q((k+1)\tau) + ((k+1)c_{ins} + c_c)P_c^q((k+1)\tau).$$

## 5 | NUMERICAL EXAMPLE

A numerical example, artificially constructed based on a realistic physical example given in Fan et al.,<sup>35</sup> will illustrate the application of the proposed method to handle the dynamic environment. The example has been constructed based on the following assumptions:

1. The lifetime of the single-unit system follows the Weibull distribution with parameters  $(\gamma, \eta)$ , that is,  $f_{\gamma, \eta}(t) = \frac{\gamma t^{\gamma-1}}{\eta^\gamma} e^{-\left(\frac{t}{\eta}\right)^\gamma}$ .
2. The system is subject to nonlethal shocks and shocks arrive according to a) homogeneous Poisson process (HPP) with intensity  $\nu$  and b) nonhomogeneous Poisson process (NHPP) with intensity  $\nu(t) = \lambda_0 + \gamma_0 t$ .
3. The magnitude of nonlethal shocks,  $W \geq 0$ , follows a gamma distribution with shape parameter  $\mu$  and scale parameter  $\theta$ , that is,  $f_W(w) = \frac{w^{\mu-1}}{\Gamma(\mu)\theta^\mu} e^{-\frac{w}{\theta}}$ .
4. Number and magnitude of shocks are independent.

Without loss of generality, we suppose that inspections are scheduled periodically at times  $(k+1)\tau; k = 0, 1, 2, \dots$ . Two assumptions are considered for  $\tau$ : (1)  $\tau$  is prefixed, (2)  $\tau$  is a parameter of the maintenance policy that should be optimized. A perfect preventive replacement is carried out when the number of nonlethal shocks exceeds a specified value  $n^*$ , or a perfect corrective replacement is made whenever a failure occurs. We set  $\alpha = \beta = 1$ ,  $c_{ins} = 1$ ,  $c_p = 2$ , and  $c_c = 3$ .

### 5.1 | Reliability function

Based on the general reliability function (11) and the above assumptions, the reliability functions under HPP and NHPP are derived as follows, respectively.

$$R^{HPP}(t) = e^{-\beta\left(\frac{t}{\eta}\right)^\gamma - \nu t + \nu \int_0^t \frac{1}{(1 - \alpha\theta((\frac{s}{\eta})^\gamma - (\frac{t}{\eta})^\gamma)^\mu)^\mu} I\left(\alpha\left((\frac{s}{\eta})^\gamma - (\frac{t}{\eta})^\gamma\right) < \frac{1}{\theta}\right) ds}, \quad (30)$$

$$R^{NHPP}(t) = e^{-\beta\left(\frac{t}{\eta}\right)^\gamma - (\lambda_0 t + \gamma_0 \frac{t^2}{2}) + \int_0^t \frac{\lambda_0 + \gamma_0 s}{(1 - \alpha\theta((\frac{s}{\eta})^\gamma - (\frac{t}{\eta})^\gamma)^\mu)^\mu} I\left(\alpha\left((\frac{s}{\eta})^\gamma - (\frac{t}{\eta})^\gamma\right) < \frac{1}{\theta}\right) ds}. \quad (31)$$

Reliability function considering different sets of parameters is drawn in Figures 2–5.

### 5.2 | Sensitivity analysis

In practice, model parameters are estimated based on the historical data and experts' opinions and the accuracy of the model is influenced by biased estimates, thus a sensitivity analysis is required.

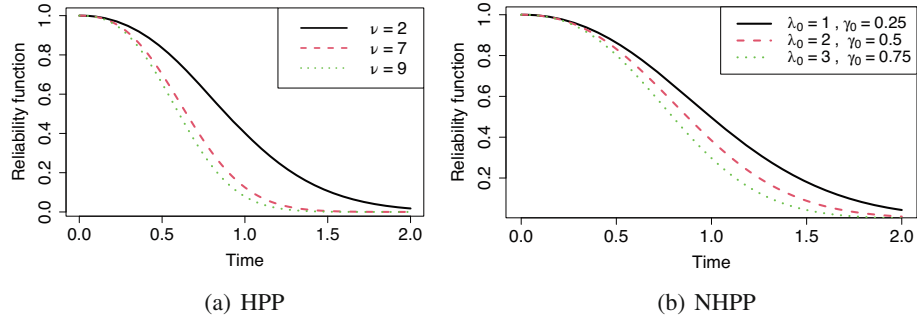


FIGURE 2 Reliability function with different values of parameters for intensity function

