Opportunistic perfect preventive maintenance policy in presence of masked data

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Abstract
In this paper, the maintenance optimization problem of multi-component system is considered. It is assumed that the exact cause of system failure might be masked. That is, the exact cause of failure is unknown, and we only know that it belongs to a set called mask set. Both opportunistic perfect preventive maintenance (OPPM) and perfect corrective maintenance are considered. Threshold of OPPM and inter-inspection interval are considered as decision parameters which are optimized using long-run cost rate criteria. The applicability of the proposed maintenance policy is investigated using an illustrative example.

Keywords
Multi-component system, masked data, opportunistic maintenance, corrective maintenance, long-run cost rate

Introduction

Maintenance activities are related with inspection, service, repair, and replacement of system. The goal of maintenance is to ensure the maximum efficiency and availability of the system at an optimal cost under safety. Generally, maintenance actions are classified into two categories: corrective maintenance (CM) where maintenance activities are carried out when the system is failed and preventive maintenance (PM) where maintenance activities are performed when the system is operating. Also, there are different types of preventive and corrective maintenance, for more details one is referred[1–5] and references therein.

Both preventive maintenance (PM) and corrective maintenance (CM) are classified based on the impact incurred by the system/unit after maintenance action as follows: perfect maintenance (PEM) which restores the system to an as good as new (AGAN) state, minimal maintenance (MM) which restores the system to an as bad as old (ABAO) state, imperfect maintenance (IM) which restores the system to a state between AGAN and ABAO state, worse maintenance which makes the system weaker than its state just before failure and worst maintenance which breaks down the system unintentionally.[6]

Maintenance of multi-component systems is different from single-unit systems because of economic, structural, and/or failure (correlated failures) dependency between components.[7,8] Thus the emergence of new categories of repairs which could support these dependencies has been the next step in classification development of maintenance. This class is based on combination of PM and CM and called opportunistic maintenance (OM). Cavalcante and Lopes[9] defined OM as a systematic method of collection, investigation, and preplanning activities for generating a set of maintenance tasks to act on in the occurrence of an opportunity. OM contains any maintenance actions that could opportunely be done in order to carry out the optimal maintenance policy. According to this definition, OM could be inter-twisted with many other types of maintenance. This combined model saves set-up costs using corrective maintenance which is not possible in planned maintenance but it is often not known in

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advance which actions are taken, and then no planning and no work preparation are possible.[10]

The opportunistic maintenance has been first studied by Radner and Jorgenson,[11] McCall,[12] and Jorgenson and McCall.[13] Since, budgets and costs were been more important over time, the advantage of OM overcame its disadvantages (no planning and no work preparation) and it has been developed in considerable academic and industrial works Such as Berg,[14,15] Vu et al.,[16] Yuan et al.,[17] Zhao et al.,[18] Yang et al.,[19] Cavalcante and Lopes,[9] and Mishra et al.[20]

In the reliability analysis of series systems, time to failure, and the exact cause of failure are collected in order to do different statistical analysis such as estimation of the reliability function and maintenance modeling. But, sometimes the exact cause of failure is unidentifiable (because of improper diagnostic equipment storage, or time and cost restrictions) and we only know that the exact cause of failure belongs to a minimum random subset (MRS) of all possible causes. These data are called to be masked.[21,22] It is worth nothing that there is difference between masked and missing cause. In missing data setup there is no information about missed cause while in masking data setup there is some partial information about masked cause, for more details about differences one is referred.[23,24]

In this paper, an opportunity based perfect preventive maintenance policy is considered in presence of masked data. A perfect preventive maintenance action restores an operating deteriorated (non-failed) component to an as good as new (AGAN) state, for instance replacing it by a new one. When the system fails besides the failed component, some endangered components are replaced by a new one thus the perfect preventive replacements are opportunistic. The difference of the presented paper with other existing ones could be mentioned as considering optimal maintenance policy in presence of masked data and applying opportunistic maintenance action. Also, inspections are applied periodically and inter-inspection interval is optimized.

