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ABSTRACT

This paper proposes a maintenance policy for a k-out-of-n: F system operating to fulfil several jobs without interruptions. The system is replaced when the Nth job completes or at the kth failure, whichever occurs first. So, there are some non-failed components when a replacement is done. These components can be sold as second-hand products to continue working in other systems for a while. The price of them is also considered in the proposed maintenance policy. An optimal maintenance policy is studied to minimise long-run average cost under the constraint of relative mean operating time. Comprehensive numerical studies are done to assess the effect of the model parameters on the optimal solutions. Also, to demonstrate the applicability of the proposed plan, a data set related to wind turbine generator failures is considered as a case study.

ARTICLE HISTORY
Received 5 October 2021
Accepted 9 July 2022

KEYWORDS
Maintenance; k-out-of-n system; N-policy; long-run average cost; second-hand component

1. Introduction

Nowadays, in the competitive market, manufacturers or suppliers can release similar products or provide the same services, but they face two competition factors: economic factors and quality. The maintenance theory has focussed on developing innovative maintenance policies to help manufacturers and suppliers to be competitive in the market. For this reason, maintenance policies have received considerable attention in recent years. An efficient maintenance policy can decrease global costs by reducing system failures and by increasing their reliability and performance. One of the most common maintenance policies is the age replacement policy, in which the system is replaced at a given age. There are several variants of age replacement policy which are all easy to implement and they have shown their efficiency, see for instance Glasser (1967), Osaki and Nakagawa (1975), Nakagawa (1984), Kumar and Westberg (1997), Zhao et al. (2010), Babishin and Taghipour (2016), Zhao et al. (2017), Safaei, Ahmadi, and Gildeh (2018), Safaei, Châtelet, and Ahmadi (2020), Safaei et al. (2019) and Sgarbossa et al. (2020). If the system is fulfilling successive jobs (products or services) without interruptions, to avoid stopping the system while fulfilling its job, it would be better to do replacement or maintenance actions after a job or several jobs are accomplished (see, Sugiura 2004). Barlow and Proschan (1996) studied a random replacement policy that is designed to replace a system at random times during its missions. In the literature, there are several replacement models with random working times. See, for example, Chen, Nakamura, and Nakagawa (2010); Chen, Mizutani, and Nakagawa (2010), Zhao and Nakagawa (2012), Nakagawa and Zhao (2013), Zhao, Nakagawa, and Zuo (2014), Zhao, Liu, and Nakagawa (2015), Chang (2014, 2018) and Sheu et al. (2018) and Sheu et al. (2019).

Many of the products we deal with daily are multi-component systems. Given that in multi-component systems, maintenance is more expensive than in single-component systems, it is worthwhile to discuss their optimal maintenance policy. Various structures have been proposed in reliability engineering for multi-component systems. Among them, k-out-of-n systems are known for their broad applicability. A k-out-of-n system can also be categorised as a k-out-of-n:F or k-out-of-n:G system. A k-out-of-n:F system fails when at least k components have failed while a k-out-of-n:G system works until at least k components are functioning. The widely appeared series and the parallel systems are the k-out-of-n:F systems with $k = 1$ and $k = n$, respectively. Tolerant systems such as oil pipeline systems, computer ring networks, spacecraft relay communication, cables in a suspension bridge, and vacuum systems in accelerators are real-world examples of k-out-of-n systems. In the last decades, extensive studies have been conducted on developing maintenance policies for k-out-of-n systems. For example, De Smidt-Destombes, van der Heijden, and van Harten (2009) developed a heuristic method for joint optimisation of condition-based maintenance frequency, spare parts inventory, and repair capabilities for k-out-of-n systems. In their model, optimisation heuristics is constructed based on approximations. Ruiz-Castro and Li (2011) modelled a discrete k-out-of-n system with multi-state components using a block-structured Markov chain, and units could undergo two types of failures, repairable or non-repairable. But, their proposed model becomes more complex in some situations, and when the distributions involved in the system are general, the results have very complex structures. Levitin, Xing, and Dai (2014) suggested a method for evaluating mission cost in systems with standby elements in 1-out-of-n non-repairable systems. They have some limitations in their model, for instance, the mission time is fixed, and their method is based on discrete approximation of time to failure distributions of the system elements.

Ito, Zhao, and Nakagawa (2017) studied the preventive replacement problems for a k-out-of-n system when k is a random variable. Using the reliability function for k-out-of-n systems, Diallo et al. (2019) formulated two non-linear models for the joint selective maintenance and repair-person assignment problem. They assumed that the components don't age during the break and components age only during operation. Zhang, Fouladirad, and Barros (2019) proposed reliability-based measures and prognostic problems of a k-out-of-n system in which the failure process of each component depends on its intrinsic characteristic and its operating environment conditions. They have some restrictions in their model, for example, the system failure is not self-announcing and can be revealed only by system inspections. Moreover, they derived asymptotic availability instead of exact availability. Sheu et al. (2019) studied two preventive replacement policies including the T-policy and N-policy for a k-out-of-n system. They supposed only two cases for components. The component is either minimally repaired (type 1 failure) or lying idle (type 2 failure). Rykov, Sukharev, and Itkin (2020) used the mathematical models of k-out-of-n systems for analysing the reliability of oil and gas facilities. Zhang et al. (2020) introduced a condition-based maintenance policy of a k-out-of-n deteriorating system with failure dependence. However, they only considered perfect periodic inspections and the supposed degradation of each component follows a pure jump Lévy process. Rykov, Kochueva, and Farkhadov (2021) also studied the preventive maintenance (PM) of pipeline transport underwater monitoring equipment based on the k-out-of-n model. Recently, Safaei, Ahmadi, and Taghipour (2022) proposed a maintenance policy for k-out-of-n systems, which includes an age replacement policy and minor repair by considering safety constraints.

When the replacement policy (planned or unplanned) is carried out for a multi-component system, it is possible that some components of the system have not failed and are still usable, are called second-hand parts. These components can be used again in another system that is not in a high-risk industry. The second-hand components can also be sold to a second-hand consumer buyer. It is logical to consider the price of such components as a function of their lifetime and the system replacement time. This issue is investigated in the related warranty-based literature on second-hand components and maintenance. For example, Kim, Lim, and Park (2015) developed a maintenance policy to sell second-hand products under a free non-renewable inspection/upgrade warranty. They supposed second-hand products had been minimally repaired and derived the optimal number of required inspections and improvement levels to minimise the expected total warranty cost. Some papers taking into account second-hand components sales in cost functions, for instance, refer to Khatab, Diallo, and Sidibe (2017), Lim, Kim, and Park (2019), and Park, Jung, and Park (2020). Darghouth, Chelbi, and Ait-Kadi (2017) proposed a cost model to determine the optimal reliability improvement level for warranted second-hand production equipment. But, their model can be used for second-hand products having an increasing failure rate. Wang, Xie, and Li (2019) investigated the optimal upgrade strategy for second-hand series systems sold with a free repair/replacement warranty. Safaei, Ahmadi, and Taghipour (2022) taken into account the income from selling second-hand parts to find optimal T-policy for k-out-of-n systems. As far as we know, no previous research work has been done on optimal N-policy by considering the sale of second-hand components in the cost function. Moreover, one of the essential concerns of managers and engineers is that they want to reduce the costs imposed on the system while increasing the system efficiency. This goal is achievable by applying an optimal maintenance policy not only minimising the global cost but also enhancing the mean operating time of the system.

In this paper, in the framework of the k-out-of-n:F system, we consider two maintenance actions for ease of convenience to the reader, we call them the first and the second types as follows.

(i) The first type includes minor repairs on non-failed components, such as lubrication, cleaning some parts, alignment, or fixing small defects. It can be considered a minor preventive repair for components and systems. It may be noted that it is assumed that the failed component is detected immediately and minor repairs increase the operating time of the components, namely the components' failure time will be delayed. For given n and k, when the r-th failure occurs ($r = 1, 2, \ldots, k-1$), the PM is implemented, namely all the remaining $n-r$ components (non-failed components) should be minor repaired. This procedure will proceed until $(r+m-1)$th failure occurs, where $m \geq 1$, and $r+m-1 < k$, where the values of r and m are obtained during optimisation process.

(ii) The second type relates to the N replacement policy where the system is replaced after the Nth successive job (PM) or at the failure of the system (corrective maintenance), whichever occurs first. The value of N is also obtained by optimising the objective function.

Moreover, the income from the sale of second-hand parts after the system replacement is taken into account in the cost function. The aim of this study is to minimise the long-run average cost of the system by finding the optimal time for starting and finishing the first type of maintenance (optimal values for r and m), and also the optimal number of successive jobs (optimal value for N). Besides, a constraint including the relative mean operating time (RMOT) is implemented in the optimisation problem. The large value of RMOT, the more efficient the policy is. Accordingly, in the presented model, the long-run average cost will be minimised, and the RMOT will be enhanced by at least $100\alpha\%$ percent.

As an example of such policy in real life industrial framework, we can refer to the maintenance of pipeline networks or wind turbine generators. The pipeline network is constituted from n pipelines and the system failure is considered as being the failure of k pipelines (see, for example, Rykov, Sukharev, and Itkin (2020) and Rykov, Kochueva, and Farkhadov (2021). Indeed, in order to transport gas from point A to point B, the transport is acceptable if a given quantity is transferred. Each pipeline has its maximum capacity of transfer and therefore, when a given number of pipelines are failed, the gas supply does not respond to the demand and the pipeline network can be considered as failed. Maintenance optimisation of pipelines and pipeline networks is of great interest since they permit stable and efficient production and economical benefits. In wind turbines, some subsystems work as a k-out-of-n system. For instance, a parallel topology can be made up of a gearbox, generator, or power converter (McDonald and Jimmy 2016). Also, wind turbines are used for doing some missions, for example, charging for auxiliary power for boats or caravans that may be used as a job for this system (see, De Broe, Drouilhet, and Gevorgian 1999; Lo, Chen, and Chang 2010). Moreover, wind turbines need regular maintenance to avoid failures that are overpriced.