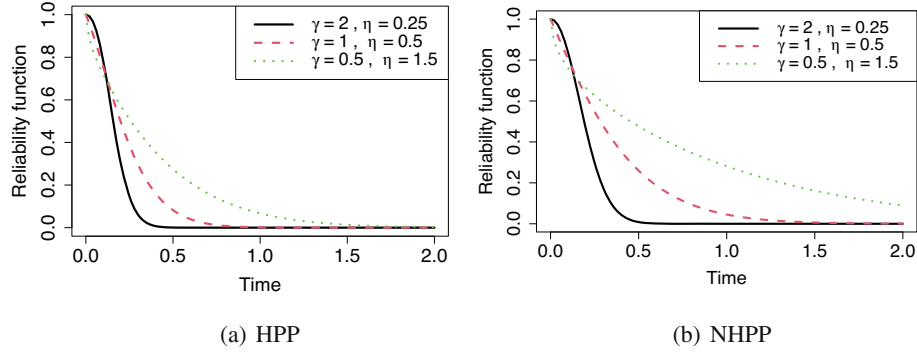


FIGURE 3 Reliability function with different values of parameters for lifetime (T) distribution

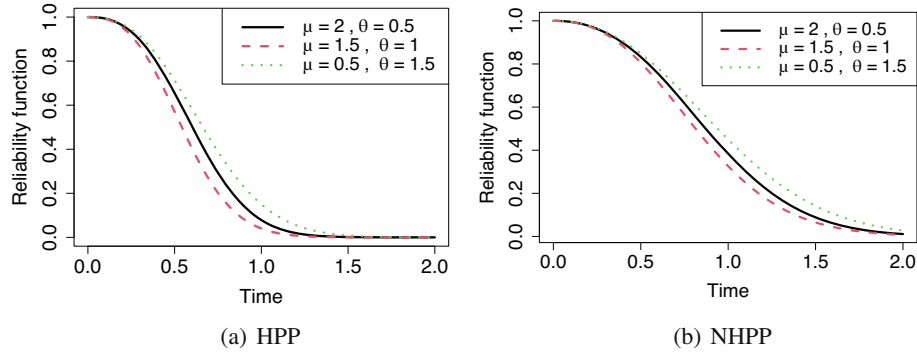


FIGURE 4 Reliability function with different values of parameters for increments ( $W_i$ ) distribution

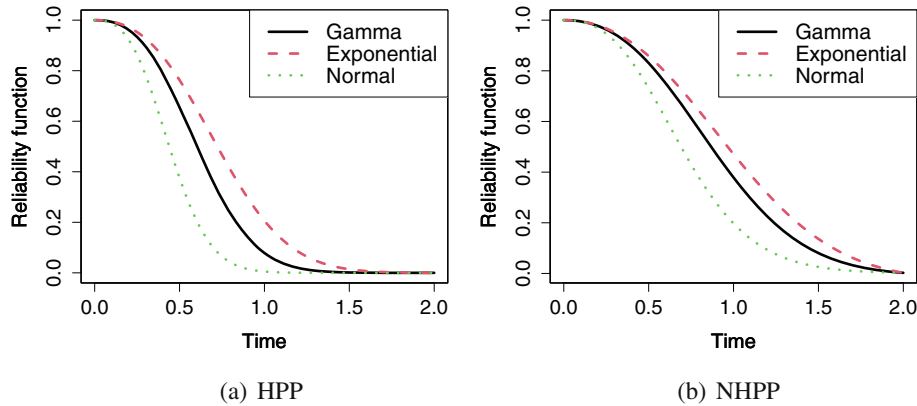


FIGURE 5 Reliability function with different choices of increments ( $W_i$ ) distribution