The main idea of the paper lies in a real problem related to water supply network of some villages that performs as a series system. The water supply network comprises many components and when a component fails the whole network is stopped. Overhauling of all network is impossible because of cost and time limitations. The best way is to find possible failure causes and repair them. When the water supply network is broken down the exact cause of failure might be unknown but there are some possible failure causes yielding in the masking problem. To repair the network a maintenance modeling is required. In this paper, an opportunity based perfect preventive maintenance policy is presented to encounter masking. Based on the opportunistic perfect preventive maintenance (OPPM) policy a probability threshold and inter-inspection interval are optimized.

The rest of the paper is organized as follows. In Section 2, the model is explained. The reliability function is characterized in Section 3. In Section 4, the maintenance model is proposed and maintenance probabilities are calculated. A numerical example is conducted in order to illustrate the applicability of the proposed method in Section 5. Finally, the conclusion is given in Section 6.

Model description

Suppose that we have a series system with J components that operates in a static environment. Moreover, we suppose that when the system fails we observe failure time, t, but the exact cause of failure might be unknown, and we only know that it belongs to MRS of $\{1, 2, ..., J\}$. Let M be the observed MRS corresponding to the failure time t for the system. The set M essentially includes components that are possible to be cause for system failure and if $M = \{1, ..., J\}$ then the system cause of failure is called to be completely masked. Thus the known information is given as follows:

$$(t, M). \tag{1}$$

To obtain the reliability function of the system and propose its maintenance modeling some assumptions and definitions have been made as follows.

Assumptions and definitions:

1. Let $T_l;\ \ l = 1, 2, ..., J$ be the lifetimes of the l th component (independent components) and assume that the system fails only due to one of the J components, therefore the system failure time T is defined to be $T = min(T_1, ..., T_J)$.
2. T_l belongs to a continuous distribution family with probability density and reliability functions denoted by $f_l(t)$ and $R_l(t)$, respectively.
3. The reliability function of T is given by

$$R(t) = R(t; \theta) = P_\theta(T > t) = \prod_{l=1}^{J} [1 - F_l(t)] \tag{2}$$

where $\boldsymbol{\theta} = (\theta_1, ..., \theta_J)$ and θ_l is the set of parameters related to the l th component and F_l is corresponding distribution function of l th component.

4. Suppose K be a random variable which indicates the cause of failure for the system. Then the joint probability density function of (T, K) is given by

$$f_{T,K}(t, l) = f_l(t) \prod_{j \neq l} [1 - F_j(t)] \tag{3}$$

where, the joint distribution of T and K can be specified in terms of the so called sub-distribution function $F(j, t) = P(K = j, T \leqslant t)$, or equivalently by the sub-reliability function $R(j, t) = P(K = j, T > t)$.[25]

5. $p_l^t(M_i) = P(M = M_i | T = t, K = l)$ is called the masking probability, where M_i is an observation of M. Some authors such as Mukhopadhyay and Basu,[26] Kuo and Yang,[27] and Cai et al.[28] assumed

$$P(M = M_i | T = t, K = l) = P(M = M_i | K = l) = p_l(M_i),$$

that is, the masking probability is independent of failure time, but is dependent to the causes of failure. Similar to a new approach that was presented to model the dependency of the masking probability on the failure time and its exact cause using the multinomial logistic regression model,[23] we also assume that the masking probability depends on the failure time and its exact cause.