The rest of this paper is given as follows. The model assumptions and descriptions are introduced in Section 2. The cost function for the proposed model is presented in Section 3. The optimisation problem and method are discussed in Section 4. A search algorithm is provided to find the global optimisation, when k or the feasible set is large, we propose a branch and bound algorithm as an alternative approach. Section 5 provides some beneficial numerical results and graphical illustrations. To illustrate the applicability of the proposed maintenance policy, a data set related to wind turbine generator failures is considered as a case study. Section 6 contains some concluding remarks.

2. Model description

This section is devoted to describing the proposed model and the idea behind it. Before that, let us briefly present the notations that will be used for the rest of the paper in Table 1. In this table, 'PM' stands for preventive maintenance.

Let us consider a n-component ($n > 1$) system so that the lifetimes of its component, say $X_1, \ldots, X_n$, are independent and identically distributed coming from the

Table 1. The variables, parameters, and notations of the proposed policy.

Symbol	Definition
X_i	The ith component lifetime
$X_{i:n}$	The ith failure time
Y_i, η_y	Random variable related to the ith PM of the components and its distribution's parameter
D_i, η_D	Random variable related to the ith working time and its distribution's parameter
$Z_{i,j}$	The operation time of the system after the ith failure and the jth PM
L_{wp}	System lifetime without proposed policy
L_p	System lifetime with proposed policy
N	Number of successive jobs in PR
r	When the rth failure occurs, minor repair as a PM action will be started
m	Number of minor repairs
F and R	CDF and RF of the ith component
f_D, F_D and R_D	PDF, CDF, and RF of $D = \sum_{i=1}^{N} D_i$
f_i^y, F_i^y	PDF and CDF of Y_i
$f_{i,j}^z$, $F_{i,j}^z$ and $R_{i,j}^z$	PDF,CDF, and RF of $Z_{i,j}$
α	Lower bound for enhancing the RMOT
c_{upl}	Cost for UR
c_{pl}	Cost for PR
c_l	The cost of minor repairs of each component
$Cost(N, r, m)$	Model cost function
$RE(N, r, m)$	RMOT function
r^*, m^*, and N^*	The optimal values for r, m, and N

cumulative distribution function (CDF) F and the reliability function (RF) R. In addition, we denote $X_{i:n}$ as the variable regarding the ith failure time with the probability density function (PDF) $f_{i:n}$, CDF $F_{i:n}$, and RF $R_{i:n}$. Clearly, $X_{i:n}$'s are in ascending order with probability one, i.e.$X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}$. The system is assumed to be k-out-of-n:F with the lifetime $\phi(X_1, X_2, \ldots, X_n) = X_{k:n}$. Hence, in the absence of any PM actions, the operating time and the mean operating time of the system are $L_{wp} = X_{k:n}$ and $\mathbb{E}(L_{wp}) = \mathbb{E}(X_{k:n})$, respectively.

Suppose the system should operate to fulfil some successive jobs without interruptions. For this system, it would be better to carry out replacement actions after the job is accomplished. The ith job is assumed to have a random duration time D_i which has the CDF G_i and PDF g_i for $i = 1, 2, \ldots$.

We apply the following two types of maintenance jointly:

(i) The first type is PM related to minor repairs before the failure of the system, which is the same as the proposed maintenance policy by Safaei, Ahmadi, and Taghipour (2022). For the convenience of the reader, we recall shortly the main steps. It can be applied as follows.

- At the time of observing $X_{r:n}$ (rth failure) where $r \in \{1, 2, \ldots k-1\}$, as a PM activity, minor repairs have to be done on all the remaining $n-r$ components. By doing this, the operating time of these components will be increased by a random amount, say Y_1, which means that their failure time will be delayed. Let F_1^y be the CDF of Y_1. Hence, the operating time of the sth non-failed component after the minor repair is $X_{s:n} + Y_1$, where $s = r+1, r+2, \ldots, n$.
- As soon as $X_{r+1:n} + Y_1$ (($r+1$)th failure) observed, minor repairs have to be done on the all the remaining $n-r-1$ components. In the same way, the operating time of these components will be increased by the random amount Y_2. Let F_2^y be the CDF of Y_2. Thus, the operating time of the s-th (out of $n-r-1$) non-failed component after the minor repair is $X_{s:n} + Y_1 + Y_2$, where $s = r+2, r+3, \ldots, n$.
- This procedure will proceed until the ($r+m-1$)th failure occurs, $m \geq 1$, and $r+m-1 < k$. Immediately after observing $X_{r+m-1:n} + \sum_{i=1}^{m-1} Y_i$ (($r+m-1$)th failure), minor repairs have to be done on the all the remaining components. Let define Y_m with CDF F_m^y as the random time extension of the $n-r-m$ non-failed components due to this action. Eventually, the operating time of the sth non-failed component after the minor repair is $X_{s:n} + \sum_{i=1}^{m} Y_i$, where $s = r+m, r+m+1, \ldots, n$.
- It is worth mentioning that the mth minor repair is the last PM action applied to the system, so afterward, the operating time of the non-failed components will not change.

We remind that the underlying system is k-out-of-n:F, so the operating time of the system will be changed to $X_{k:n} + \sum_{i=1}^{m} Y_i$, where $r+m-1 < k$. It should be mentioned that the optimal values of r and m will be found by optimisation. Moreover, if $Y_1, Y_2, \ldots, Y_m$ are stochastically small, the effect of PM on the components is small and the components' failure time will be delayed slightly. As a result, the proposed optimisation algorithm selects the optimal parameters of PM (r and m) according to distributions of $Y_1, Y_2, \ldots, Y_m$. The distribution of $Y_1, Y_2, \ldots, Y_m$ will appear in objective function as well as constraints.

(ii) The second type of maintenance is related to the N-policy where the system is replaced at the time after the N-th job is fulfilled (i.e, $\sum_{i=1}^{N} D_i$) or at the failure time of the system, namely $X_{k:n} + \sum_{i=1}^{m} Y_i$, whichever occurs first. We are going to find r, m, and N such that to minimise the overall costs of the proposed plan.

Let $Z_{i,j} = X_{i:n} + \sum_{s=1}^{j} Y_s$ be the occurrence time of the ith failure and so far j times PM have been performed, with PDF, CDF, and RF $f_{i,j}^z$, $F_{i,j}^z$ and $R_{i,j}^z$, respectively, for $i = r+j, r+j+1, \ldots, n$.

The system is replaced when the Nth successive jobs are accomplished or at $X_{k:n} + \sum_{i=1}^{m} Y_i$, whichever occurs first. The time of finishing Nth successive job is $\sum_{i=1}^{N} D_i$. The replacement cycle is considered as the time interval between two replacements of the system due to an unplanned replacement (UR) or by a planned replacement (PR) at the time of finishing Nth successive job. For a better explanation, Figure 1 illustrates the proposed maintenance policy. This figure describes the decision-making procedure of the policy. Notice that the flowchart shows how to implement the proposed policy by managers after finding the optimal r, m and N. We remind that the optimal parameters will be found by minimising a cost function under some constraints.

Under the assumptions of the proposed policy, the time to the replacement of the system (the operating time of the system) is given by

$$L_p = \min\left\{X_{k:n} + \sum_{i=1}^{m} Y_i, \sum_{i=1}^{N} D_i\right\}. \tag{1}$$