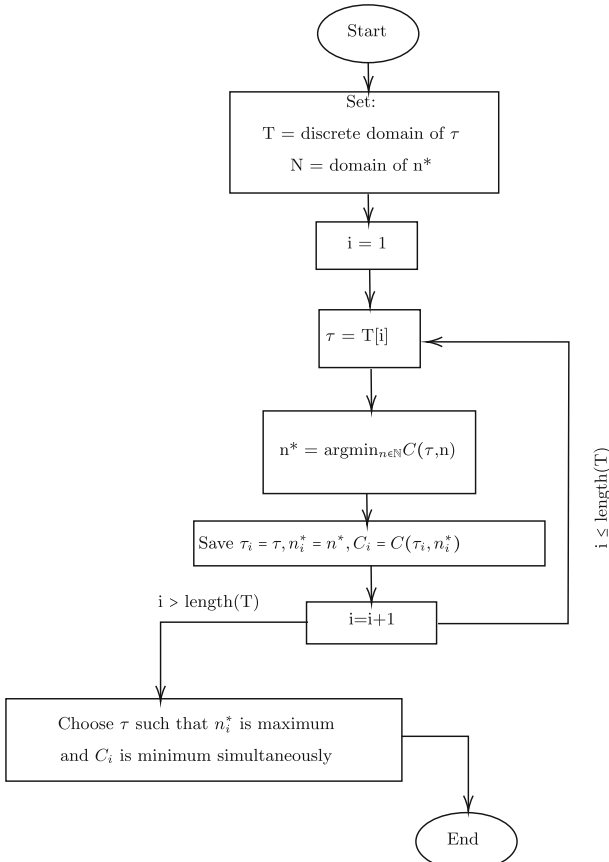
Variability of the reliability functions under HPP and NHPP with different ranges of intensity parameters is depicted in Figure 2 while other parameters of the model  $[(\mu, \theta, \gamma, \eta) = (2, 0.5, 0.2, 1.5)]$  are fixed. As expected, the reliability function shifts to the left when the intensity parameters increase. Fluctuation of the reliability function due to variations of lifetime distribution parameters is depicted in Figure 3 under HPP and NHPP, where parameters  $(\mu, \theta, \lambda_0, \gamma_0, \nu) = (2, 0.5, 2, 0.5, 9)$  are fixed. It is observed that by increasing  $\eta$  and decreasing  $\gamma$  the reliability function move to the right. Moreover, reliability function under HPP and NHPP with different values of parameters for magnitude of shocks distribution in presence of fixed parameters  $[(\gamma, \eta, \lambda_0, \gamma_0, \nu) = (0.2, 1.5, 2, 0.5, 9)]$  is drawn in Figure 4. By increasing  $\mu$  when  $\mu$  decreases from 2 to 1.5 the reliability function shifts to the left and when  $\mu$  decreases to 0.5 the reliability function shifts to the right.

The influence of model selection on the reliability function under HPP and NHPP is surveyed in Figure 5. The exponential and normal distributions result in a shift to the right and left of the reliability function, respectively. Thus, inappropriate model selection leads to inaccurate reliability functions.

### 5.3 | Optimal maintenance policy

In this section, minimum cost rate and optimal number of nonlethal shocks  $n^*$  are obtained by two different approaches. The inspection times are assumed to be periodic,  $(k + 1)\tau$ ;  $k = 0, 1, 2, \dots$ . In the first approach, it is assumed that  $\tau$  is known (prespecified) and  $\tau = 2$ . In the second approach, it is assumed that inspection interval  $\tau$  is unknown, and its optimal value is adjusted based on Algorithm 1. The optimal value of  $\tau$  is a trade-off between  $n^*$  and  $C(\tau, n^*)$ , and it should be chosen such that inspection times arrange professionally despite the affordable cost in order to catch maximum efficiency. We consider discrete values for  $\tau$  such that  $\tau \in \{1, 1.1, 1.2, 1.3, \dots, 2.3, 2.4, \dots\}$  and find the optimal value of  $n^* \in \mathbb{N}$  by minimizing the long-run cost rate function. As presented in Table 1, if  $\tau \in [1, 1.7] \Rightarrow n^* = 3$  and  $C(\tau, n^*) \geq 1.886$ , if  $\tau \in (1.7, 2.4] \Rightarrow n^* = 4$  and  $C(\tau, n^*) \in [1.499, 1.807]$ , if  $\tau = 2.5 \Rightarrow n^* = 3$  and  $C(\tau, n^*) = 1.405$ , and if  $\tau \geq 2.6 \Rightarrow n^* = 1$  and  $C(\tau, n^*)$  has an increasing trend, where  $n^* = \operatorname{argmin}_{n \in \mathbb{N}} C(\tau, n)$ . Thus,  $\tau = 2.4$  is selected because of its minimum cost rate (1.450) in spite of receiving the maximum number of shocks (4) in terms of the most functionality.