6. Some constraints are considered for conditional masking probabilities. Suppose $\mathfrak{M}$ be the set of all nonempty subsets of $\{1, ..., J\}$ that have $2^J - 1$ members. For $l = 1, ..., J$, define $\mathfrak{M}_l = \{M \in \mathfrak{M} : l \in M\}$ (i.e. members of $\mathfrak{M}$ that include l) thus

$$\begin{aligned} p_l^t(M_i) &= P(M = M_i | K = l, T = t) \\ &= 0 \;\; \forall \;\; M_i \in \mathfrak{M}_l^c = \mathfrak{M} - \mathfrak{M}_l \end{aligned}$$

and

$$\sum_{M_i \in \mathfrak{M}} p_l^t(M_i) = \sum_{M_i \in \mathfrak{M}_l} p_l^t(M_i) = 1, \quad l = 1, ..., J \tag{4}$$

denote $\mathbf{P}_l = \{_l^t(M_i) : M_i \in \mathfrak{M}_l\}, \quad l = 1, 2, ..., J$ then the set of all masking probabilities is $P = (P_1, ..., P_J)$.

Reliability modeling

The following theorem provides an expression for the reliability function of a series system in presence of masked data under the mentioned assumptions in the previous section.

Theorem 1. *Consider a series system with J component where the exact cause of failure might be unknown. Assume that T is the lifetime of the system, $\mathfrak{M}$ is the set of all nonempty subsets of $\{1, 2, ..., J\}$ and K indicates the exact cause of failure. The reliability function is given by*

$$R(t) = \int_t^\infty \sum_{l=1}^{J} f_l(u) \prod_{j \neq l} R_j(u) du. \tag{5}$$

Proof of theorem 1: The reliability function is driven as follows:

$$\begin{aligned} R(t) &= \int_t^\infty \sum_{M_i \in \mathfrak{M}} \sum_{l \in M_i} P(M = M_i | u < T < u + \delta u, l) \\ &\quad f_{T,K}(u, l) du \\ &= \int_t^\infty \sum_{l=1}^{J} f_{T,K}(u, l) \sum_{M_i \in \mathfrak{M}} \\ &\quad P(M = M_i | u < T < u + \delta u, l) du \\ &= \int_t^\infty \sum_{l=1}^{J} f_{T,K}(u, l) du = \int_t^\infty \sum_{l=1}^{J} f_l(u) \prod_{j \neq k} R_j(u) du \end{aligned}$$

where the third equality is held since

$$\forall \; l \in \{1, 2, ..., J\} \Rightarrow \sum_{M_i \in \mathfrak{M}} P(M = M_i | u < T < u + \delta u, l) = 1$$

and other equalities are straightforward. Further, from (2), it is concluded that

$$\prod_{l=1}^{J} R_l(t) = \int_t^\infty \sum_{l=1}^{J} f_l(u) \prod_{j \neq k} R_j(u) du.$$

Maintenance modeling

In this section, a perfect preventive maintenance (PPM) policy is presented based on an opportunistic action and an optimal maintenance policy is derived using long-run cost rate criteria for a series system with J components. Inspections are assumed to be periodically at times $k\tau; \;\; k = 1, 2, ...$, with cost c_{ins} for the system and $M_k; \;\; k = 1, 2, ...$ are corresponding masked sets. The time interval $((k-1)\tau, k\tau]$ is called the k th period. Maintenance actions are applied based on some assumptions including:

- Inspection is performed at the end of each period
- Time needed for inspection and maintenance actions is negligible
- The system failure is not self-announced
- Components are maintained independently

At k th inspection time, $k\tau$, a maintenance action is performed if the system has been failed during $((k-1)\tau, k\tau]$ interval, that is,

$$T > (k-1)\tau \quad \& \quad T < k\tau.$$

Since it is assumed that exact cause of failure is unknown and it belongs to possibly masked set, $M_k \subseteq \{1, 2, ..., J\}$, thus the probability of each cause in M_k given possibly masked set and interval censored failure time is given by

$$p_{jM_k} = P(K=j|M_k, u\in((k-1)\tau, k\tau]) = \frac{\int_{(k-1)\tau}^{k\tau} P(M_k|j,u) f_{T,K}(u,j)du}{\int_{(k-1)\tau}^{k\tau} \sum_{l'\in M_k} P(M_k|l',u) f_{T,K}(u,l')du} \quad (6)$$

where u is the exact failure time. Note that $p_{jM_k} = 0$ for $j \notin M_k$.