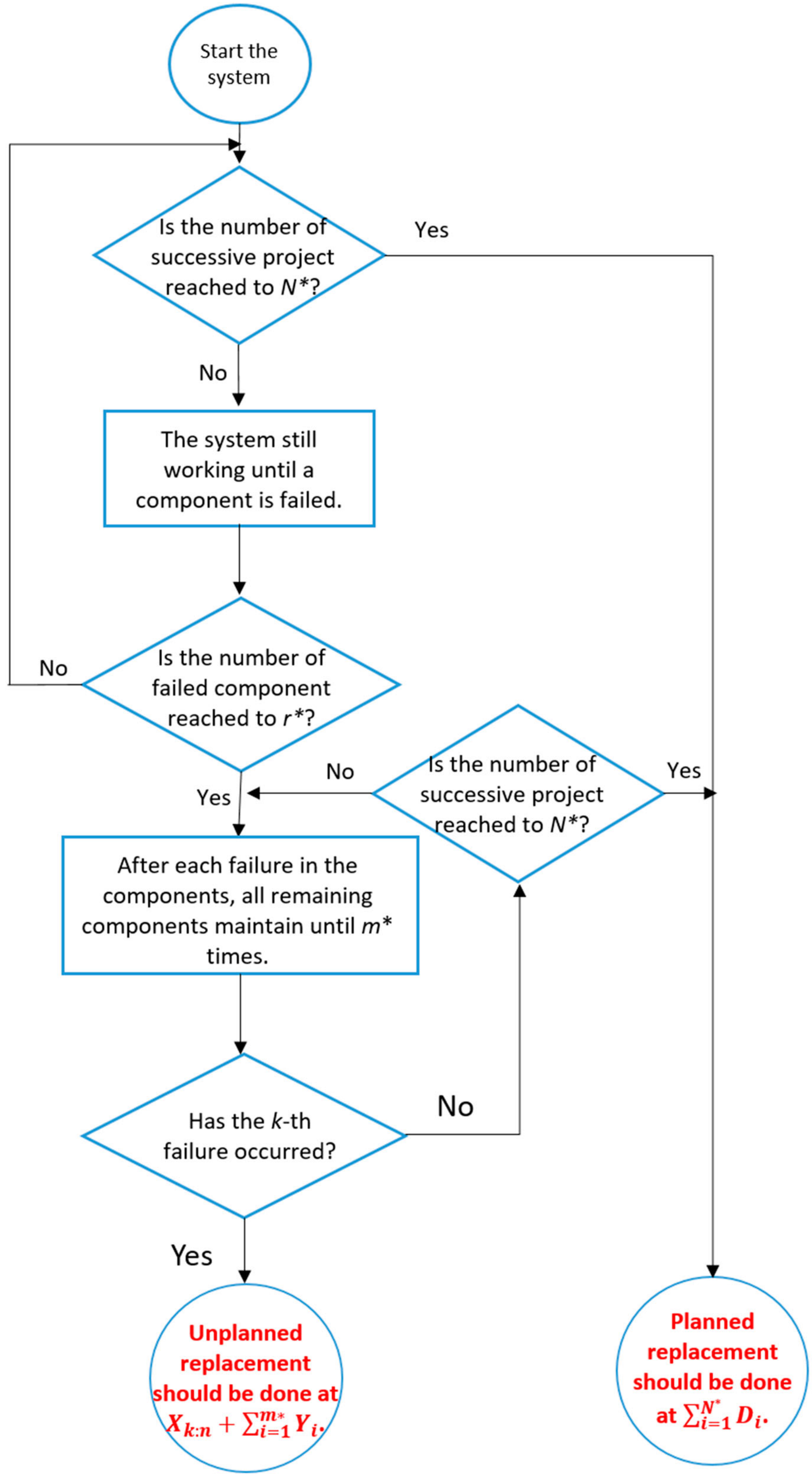


Figure 1. The flowchart to describe the decision-making procedure of the proposed policy.

Take $D = \sum_{i=1}^{N} D_i$ and let us denote by f_D, F_D, and R_D the PDF, CDF,and RF of D, respectively. Then, it is not difficult to show that the mean operating time of the system is as follows:

$$\mathbb{E}(L_p) = \int_0^\infty tR^z_{k,m}(t) f_D(t)\,\mathrm{d}t + \int_0^\infty tR_D(t) f^z_{k,m}(t)\,\mathrm{d}t$$
$$= \int_0^\infty R^z_{k,m}(t) R_D(t)\,\mathrm{d}t, \tag{2}$$

where $Z_{k,m} = X_{k:n} + \sum_{i=1}^{m} Y_i$, and $R^z_{k,m}(t) = R_{k:n}(t) + \int_0^t (1 - F^{*m}(t-x)) f_{k:m}(x)dx$, in which $F^{*m} = F^y_1 * F^y_2 * \ldots * F^y_m$ and the symbol $*$ stands for the convolution operator.

3. Cost function

We first recall the long-run average cost (expected cost rate) from Barlow and Proschan (1996, Chapter 4). Let $C(t)$ be the total cost at time t, the random variable L_p be the length of a replacement cycle, and random variable C_{L_p} be the total cost associated with this replacement cycle. Due to the fact that the sequence of replacement cycles (PR or UR) forms a renewal process, hence we examine one replacement cycle. In this case, it is known that for a cycle the long-run average cost incurred can be written as

$$\lim_{t\to\infty} \frac{C(t)}{t} = \frac{\mathbb{E}(C_{L_p})}{\mathbb{E}(L_p)}. \tag{3}$$

To calculate the cost given in (3), one needs to derive the expected total cost of a cycle for the proposed policy as an explicit expression. To this end, let us define c_{upl} and c_{pl} as the costs regarding the system replacement at the random times $Z_{k,m} = X_{k:n} + \sum_{i=1}^{m} Y_i$ and $\sum_{i=1}^{N} D_i$, respectively. Moreover, denote the minor repair cost of each component by c_I.

Moreover, when a multi-component system is replaced, no matter whether is PR or UR, its components can be released to the market as second-hand products because they are still usable. For instance, they can go back to work as components of another system but probably not in high-risk industries. It is logical to assume the price of these components depends on their lifetime and the system's replacement time. Chattopadhyay and Murthy (2004) developed a purchasing cost model for a second-hand product when it is at a special age. In accordion to the Chattopadhyay and Murthy's model, the item's price in age x $(x < L)$ is $p_0 c_0 (1 - \frac{x}{L})$ where p_0 $(0 < p_0 < 1)$ models the immediate loss in resale value after the sale of a new item, c_0 is the sale price of a new item, and L is the expected lifetime of the new product. In addition, their model states that a component with age $x > L$ can not be sold, so its corresponding income is zero. This model has been used by some authors in studies on the construction of cost functions for the second-hand product (see; Saidi-Mehrabad, Noorossana, and Shafiee 2010). For our PM policy, we applied Chattopadhyay and Murthy's model as follows: the sale price of a new system with n component is c_{pl}; thus, the sale price of a new component is considered to be $\frac{c_{pl}}{n}$. It is also assumed that the components that have worked less than their average lifetime can be released for sale to the market. Hence according to Chattopadhyay and Murthy's approach, we have $L = \mathbb{E}(X)$ where X is the random lifetime of a new component. On the other hand, since the components are assumed to be identically distributed, we have $\mathbb{E}(X) = \mathbb{E}(X_i)$, for $i = 1, 2, \ldots, n$. If the system is replaced at the given time $D = d$ and a component still survives until this time, we suppose the income regarding this component is given by

$$p_0 \frac{c_{pl}}{n} \left(1 - \frac{d}{\mathbb{E}(X)}\right)^+$$

where $a^+ = \max\{0, a\}$. Also, if the system is replaced at the given time $Z_{k,m} = z_{k,m}$, then the income is

$$p_0 \frac{c_{pl}}{n} \left(1 - \frac{z_{k,m}}{\mathbb{E}(X)}\right)^+.$$

For one cycle, if we denote the replacement and minor repairs costs by $C_1(N, r, m)$ and the income from the sale of second-hand parts by $C_2(N, m)$, then the system's cost for one cycle, denoted by $C(N, r, m)$, is given by

$$C(N, r, m) \equiv C_1(N, r, m) - C_2(N, m).$$

To obtain an explicit expression for $C_1(N, r, m)$, we have:

- If replacement is done due to a PR and before starting the repair and minor repair actions, i.e, $D < X_{r:n}$, only the incurred cost of a PR just should be paid, and therefore, $C_1(N, r, m) = c_{pl}$.
- If the system is replaced due to a PR after the first repair and minor repair action and before the second PM and minor repair action, i.e, $X_{r:n} < D < Z_{r+1,1}$, only the incurred cost of a PR and $(n - r)$ PM and minor repair should be paid and therefore, $C_1(N, r, m) = c_{pl} + (n - r)c_I$.
- This procedure will proceed until the $(r + m - 1)$-th failure occurs, where $m \geq 1$, and $r + m - 1 < k$, then the total cost $c_{pl} + c_I \sum_{i=0}^{m} (n - r - i)$ should be paid.
- If the system replaces due to an UR, i.e, $D > Z_{k,m}$, the incurred cost is as follows, $C_1(N, r, m) = c_{upl} + c_I \sum_{i=0}^{m} (n - r - i)$.

Summing-up, we can write the following explicit expression for $C_1(N,r,m)$:

$$C_1(N,r,m) = \begin{cases} c_{pl}, & \text{if } D < X_{r:n} \\ c_{pl} + (n-r)c_I, & \text{if } X_{r:n} < D < Z_{r+1,1} \\ c_{pl} + c_I \sum_{i=0}^{l} (n-r-i), & \text{if } Z_{r+l,l} < D < Z_{r+l+1,l+1},\ 1 \le l < m \\ c_{pl} + c_I \sum_{i=0}^{m} (n-r-i), & \text{if } Z_{r+m,m} < D < Z_{k,m} \\ c_{upl} + c_I \sum_{i=0}^{m} (n-r-i), & \text{if } D > Z_{k,m}. \end{cases} \quad (4)$$

As mentioned earlier, if the system is replaced at a given time $D = d$, the income of selling a component that is still working until this time is $p_0 \frac{c_{pl}}{n}(1 - \frac{d}{\mathbb{E}(X)})^+$. Moreover, the number of components that worked up to time $D = d$ has a binomial distribution $B(n, R(d))$, so its expected value is given by $nR(d)$. Consequently, the total income from the sale of the second-hand parts at a given time can be presented as $p_0 c_{pl} R(d)(1 - \frac{d}{\mathbb{E}(X)})^+$. Otherwise, if the system is replaced at a given time $Z_{k,m} = z_{k,m}$, the income for a component is $p_0 \frac{c_{pl}}{n}(1 - \frac{z_{k,m}}{\mathbb{E}(X)})^+$, and the number of components that still working until this time is $n-k$. Therefore, the total income from selling the second-hand components can be presented as $p_0 c_{pl} \frac{(n-k)}{n}(1 - \frac{z_{k,m}}{\mathbb{E}(X)})^+$. Summing-up, we have

$$C_2(N,m) = \begin{cases} p_0 c_{pl} R(D) \left(1 - \frac{D}{\mathbb{E}(X)}\right)^+, & \text{if } D < Z_{k,m} \\ p_0 \frac{c_{pl}}{n}(n-k)\left(1 - \frac{Z_{k,m}}{\mathbb{E}(X)}\right)^+, & \text{if } D > Z_{k,m}, \end{cases} \quad (5)$$