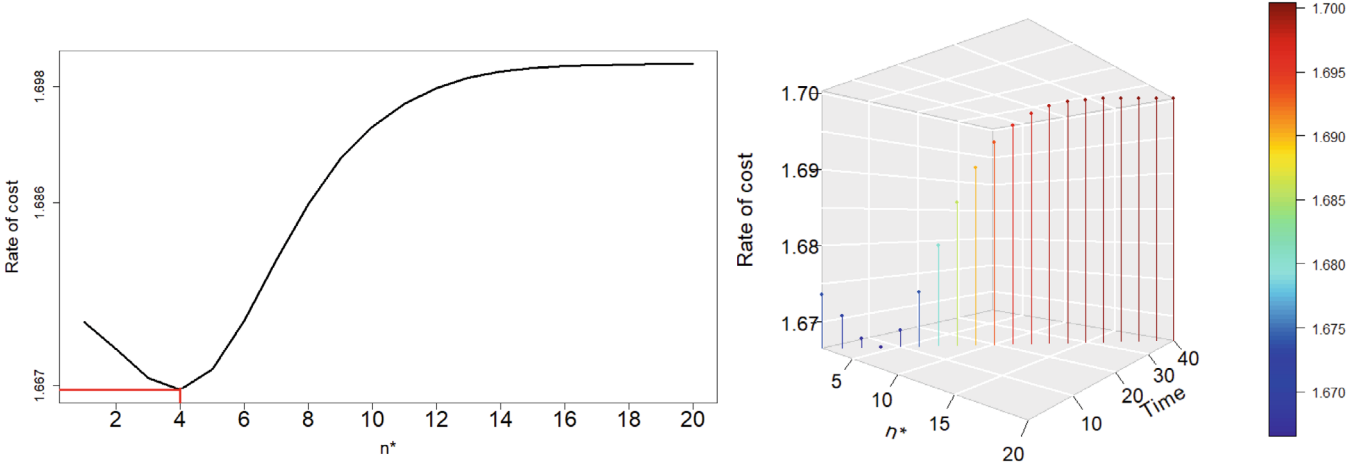
**Algorithm 1.** The optimal value of  $\tau$  as a decision parameter



**TABLE 1** The optimal interinspection interval value ( $\tau$ ) as decision parameter with different cost rates and number of shocks

$\tau$	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	<b>2.4</b>	2.5	$\geq 2.6$
$n^*$	3	3	3	3	3	3	3	3	4	4	4	4	4	4	<b>4</b>	3	1
$C(\tau, n^*)$	2.753	2.570	2.427	2.924	2.175	2.069	1.973	1.886	1.807	1.735	1.667	1.607	1.551	1.499	<b>1.450</b>	1.405	Increasing

Note: Bold values represents the these are best decision parameters.