The average mean of the total cost, denoted by ETC (N,r,m), is given by

$$etc(N,r,m) = \mathbb{E}[C_1(N,r,m)] - \mathbb{E}[C_2(N,m)]. \quad (6)$$

From (4), we obtain the following expression for $E(C_1(N,r,m))$,

$$\begin{aligned} \mathbb{E}(C_1(N,r,m)) &= c_{pl}\mathbb{P}(D < X_{r:n}) \\ &+ \left(c_{pl} + (n-r)c_I\right) \\ &\times \mathbb{P}(X_{r:n} < D < Z_{r+1,1}) \\ &+ \sum_{l=1}^{m-1} \left(c_{pl} + c_I \sum_{i=0}^{l}(n-r-i)\right) \\ &\times \mathbb{P}(Z_{r+l,l} < D < Z_{r+l+1,l+1}) \\ &+ \left(c_{pl} + c_I \sum_{i=0}^{m}(n-r-i)\right) \\ &\times \mathbb{P}(Z_{r+m,m} < D < Z_{k,m}) \\ &+ \left(c_{upl} + c_I \sum_{i=0}^{m}(n-r-i)\right) \\ &\times \mathbb{P}(D > Z_{k,m}) \\ &= A_1(r,m) + \int_0^\infty A_2(N,r,m,t) f_D(t)\,\mathrm{d}t, \end{aligned} \quad (7)$$

where

$$A_1(r,m) = c_{upl} + c_I \sum_{i=0}^{m}(n-r-i)$$

and

$$\begin{aligned} A_2(N,r,m,t) &= c_{pl}R_{r:n}(t) + \left(c_{pl} + (n-r)c_I\right) \\ &\times \{R^z_{r+1,1}(t) - R_{r:n}(t)\} \\ &+ \sum_{l=1}^{m-1}\left(c_{pl} + c_I \sum_{i=0}^{l}(n-r-i)\right) \\ &\times [R^z_{r+l+1,l+1}(t) - R^z_{r+l,l}(t)] \\ &+ \left(c_{pl} + c_I \sum_{i=0}^{m}(n-r-i)\right) \\ &\times [R^z_{k,m}(t) - R^z_{r+m,m}(t)] \\ &- \left(c_{upl} + c_I \sum_{i=0}^{m}(n-r-i)\right) R^z_{k,m}(t). \end{aligned}$$

Also, from (5), we obtain

$$\begin{aligned} \mathbb{E}(C_2(N,m)) &= p_0 \frac{c_{pl}}{n}(n-k) \int_0^\infty \left(1 - \frac{t}{\mathbb{E}(X)}\right)^+ \\ &\times f^z_{k,m}(t) R_D(t)\,\mathrm{d}t \\ &+ p_0 c_{pl} \int_0^\infty R(t)(1 - \frac{t}{\mathbb{E}(X)})^+ \\ &\times R^z_{k,m}(t) f_D(t)\,\mathrm{d}t \\ &= B_1(N,r,m) + \int_0^\infty B_2(t) R^z_{k,m}(t) f_D(t)\,\mathrm{d}t, \end{aligned} \quad (8)$$

where

$$B_1(N, r, m) = p_0 \frac{c_{pl}}{n}(n-k) \int_0^\infty \left(1 - \frac{t}{\mathbb{E}(X)}\right)^+ \times f^z_{k,m}(t) R_D(t)\, \mathrm{d}t$$

and

$$B_2(t) = p_0 c_{pl} R(t)(1 - \frac{t}{\mathbb{E}(X)})^+.$$

For the proposed policy, by using (2), (7), and (8), the long-run average cost can be presented as follows

$$Cost(N, r, m) = \frac{\begin{array}{c} A_1(r, m) - B_1(N, r, m) \\ + \int_0^\infty [A_2(N, r, m, t) \\ -B_2(t) R^z_{k,m}(t)] f_D(t)\, \mathrm{d}t \end{array}}{\int_0^\infty R^z_{k,m}(t) R_D(t)\, \mathrm{d}t}. \quad (9)$$

We are interested in finding the optimal value of (N, r, m) so that the cost function in (9) be minimised.

4. Optimization

An interesting feature of a maintenance policy, adding to ensuring the minimum cost, is the guarantee of a minimum specified amount of increase in system performance. To address this issue, the novel measure RMOT proposed by Safaei, Ahmadi, and Taghipour (2022) is applied. This measure quantifies the relative difference between the mean operating time of the system in the presence and absence of PM action. The larger value of this measure, the more efficient policy is. Now, if an enhancement of at least size $100\alpha\%$ ($\alpha > 0$) is aimed in the mean operating time of the system after PM implementation, i.e. $\mathbb{E}(L_p) > (1+\alpha)\mathbb{E}(L_{wp})$, then we have

$$RE \equiv 100\% \frac{\mathbb{E}(L_p) - \mathbb{E}(L_{wp})}{\mathbb{E}(L_{wp})} > 100\alpha\%.$$

Accordingly,

$$RE(N, r, m) = 100\% \frac{\int_0^\infty R^z_{k,m}(t) R_D(t)\, \mathrm{d}t - \mathbb{E}(X_{k:n})}{\mathbb{E}(X_{k:n})} > 100\alpha\%.$$

Finally, the optimal value for r, m, and N will be obtained by minimising the following cost function with the given constraint

$$\underline{\text{minimize}} \quad Cost(N, r, m) = \frac{\begin{array}{c} A_1(r, m) - B_1(N, r, m) \\ + \int_0^\infty [A_2(N, r, m, t) - B_2(t) R^z_{k,m}(t)] f_D(t)\, \mathrm{d}t \end{array}}{\int_0^\infty R^z_{k,m}(t) R_D(t)\, \mathrm{d}t}, \quad (10)$$

$$\underline{\text{subject to}} \quad RE(N, r, m) = 100\% \frac{\int_0^\infty R^z_{k,m}(t) R_D(t)\, \mathrm{d}t - \mathbb{E}(X_{k:n})}{\mathbb{E}(X_{k:n})} > 100\alpha\%,$$
$$r \in \{1, 2, \ldots, k-1\},\ m \in \{1, 2, \ldots, k-r\},\ N \in \{1, 2, \ldots\}. \quad (11)$$

As it can be seen from (10) to (11), we are dealing with a discrete optimisation problem. In our proposed policy, the domain for the parameter r is limited ($r = 1, 2, \ldots, k-1$) and for the parameter m is $\{1, 2, \ldots, k-r\}$, and finally, the domain for parameter N is $\{1, 2, \ldots\}$. So the search algorithm is used for global optimisation. The statistical software **R** version 4.0.3 is used to obtain the results and due to applying a search algorithm, it is supposed that $N \in \{1, 2, \ldots, N_0\}$, where N_0 is a large enough number. We remind that a search algorithm tries to solve the optimisation problem by searching along the space of the problem domain subject to constraints. With this in mind, for given k and n, we propose Algorithm 1 (below) to find the optimal values of r^*, m^*, and N^*.

Algorithm 1: Finding optimal values of r^*, m^* and N^* such that $Cost(N^*, r^*, m^*) = \min_\Omega Cost(N, r, m)$ and $RE(N, r, m) > 100\alpha\%$, where $\Omega = \{(N, r, m) : N \in \{1, 2, \ldots, N_0\}, r \in \{1, 2, \ldots, k-1\}, m \in \{1, 2, \ldots, k-r\}\}$.

Input : A set of integers r, m and N
Output: Optimal r^*, m^* and N^*

1 $r \longleftarrow 1$
2 $m \longleftarrow 1$
3 $N \longleftarrow 1$
4 Compute $Cost(N, r, m)$ and $RE(N, r, m)$
5 **if** $RE(N, r, m) > 100\alpha\%$ **then**
6   Save $(N, r, m, Cost(N, r, m))$ in feasible set;
7 **end**
8 **if** $N \leq N_0$ **then**
9   Set $N = N + 1$ and go to *Step 4*;
10 **end**
11 **if** $m \leq k - r$ **then**
12   Set $m = m + 1$ and go to *Step 3*;
13 **end**
14 **if** $r \leq k - 1$ **then**
15   Set $r = r + 1$ and go to *Step 2*;
16 **end**
17 Consider the saved set in *Step 6*, and select min $Cost(N, r, m)$ and corresponding (N, r, m) say (N^*, r^*, m^*).

Exhaustive search strategies are known to evaluate all the possible candidates for the problem's solution to find the satisfying solution(s). So, when k or the feasible set is large, we propose a branch and bound algorithm as an alternative that can be used to find approximately optimal solutions. It is known that this algorithm does not need to check all possible combinations to find the optimal values. More details are provided in Section 5.5.

5. Numerical computations

The theoretical matters regarding the proposed policy have been derived and presented in the previous sections. The current section provides a numerical study to obtain the optimal policies and investigate the effect of model parameters. The effect of the constraint (11) is also discussed. Also, a data set related to wind turbine generator failures is considered a case study.

5.1. Optimal solutions

Let us consider a k-out-of-n:F system where the lifetime of its components is Weibull random variables with parameters (λ, β), i.e. $F(x) = 1 - e^{-(\frac{x}{\lambda})^{\beta}}$, $x > 0, \lambda > 0, \beta > 0$. Also the duration for a duty which is D_i follows an exponential distribution with parameters η_D and Y_i have an exponential distribution with parameter η_y. Let us take

$$M_1 := \{n = 10,\ k = 6,\ c_{upl} = 90,\ c_{pl} = c_0 = 70,\ c_I = 1,\ \eta_y = 0.15,\ \eta_D = 0.3,\ \alpha = 0.1,\ \beta = 3,\ \lambda = 2,\ p_0 = 2\}. \tag{12}$$

By considering the given parameters in (12), and using Algorithm 1, we have obtained $Cost(N^*, r^*, m^*) = 47.9360$, and $RE(N^*, r^*, m^*) = 17.31\%$ for $r^* = 3$, $m^* = 3$, and $N^* = 10$. Therefore, if the system doesn't fail before 10th successive job, it should be replaced when the 10th job is accomplished. Since $r^* = 3$, so, as soon as the third failure occurs, all remaining components $(n-3)$ should be minor repaired. Moreover, since $m^* = 3$ the minor repairs are carried out at the time of the third, fourth and fifth failures. Thus, the total incurred cost 47.9360 and the RMOT is improved by 17.31%. This means that by paying 47.9360 cost unit per time unit, the operator will increase the mean operating time of the system by 17.31% compared to the case where no preventive action is implemented.

As another set of parameters, let us take

$$M_2 := \{n = 10,\ k = 6,\ c_{upl} = 90,\ c_{pl} = c_0 = 70,\ c_I = 1,\ \eta_y = 0.3,\ \eta_D = 0.3,\ \alpha = 0.1,\ \beta = 3,\ \lambda = 2,\ p_0 = 2\}. \tag{13}$$

By (13), we have obtained $Cost(N^*, r^*, m^*) = 38.3503$, and $RE(N^*, r^*, m^*) = 75.64\%$ for $r^* = 1$, $m^* = 5$, and $N^* = 16$. Therefore, based on the proposed model, if the system doesn't fail before 16th successive job, it should be replaced when the 16th job is accomplished. Since $r^* = 1$, so, as soon as the first failure occurs, all remaining components $(n-1 = 9)$ should be minor repaired. Moreover, since $m^* = 5$ the minor repairs are carried out until the failure of the fifth component. Thus, the total incurred cost 38.3503 and the RMOT is improved by 75.64%. This means that by paying 38.3503 cost unit per time unit, the operator will increase the mean operating time of the system by 75.64% compared to the case where no preventive action is implemented.

Figure 2 displays the cost function for a feasible set when the parameters are as given in (13). The points on Figure 2 are obtained based on Algorithm 1, taking into account the range of r, m and N given by $r \in \{1, 2, \ldots, k-1\}$, $m \in \{1, 2, \ldots, k-r\}$, $N \in \{1, 2, \ldots N_0\}$, where N_0 is a large enough number. Let us explain how this graph is created. For given fixed k and n, for each triple of (N, r, m) started from $(1, 1, 1)$, which satisfies a constraint $RE(N, r, m) > 100\alpha\%$, a positive integer number $x_{(N,r,m)}$, started from 1, is assigned and displayed on the X-axis. So, each integer number $x_{(N,r,m)}$ shows an element in a feasible set. Eventually, the cost function $Cost(N, r, m)$ for each element in the feasible set is calculated and stated on the Y-axis of the plot. For clarity, Figure 3 plotted that represents a work-flow for finding coordinates of points.

5.2. Sensitivity analysis

We also studied numerically the impact of the model parameters on r^*, m^*, N^*, $Cost(N^*, r^*, m^*)$ and $RE(N^*, r^*, m^*)$. The results are presented in Tables 2–6, also displayed in Figure 4.

Figure 4 displays the variations of the feasible region subject to α, other parameters are fixed as given in (12). Let $\alpha \in \{0.1, 0.3, 0.4\}$, the feasible sets which take into account the constraint $RE(N, r, m) > 100\alpha\%$ are shown in Figure 4. Similar to Figure 2, for each triple of (N, r, m) started from $(1, 1, 1)$, which satisfies constraint $RE(N, r, m) > 100\alpha\%$, a positive integer number $x_{(N,r,m)}$, started from 1, is assigned and displayed on the X-axis. Finally, the cost function $Cost(N, r, m)$ for each element in the feasible set is calculated and stated on the Y-axis of the plot. Based on Figure 4, when α increases the feasible region decreases.

When $\alpha = 0.1$ (i.e. $RE(N, r, m) > 10\%$), the feasible region is shown in Figure 4(a). The optimal N, m, and r are $N^* = 10$, $m^* = 3$, and $r^* = 3$, respectively, and also $Cost(N^*, r^*, m^*) = 47.9360$. If we consider $\alpha = 0.3$, the previous optimal point doesn't belong to the feasible region and we should select the minimum cost in

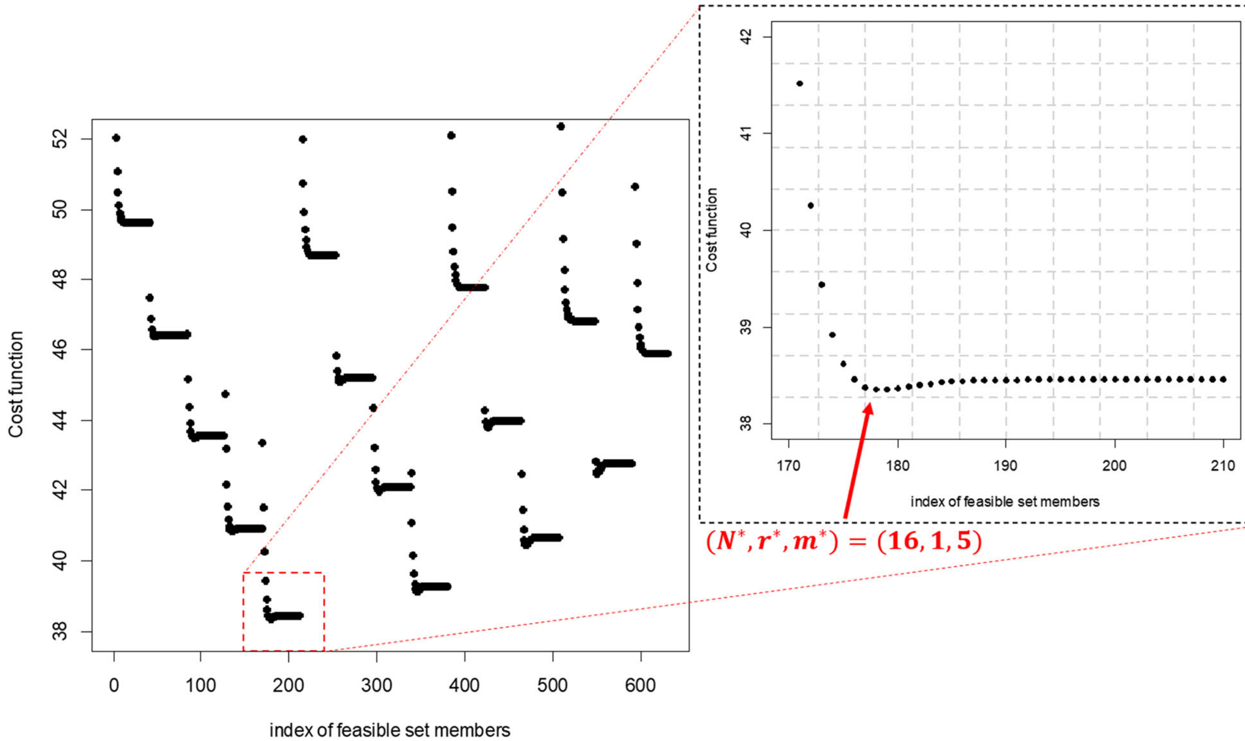


Figure 2. Cost function for feasible set.

the corresponding region which will lead to $N^* = 13$, $m^* = 4$, $r^* = 2$ and $Cost(N^*, r^*, m^*) = 48.5878$. Finally, if we consider $\alpha = 0.4$, the feasible region is smaller than the previous ones and the previous optimal points do not belong to this feasible region and we should select the minimum cost in the corresponding region which will lead to $N^* = 17$, $m^* = 5$, $r^* = 1$ and $Cost(N^*, r^*, m^*) = 49.4655$.

To assess the impact of η_D, η_y and p_0 on the optimal solutions, the other parameters are fixed as given in (12). Results are presented in Table 2. Let us remind that, η_y is the mean of the amount of delay in failure due to PM ($\mathbb{E}(Y_i)$). Table 2 shows that, r^* decreases and m^* increases as η_y increases. Moreover, the minimum cost function decrease when η_y increases. So, by fixing the other factors if η_y increases (PM has more impact on the amount of delay in failure), we should perform more PM action. Also, N^* decreases as η_D, the mean of the time for finishing a job ($\mathbb{E}(D_i)$), increases. Hence when η_D increases the jobs are time-consuming and less jobs are accomplished. In addition, the optimal cost function grows with η_D. Table 2 also shows that, N^* decreases as the immediate loss in resale value subsequent to the sale of a new item (p_0) increases and has no significant effect on r^* and m^*.

The impact of c_{pl}, c_{upl} and c_I on the optimal solutions is presented in Table 3. Table 3 shows that, m^* and N^* increase and r^* decreases as UR cost (c_{upl}) increases. This indicates that when UR are expensive, the number of PM and jobs carried out before replacement increases. As expected, the minimum cost increases.

Table 3 also shows that, $Cost(N^*, r^*, m^*)$, N^* and r^* increase as PR cost (c_{pl}) increases. This indicates that when PRs are expensive, they should be carried out later (N^* increases) and the optimal number of PM (m^*) decreases to prevent UR, and above all $RE(N^*, r^*, m^*)$ increases. In this table, the behaviour of r^*, m^*, N^*, $Cost(N^*, r^*, m^*)$ and $RE(N^*, r^*, m^*)$ with respect to the cost of minor repair (c_I) are also examined. When c_I increases, r^* and the optimal long-run average cost increase and m^* and N^* decrease. So, when the minor repairs are expensive, fewer minor repairs should be done and the system can perform fewer job.