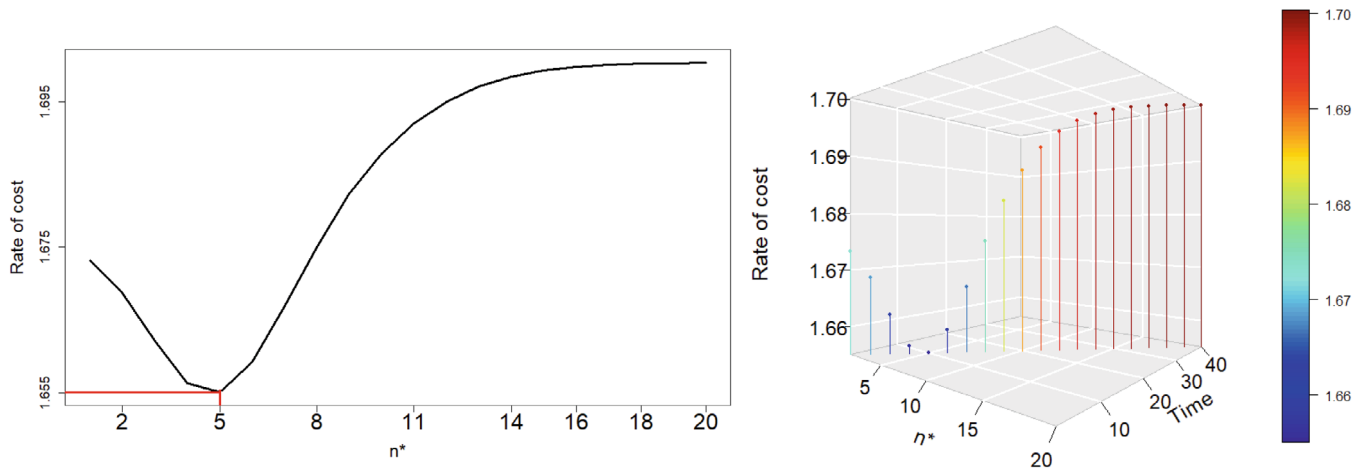
**FIGURE 6** The optimal number of nonlethal shocks as decision parameter in maintenance policy *I* under NHPP considering  $\tau = 2$

At the end of each interval, the system is inspected. If the system fails during the inspection interval, it should be replaced by a new one (perfect corrective maintenance) at inspection time. Moreover, the system, which does not fail at the end of the inspection interval, but its number of received nonlethal shocks exceeds a prefixed level  $n^*$ , should be also replaced as a precaution (perfect preventive maintenance). Now, considering two mentioned approaches, the problem is to find the optimized number of nonlethal shocks that a system receives without failing despite minimum rate of cost. This value of  $n^*$  provides an optimal shock number-based maintenance policy under the presented model. Using the formula associated with Equation (29) and numerical methods to obtain an optimal value of  $n^*$  under NHPP, it is assumed that all the model parameters are known and  $(\mu, \theta, \gamma, \eta, \lambda_0, \gamma_0) = (2, 0.5, 0.2, 1.5, 2, 0.5)$ .

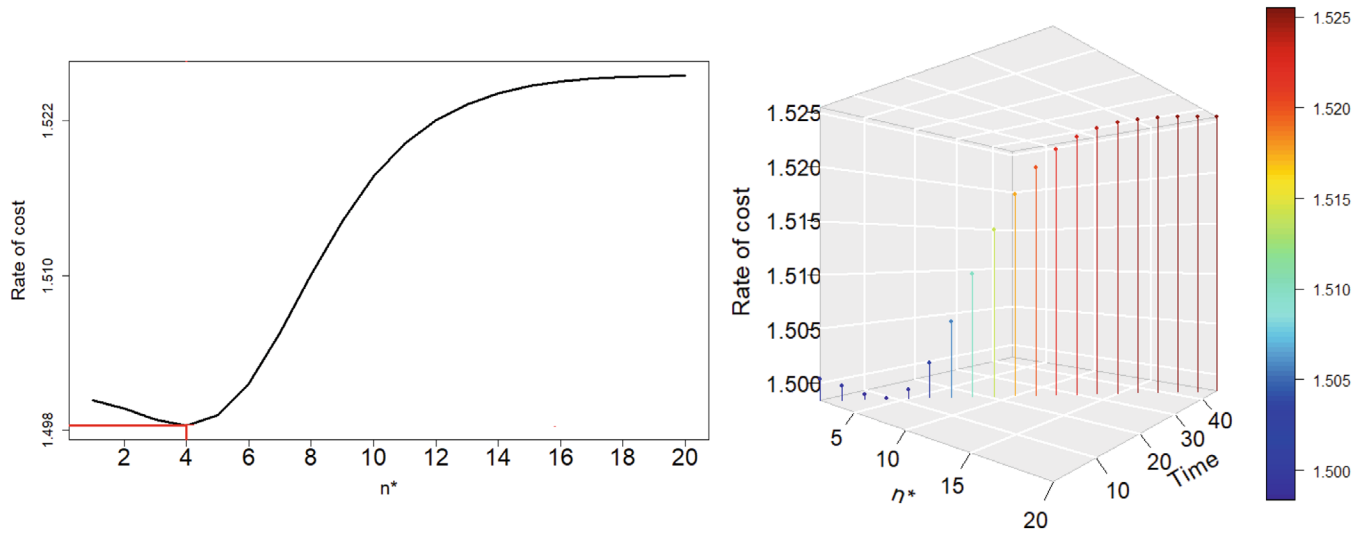
Regarding the first approach,  $\tau = 2$ , Figures 6 and 7 represent the optimal value of  $n^*$  and the associated cost rate for policies *I* and *II*, respectively. As expected, imperfect preventive repairs have postponed preventive maintenance time such that in maintenance policies without imperfect inspections the PM is applied after incurring at least four nonlethal shocks while in maintenance policies with imperfect preventive repairs PM is applied after incurring at least five nonlethal shocks ( $n^* : 4 \rightarrow 5$ ) and also reduced the rate of cost from 1.667 to 1.655. Moreover, considering the optimal value of  $\tau = 2.4$  as the second approach, Figures 8 and 9 represent the optimal value of  $n^*$  and the associated cost rate for policies *I* and *II*, respectively. Indeed, using optimum value of  $\tau$  has postponed PM time more than the case when  $\tau$  was prespecified ( $n^* : 4 \rightarrow 6$ ) and increased useful functionality. Also, the cost rate has an impressive reduction, from 1.667 to 1.498 in maintenance policies without imperfect repairs and from 1.655 to 1.487 in maintenance policies with imperfect preventive repairs. Eventually, optimal numbers of  $n^*$  and associated cost rates in different amounts of  $q$  are represented in Figure 10, and as it is expected when the effect of imperfect preventive maintenance decreases ( $q$  increases) the cost rate increases.

## 6 | CONCLUSION

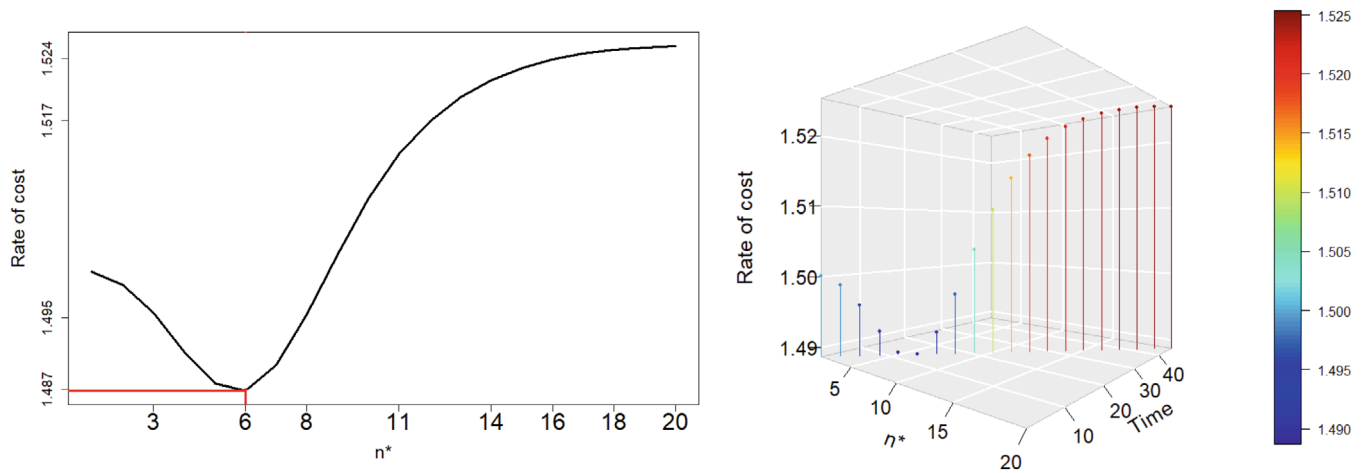
In this article, we first developed a failure model in which random shocks affect the failure rate of the system based on a multiplicative failure rate model. The idea comes from the tampered failure rate (TFR) model, where the acceleration of failure is reflected in the failure rate function when the stress is raised from a lower level to a higher level. However,