Similarly, Table 4 depicts the impact of λ and β on optimal solutions. As λ and β grow, the lifetime of components grows and therefore less replacements will be carried out. That is why the total costs $Cost(N^*, r^*, m^*)$ and $RE(N^*, r^*, m^*)$ both decrease. In addition, the optimal N^* decreases as β increases while N^* increases when λ increases.

Although the aim of this study is not to investigate the structure of the system, namely to determine the optimal k and n, however, in Tables 5 and 6, sensitivity analysis is also analysed to the parameters of the system structure. Table 5 shows that, r^*, m^* and N^* increase as the number of failed components for failing the system (the system is k-out-of-n) increases and also the minimum

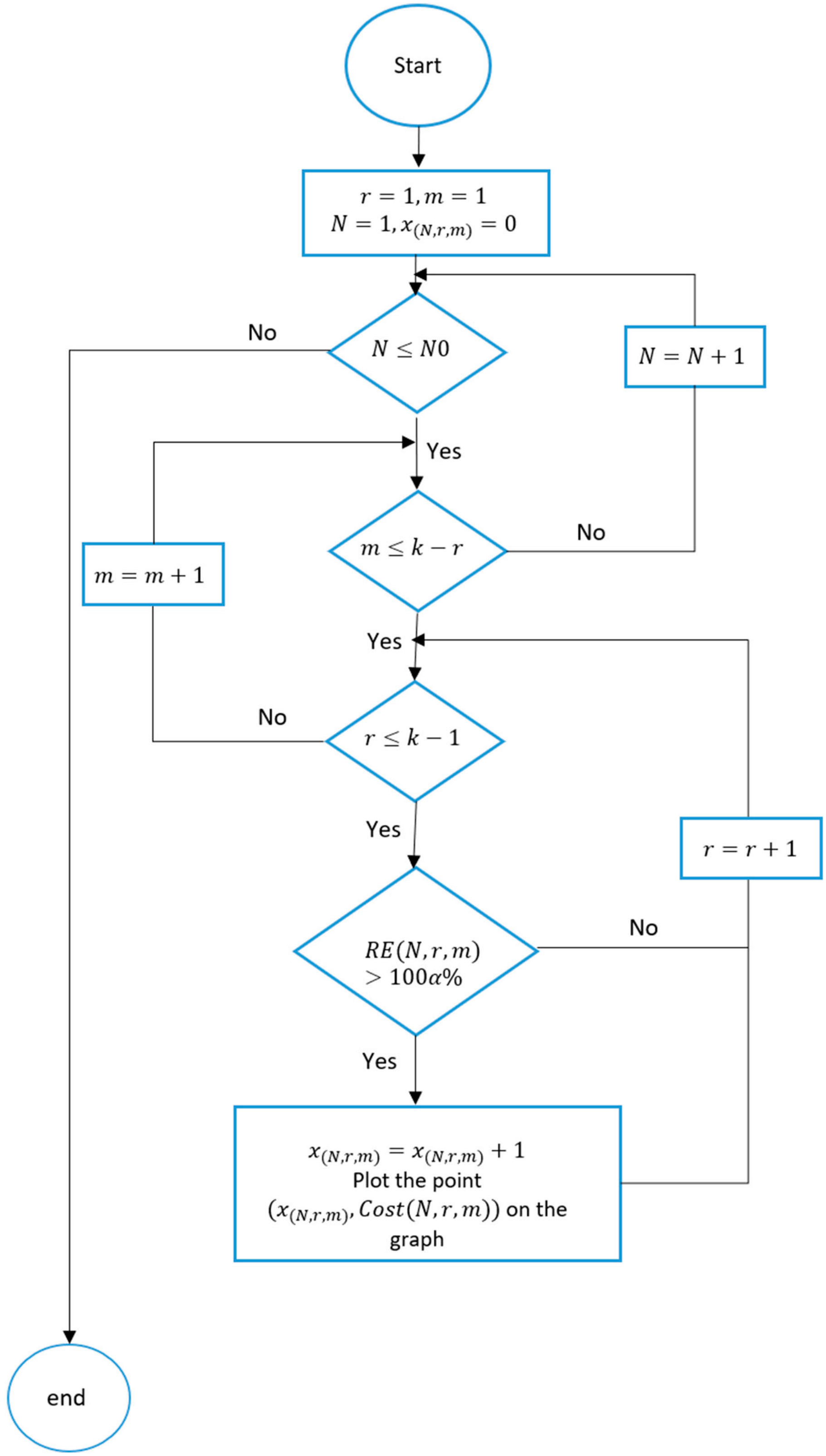


Figure 3. The steps in a process for finding coordinates of points in Figure 2.

Table 2. The effect of η_D, η_y and p_0 on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$	$RE(N^*, r^*, m^*)$
	0.05	3	3	44	46.5663	10.61%
	0.1	3	3	23	47.0698	11.19%
η_D	0.7	3	3	6	48.3502	20.61%
	1	3	3	5	48.4461	21.49%
	1.5	3	3	4	48.5198	21.78%
	0.05	2	4	13	58.1867	10.30%
η_y	0.15	3	3	10	47.9360	17.31%
	0.3	1	5	16	38.3503	75.64%
	1.2	1	5	50	16.4363	320.81%
	0.01	3	3	10	48.0024	17.31%
p_0	0.2	3	3	10	47.9360	17.31%
	0.5	3	3	9	47.7241	13.14%
	0.8	3	3	9	47.5118	13.14%

Table 3. The effect of c_{pl}, c_{upl} and c_l on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$	$RE(N^*, r^*, m^*)$
	80	4	2	12	44.0217	14.65%
c_{upl}	90	3	3	10	47.9360	17.31%
	110	2	4	8	53.3934	10.81%
	170	1	5	8	65.1276	13.92%
	50	2	4	8	43.7394	10.81%
c_{pl}	60	3	3	9	46.2114	13.14%
	70	3	3	10	47.9360	17.31%
	80	3	3	15	48.5779	23.89%
	0.05	1	5	12	35.0930	35.33%
c_l	0.1	1	5	12	35.8342	35.33%
	1	3	3	10	47.9360	17.31%
	2	4	2	10	54.4953	11.31%

cost decreases. Similarly, Table 6 presents the impact of n on optimal solutions. Also, r^* and N^*, the minimum cost increase and m^* decreases when the number of components of system increases.

5.3. Without constraint

When we removed the constraint from the optimisation problem, as we expected, the set in which we were looking for the minimum cost has become larger. As a result, the minimum cost function becomes less than or equal to the case with a constraints optimisation problem. In short, two cases occur:

Case I. The optimal solution changes and the minimum value of the cost function becomes smaller.

Case II. The optimal solution and the minimum value of the cost function does not change.

For example, let us take the parameters as given in (12) except the parameter related to the constraint, α. By minimising the cost function we have $Cost(N^*, r^*, m^*) =$

Table 4. The effect of λ and β on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$	$RE(N^*, r^*, m^*)$
	1.5	2	4	10	59.9520	38.54%
λ	2	3	3	10	47.9360	17.31%
	2.5	4	2	13	39.5921	10.55%
	3	4	2	17	33.8411	10.05%
	2	3	3	11	49.4336	20.62%
β	2.5	3	3	10	48.5676	17.58%
	3	3	3	10	47.9360	17.31%
	3.5	3	3	9	47.4060	12.86%

Table 5. The effect of k on optimal solution.

k	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$	$RE(N^*, r^*, m^*)$
3	1	2	8	68.0697	16.28%
4	2	2	8	59.7513	11.50%
5	3	2	9	53.3614	11.46%
6	3	3	10	47.9360	17.31%

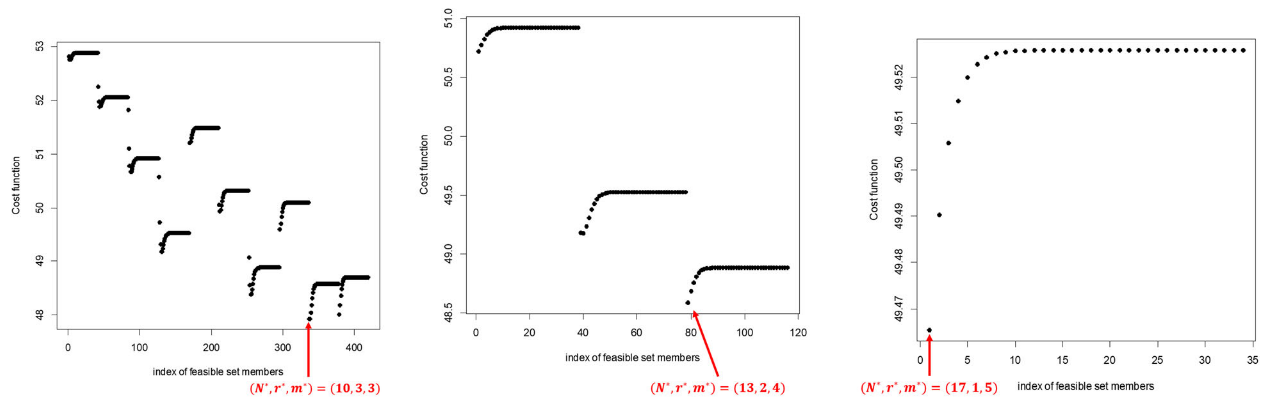
Table 6. The effect of n on optimal solution.

n	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$	$RE(N^*, r^*, m^*)$
10	3	3	10	47.9360	17.31%
20	4	2	7	76.4311	11.04%
30	5	1	36	102.1069	12.68%

47.9099, for $r^* = 4$, $m^* = 2$, and $N^* = 9$, which is different from considering the constraint case and the minimum cost function is less than the previous one. But if we consider the parameters as given in (13), then $r^* = 1$, $m^* = 5$, and $N^* = 16$, and $Cost(N^*, r^*, m^*) = 38.3503$ which is equal to the constraint case. See Tables 7–11 and compare to Tables 2–6. For example, Tables 2 and 7 show the impact of η_D, η_y and p_0 on the optimal solutions with and without constraint, respectively. As mentioned, in some cases the optimal parameter and the minimum value of the cost function change.

5.4. Illustrative case study: generators in wind turbines

In order to illustrate the applicability of the proposed plan in a practical case, a data set related to wind turbine generator failures is considered for the case study. Let us first give a brief overview of the importance of wind turbines. The wind is one of the oldest and most widely used sources of electricity. This renewable energy source is one of the promising sources that will be able to meet the growing worldwide demand for electricity. Wind turbines operate on a simple principle: instead of using power to generate wind (like a fan does), they utilise the wind to generate electricity. The wind turns the propeller-like blades of a turbine around a rotor, which spins a generator and creates electricity. Wind turbine needs regular maintenance to stay reliable and available. In the best case turbines are available to generate energy



(a) $RE(N, r, m) > 10\%$.
(b) $RE(N, r, m) > 30\%$.
(c) $RE(N, r, m) > 40\%$.

Figure 4. Sensitivity analysis for α. (a) $RE(N, r, m) > 10\%$. (b) $RE(N, r, m) > 30\%$ and (c) $RE(N, r, m) > 40\%$.

Table 7. The effect of η_D, η_y and p_0 on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$
	0.05	4	2	40	46.2295
	0.1	4	2	21	46.8615
η_D	0.7	3	3	6	48.3502
	1	3	3	5	48.4461
	1.5	3	3	4	48.5198
	0.05	5	1	40	51.8849
η_y	0.15	4	2	9	47.9099
	0.3	1	5	16	38.3503
	1.2	1	5	50	16.4363
	0.01	3	3	10	48.0024
p_0	0.2	4	2	9	47.9099
	0.5	4	2	8	47.5510
	0.8	4	2	8	47.0814

Table 8. The effect of c_{pl}, c_{upl} and c_l on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$
	80	4	2	12	44.0217
c_{upl}	90	4	2	9	47.9099
	110	3	3	8	53.3115
	170	1	5	7	63.1377
	50	2	4	7	43.1069
c_{pl}	60	3	3	8	45.7727
	70	4	2	9	47.9099
	80	3	3	15	48.5779
	0.05	1	5	12	35.0930
c_l	0.1	1	5	12	35.8342
	1	4	2	9	47.9099
	2	4	2	8	53.7465

Table 9. The effect of λ and β on optimal solution.

Parameters	Values	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$
	1.5	2	4	10	59.9520
λ	2	4	2	9	47.9099
	2.5	4	2	10	39.1557
	3	4	2	12	33.1194
	2	3	3	11	49.4336
β	2.5	3	3	10	48.5676
	3	4	2	9	47.9099
	3.5	4	2	9	47.3972

Table 10. The effect of k on optimal solution.

k	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$
3	1	2	8	68.0697
4	2	2	8	59.7513
5	3	2	8	53.2931
6	4	2	9	47.9099

Table 11. The effect of n on optimal solution.

n	r^*	m^*	N^*	$Cost(N^*, r^*, m^*)$
10	4	2	9	47.9099
20	4	2	6	75.3711
30	4	2	5	99.8304

98% of the time (see, for example, Van Bussel and Zaaijer 2001). Many wind turbine powertrains are structured with one torque/speed conversion device (e.g. gearbox) coupled to the generator which in turn is connected to a power converter. Generator is one of the most important parts of a wind turbine that is driven by the high-speed shaft. Copper windings turn through a magnetic field in the generator to produce electricity. Extensive research has been done on the various powertrain configurations (see, Polinder et al. 2006; Tavner et al. 2010). A parallel powertrain topology has at least one of its subsystems (e.g. gearbox, generator, power converter) made up of parallel components so that if a failure occurs in one such parallel subsystem, some power can still be converted by the other subsystems that still function. This parallelism can be introduced in the generator McDonald and Jimmy (2016). Bywaters et al. (2004) and Cotrell (2002) considered parallel powertrains with six parallel generators. A system of parallel generators can help the design of certain power converters to reach different ranges of voltage Astad and Molinas (2010). Some wind turbines are used for applications such as battery charging for auxiliary power for boats or caravans (see, De Broe, Drouilhet, and Gevorgian 1999; Lo, Chen, and Chang 2010). Producing electricity to charge a battery by a turbine can be considered a job for that turbine.

Here, we consider a wind turbine that consists of 6 identical parallel generators that operate independently of each other, as shown in Figure 5. In fact, we have 6-out-of-6:F (parallel) system for the generator part in a wind turbine. Suppose a maintenance service centre is responsible for maintaining the generator section (including of 6 parallel generators) such that it can charge N batteries. Charging each battery is considered a job. When each of the generators fails, PM (minor repair) can be done for the remaining generators. This maintenance service centre is interested in determining the optimal number of batteries to be charged (number of jobs) as well as the optimal time to begin maintenance and end it for the generator part. To illustrate, we consider the failure data of generators in the particular type of 600 kW wind turbine used in Andrawus, Watson, and Kishk (2007). The data were extracted from Supervisory Control and Data Acquisition (SCADA) systems over a period of 9 years. By using the ReliaSoft Weibull ++7 software, the authors showed that the Weibull distribution, with shape parameter $\beta = 1.1$ and scale parameter $\lambda = 17541$ (48.058 years), is adequate to fit the failure data (see, Table 5 in Andrawus, Watson, and Kishk 2007). As they presented the direct cost (planed replacement) for a generator part and for replacing (corrective maintenance) in catastrophic failure (UR) are 23441$ and 35965$, respectively. So, based on our notations, we have $n = 6$, $k = 6$, $c_{upl} = 35965\$$, $c_{pl} = 23441\$$, $\eta_D = 0.2$, $\beta = 1.1$, $\lambda = 48.058$. For other parameters let us take $\alpha = 0.1, p_0 = 2, \eta_y = 5, c_I = 10$. Then, from (10), (11), and using Algorithm 1, we obtained $r^* = 1$, $m^* = 5$ and $N^* = 773$, $Cost(N^*, r^*, m^*) = 269.9103\$$ and $RE(N^*, r^*, m^*) = 12.50\%$. Therefore, based on the proposed model, the generator part should be replaced at the completion of the 773th job or at the time of system failure whichever occurs first. Since $r^* = 1$, so, as soon as the first failure occurs, all remaining components (5 components) should be minor repaired. Moreover, since $m^* = 5$ the minor repairs are carried out until the failure of the fifth component. Thus, the total incurred cost 269.9103$ and the RMOT is improved by 12.50%. This

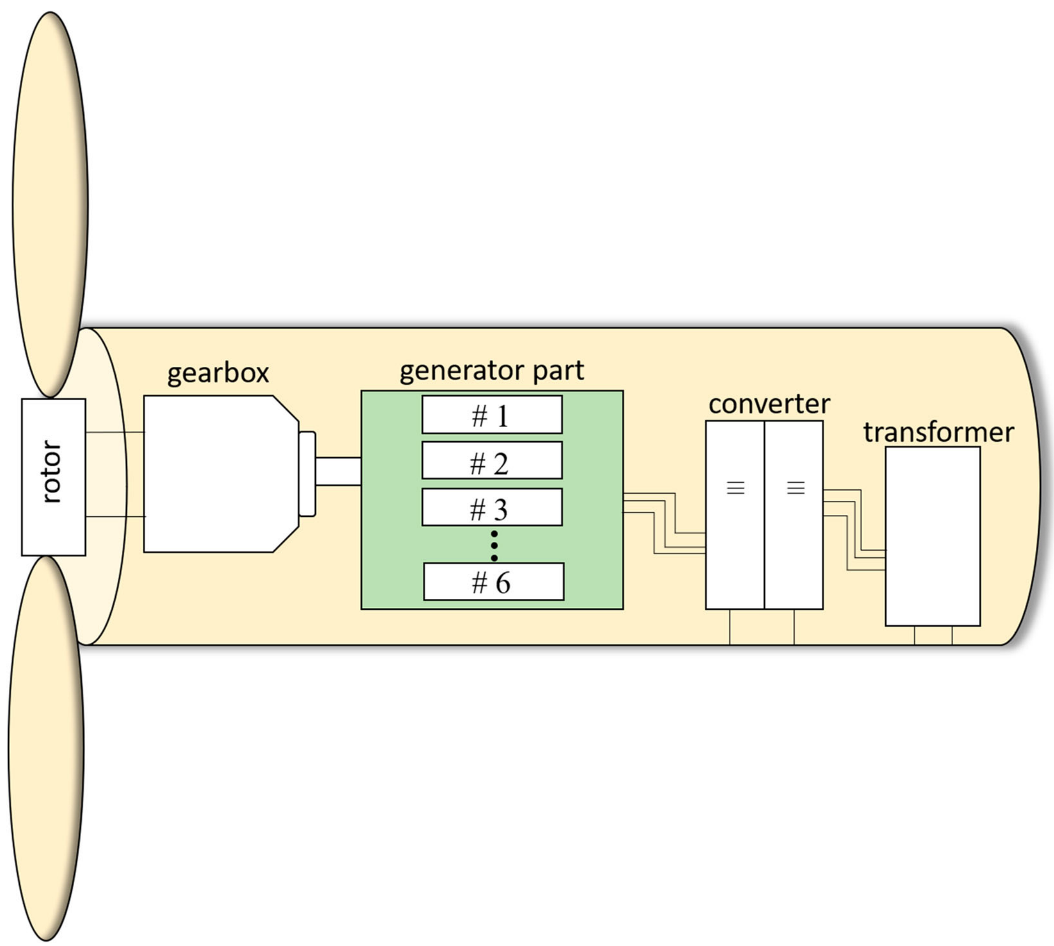


Figure 5. Wind turbine configuration.

means that by paying 269.9103$ per year, the operator will increase the mean operating time of the generator part by 12.50% compared to the case where no preventive action is implemented.