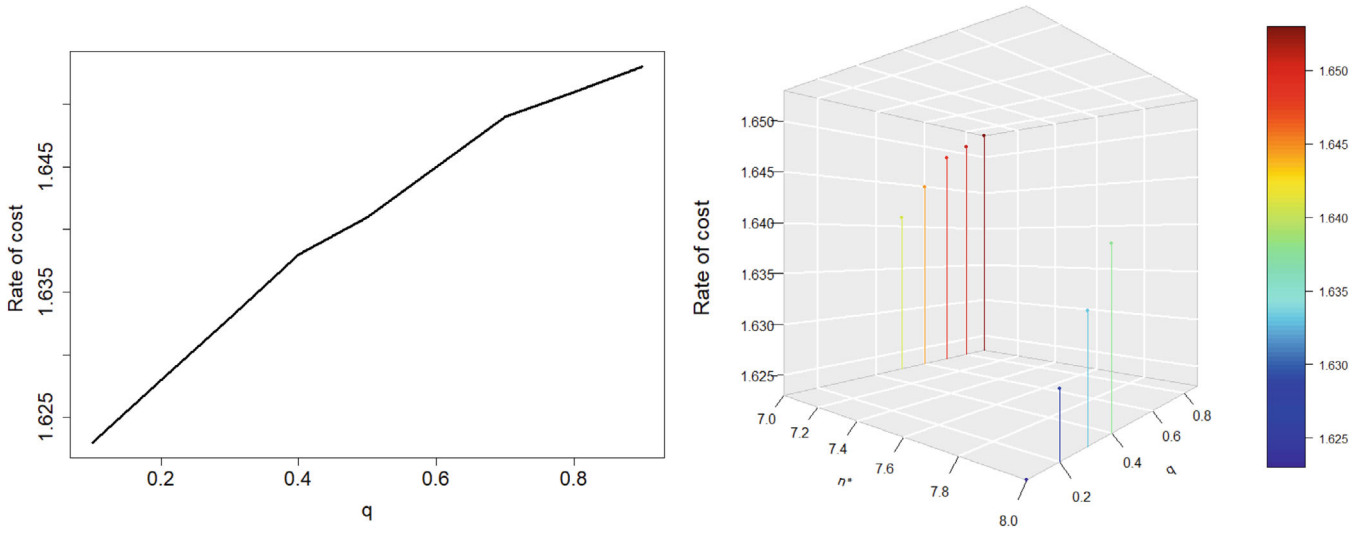
**FIGURE 7** The optimal number of nonlethal shocks as decision parameter in maintenance policy II ( $q = 0.1$ ) under NHPP considering  $\tau = 2$



**FIGURE 8** The optimal number of nonlethal shocks as decision parameter in maintenance policy I under NHPP considering optimal  $\tau = 2.4$



**FIGURE 9** The optimal number of nonlethal shocks as decision parameter in maintenance policy II ( $q = 0.1$ ) under NHPP considering optimal  $\tau = 2.4$



**FIGURE 10** The optimal number of nonlethal shocks as decision parameter and consequent cost rate in different amount of  $q$

the failure rate model proposed in this article differs from the TFR model in some aspects. Despite the TFR model, (1) the proposed failure rate model completely depends on the history of the load applied to the system, (2) the load change point is random (shock arrival time). Further, the proposed model is relatively simple, and its baseline failure rate follows an arbitrary distribution such as Weibull, Gaussian, log-normal, and gamma. According to the proposed failure model, two maintenance models have been proposed, considering both corrective and preventive replacements. The simulation studies show that the maintenance model taking into account imperfect preventive repairs at each inspection time is less costly. It is worth noting that although inspection times are assumed to be periodic, the length of inspection interval is considered as a decision variable and obtained via the optimization method. The maintenance model proposed in this article has attraction to take into account current information on the shocks and their magnitudes, which is not the case in the age-based maintenance and block or calendar-based maintenance models. From the practical point of view, one challenge to apply this maintenance model is that it requires the number of shocks and their magnitudes over time. Although, it is often possible to gather information on the received shocks with the development of sensor technology, but there are some situations in which collecting information on the shocks is difficult and costly.

This article could be developed at least in three following aspects as future works. First, in the maintenance model represented in this article, it is assumed that the failure model parameters are known. It is interesting to study the optimal maintenance policy under uncertainty about the model parameters. Mostly, system lifetime distribution and its failure rate function are unknown in practice, thus proposing a nonparametric structure could be the second aspect of developing current work. In this article, a shock number-based PM has been investigated and a shock damage-based PM could be the third development of this study.

## NOTATIONS

$T$	Lifetime of single-unit system
$R(t)$	Reliability function
$N(t)$	Number of received shocks in $[0, t]$
$W$	Magnitude of each shock
$\lambda(t)$	Failure rate function
$\lambda_t$	Failure rate process
$HPP$	Homogeneous Poisson process
$NHPP$	Non-homogeneous Poisson process
$HCPP$	Homogeneous compound Poisson process
$CM$	Corrective maintenance
$PM$	Preventive maintenance
$PCM$	Perfect corrective maintenance



$PPM$	Perfect preventive maintenance
$P_p(.)$	Probability of PM
$P_c(.)$	Probability of CM
$P_r(.)$	Probability of replacement
$\tau$	Interinspection interval
$c_{ins}$	Inspection cost
$c_p$	Preventive maintenance cost
$c_c$	Corrective maintenance cost
$L$	Long-run cost rate function
$GPP$	Generalized Polya Process

## DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

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## APPENDIX

The joint distribution function of  $(T_1, \dots, T_{N(t)}, W_1, \dots, W_{N(t)}, N(t))$  is given by

$$\begin{aligned}
 & f_{T_1, \dots, T_{N(t)}, W_1, \dots, W_{N(t)}, N(t)}(t_1, \dots, t_n, w_1, \dots, w_n, n) \\
 &= e^{-\int_0^{t_1} v(s)ds} v(t_1) f(w_1) e^{-\int_{t_1}^{t_2} v(s)ds} v(t_2) f(w_2) \dots v(t_n) f(w_n) e^{-\int_{t_n}^t v(s)ds} \\
 &= e^{-\int_0^t v(s)ds} \prod_{i=1}^n v(t_i) f(w_i), \quad 0 \leq t_1 \leq \dots \leq t_n \leq t, w_i \geq 0; \quad i = 1, \dots, n, n = 0, 1, 2, \dots, \quad (A1)
 \end{aligned}$$

where  $e^{\int_{t_{i-1}}^{t_i} v(s)ds}$  represents the probability that there is no shock in  $(t_{i-1}, t_i)$ ,  $v(t_i)$  represents the probability that a shock occurs at  $t_i$  and  $f(w_i)$  indicates the probability density function for magnitude of the  $i$ th shock at  $t_i$ ;  $i = 1, 2, \dots, n$ .

Combining (9) and (A1) comes to an end in

$$P(T > t, t_1, \dots, t_n, w_1, \dots, w_n, N(t) = n) = e^{-\beta \int_0^t \lambda(s)ds - \int_0^t v(s)ds} \prod_{i=1}^n e^{-\alpha w_i \int_{t_i}^t \lambda(s)ds} f(w_i) v(t_i) \quad (A2)$$

and then integrating out  $W_1, \dots, W_n$  results in

$$\begin{aligned}
 P(T > t, t_1, \dots, t_n, N(t) = n) &= e^{-\beta \int_0^t \lambda(s)ds - \int_0^t v(s)ds} \left( \prod_{i=1}^n v(t_i) \right) \int_0^\infty \int_0^\infty \dots \int_0^\infty \prod_{i=1}^n e^{-\alpha w_i \int_{t_i}^t \lambda(s)ds} f(w_i) dw_1 dw_2 \dots dw_n \\
 &= e^{-\beta \int_0^t \lambda(s)ds - \int_0^t v(s)ds} \prod_{i=1}^n v(t_i) M_W(\alpha(\Lambda(t_i) - \Lambda(t))), \quad (A3)
 \end{aligned}$$

where  $M_W(\cdot)$  is the moment generating function of  $W$ .