5.5. Alternative algorithm: branch and bound

In our proposed policy, the objective function and constraints are a function of the parameters that take the integer values. In the previous sections, we used search algorithm (Algorithm 1), which checks all possible combinations for the parameters (r, m, and N) and finds the optimal solution for the optimisation problem. In this section, an algorithm called 'Branch and Bound Algorithm' will be implemented. This algorithm does not need to check all possible combinations to find the optimal values for r, m, and N.

Branch and bound which is a method for handling discrete problems is an algorithm design paradigm for discrete and combinatorial optimisation problems, as well as mathematical optimisation. The method was first proposed by Ailsa Land and Alison Doig whilst carrying out research at the London School of Economics sponsored by British Petroleum in 1960 for discrete programming (Land and Doig 2010) and has become the most commonly used tool for solving NP-hard optimisation problems. The name 'branch and bound' was first used in Little et al. (1963) on the travelling salesman problem. The branch and bound strategy work by developing a tree structure. Initially, at the root of the tree, only one discrete variable is allowed to take on discrete values: other discrete variables are modelled as continuous. At each level in the tree one more discrete variable is made discrete.

In our proposed policy, there are 3 discrete variables namely r, m, and N which variable r has $k-1$ possible discrete values, variable m has $k-r$ possible discrete values, and variable N is a positive integer value. To compare the performance of the two algorithms, we consider the parameters as given in (12), then the corresponding branch and bound tree becomes like Figure 6.

In Figure 6, in 'Level 1' variable r is allowed to be discrete and variables m and N are continuous. For 'Level 2', variables r and m are discrete; only variable N is continuous. In 'Level 3', all variables are discrete. The number shown at the upper of each circle is the optimum cost function for the optimisation problem. In 'Level 3', we selected discrete value neighbourhoods around the continuous optimum. An asterisk means no feasible solution could be found to the optimisation problem; a double underscore indicates the branch was pruned. The continuous optimisation problems have been solved by `mle2` function of `bbmle` package in **R** software. The results of the branch and bound algorithm are the same as we obtained by using search algorithm (Algorithm 1) given in Section 5.1. Let us also check an example when k is enough large. Suppose we have a 60-out-of-100:F system such that its components have Weibull distribution with

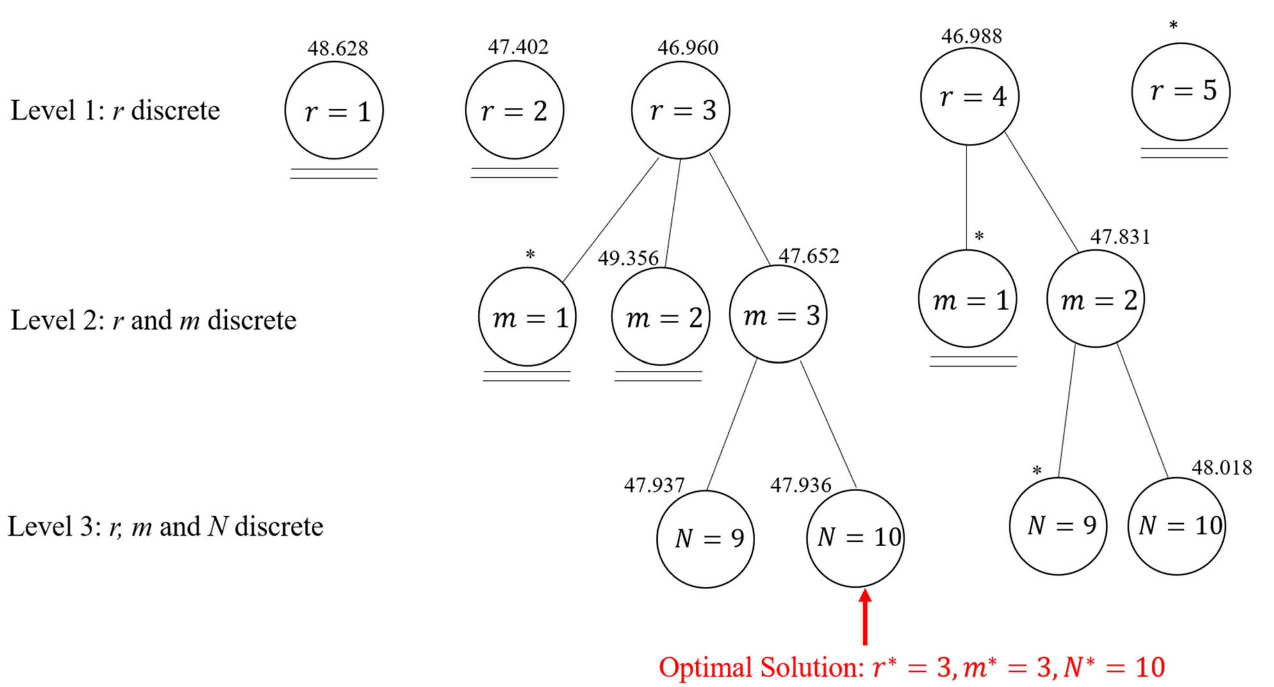


Figure 6. Branch and bound tree.

parameters (λ, β). Let us take

$$c_{upl} = 900, \quad c_{pl} = c_0 = 700, \quad c_I = 1, \quad \eta_y = 0.15,$$
$$\eta_D = 0.07, \quad \alpha = 0.1, \quad \beta = 3, \quad \lambda = 2, \quad p_0 = 2.$$

Then, using branch and bound algorithm, we have obtained $r^* = 42$, $m^* = 18$, and $N^* = 60$ also $Cost(N^*, r^*, m^*) = 380.182$ and $RE(N^*, r^*, m^*) = 109.25\%$. Therefore, if the system doesn't fail before 60th successive job, it should be replaced when the 60th job is accomplished. Since $r^* = 42$, so, as soon as the 42th failure occurs, all remaining components $(100-42)$ should be minor repaired.

Summing-up, when dealing with optimisation problems with a large feasible set, as the search algorithm examines all possible combinations, with this in mind the branch and bound algorithm is preferred because in less time we will reach an appropriate approximate solution. We remind that branch and bound algorithm gains efficiency by pruning branches of the tree that have higher cost functions. In fact, the majority of branches are pruned.

6. Concluding remarks

In this paper, the problem of optimal N-policy to maintain of k-out-of-n:F system has been tackled. Two types of maintenance actions are jointly discussed. The first is related to minor repairs and the second PM is related to the N replacement policy. The cost of the proposed policy has been constructed based on the income from the sale of the second-hand components along with the planned and unplanned costs. This cost is considered the objective cost function of the optimisation problem. A constraint related to enhancing the RMOT is also added to the optimisation problem. Accordingly, the proposed policy is optimised to minimise the long-run average cost while the RMOT is enhanced by at least $100\alpha\%$ percent. The aim is to minimise the long-run average cost of the system by finding the optimal number of successive jobs before PR (optimal N), and the optimal time for starting and finishing the first type of PM (optimal r and m).

The numerical computation showed that, m^* increases as UR cost increases, so when UR are expensive, the number of PM increases. As expected, the minimum cost increases. Moreover, $Cost(N^*, r^*, m^*)$, r^* and N^* increases as PR cost increases, hence when PRs are expensive, they should be carried out later and the optimal number of PM increases to prevent early PRs. For cost of minor repairs, based on numerical computation insight, when c_I increases, the optimal value r^* increases and the optimal m^* and N^* decrease. In this case, the optimal long-run average cost increases.

The numerical computation also appeared as λ and β grow, the lifetime of components grows and therefore replacements will be carried out. That is why the total costs $Cost(N^*, r^*, m^*)$ and $RE(N^*, r^*, m^*)$ both decrease. In addition, the optimal N^* decreases as β and λ increases.

The following issues can be considered for future research in this area:

- It is assumed that the components are independent, this assumption can be extended to the dependent cases. The copula-based approaches can be used to describe the model.
- There are various maintenance policies in the literature, and each strategy has its characteristics, advantages, and disadvantages. A predictive maintenance strategy could be considered for the maintenance of such systems.

Acknowledgments

The authors would like to thank the Editor, AE and reviewers for their useful comments and suggestions on the previous version.

Data availability statement

All data generated or analysed during this study are included in this paper.

Disclosure statement

The authors declare no conflict of interest.

Funding

The second author's research is partially supported by a Grant from Ferdowsi University of Mashhad [grant number 1/56350.].

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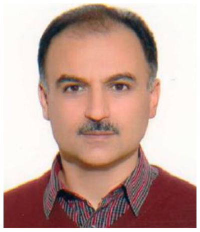

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