Eventually, when the system is failed at $((k-1)\tau, k\tau]$ a maintenance action is carried out for each component in M_k according to a predetermined value of ρ; $0<\rho<1$, as follows:

- If $T_l > (k-1)\tau \quad \& \quad T_l < k\tau$ then perfect corrective maintenance (PCM) action is done for component l with cost c_{lc} (i.e. the failed component l is replaced by a new one).
- If $T_l > k\tau \quad \& \quad p_{lM_k} > \rho$ then opportunistic perfect preventive maintenance (OPPM) action is done for component l with cost $c_{lp} < c_{lc}$ (i.e. the degraded component l is replaced by a new one).

Otherwise, no maintenance action is done.

Maintenance probabilities calculations

Proposition 1. *A perfect corrective repair (PCR) is done for component l, $l\in M_k$, if the system fails at $((k-1)\tau, k\tau]$ and*

$$T_l > (k-1)\tau \quad \& \quad T_l < k\tau.$$

Define $P_{cl}(k\tau)$ as probability of perfect corrective repair for component l; $l = 1, 2, ..., J$, at k th inspection time, thus

$$\begin{aligned} P_{cl}(k\tau) &= P(l\in M_k \ \& \ T>(k-1)\tau \ \& \ T_l<k\tau) \\ &= P(T>(k-1)\tau \ \& \ T_l<k\tau) \\ &\left[\sum_{M\in\mathfrak{M}} P(M_k=M|T>(k-1)\tau \ \& \ T_l<k\tau) I_l\{M\}\right] \\ &= P(T>(k-1)\tau \ \& \ T_l<k\tau) \\ &\left[\sum_{M\in\mathfrak{M}} P(M_k=M|(k-1)\tau<T<k\tau \ \& \ K=l) I_l\{M\}\right] \\ &= (R_l((k-1)\tau)-R_l(k\tau)) \prod_{j\neq l, j=1}^{J} R_j(k\tau) \end{aligned} \quad (7)$$

where forth equation is hold since $\sum_{M\in\mathfrak{M}} P(M_k = M|T>(k-1)\tau \ \& \ K=l) I_l\{M\} = 1$ and

$$I_l\{M\} = \begin{cases} 1 & l\in M \\ 0 & l\notin M \end{cases}$$

Proposition 2. *An opportunistic perfect preventive repair (OPPR) is done for component l, $l\in M_k$, if the system fails at $((k-1)\tau, k\tau]$ and*

$$T_l > k\tau \quad \& \quad p_{lM_k} > \rho.$$

Define $P_{pl}(k\tau)$ as the probability of the opportunistic perfect preventive repair for component l; $l = 1, 2, ..., J$, at k th inspection time, thus

$$\begin{aligned} P_{pl}(k\tau) &= P(l\in M_k \ \& \ (k-1)\tau<T<k\tau \ \& \ T_l>k\tau \ \& \ p_{lM_k}>\rho) \\ &= P((k-1)\tau<T<k\tau \ \& \ T_l>k\tau) \\ &\times\left[\sum_{M\in\mathfrak{M}} P(M_k=M|(k-1)\tau<T<k\tau \ \& \ T_l>k\tau) I_l\{M\} I(p_{lM}>\rho)\right] \\ &= P(T_l>k\tau)\left[[\sum_{j\neq l; j=1}^{J} P((k-1)\tau<T_j<k\tau) \prod_{i\neq j} R_i(k\tau)\right] \\ &\times\left[\sum_{M\in\mathfrak{M}} P(M_k=M|(k-1)\tau<T<k\tau \ \& \ K\neq l) I_l\{M\} I(p_{lM}>\rho)]\right] \\ &= P(T_l>k\tau)\left[[\sum_{j\neq l; j=1}^{J} P((k-1)\tau<T_j<k\tau) \prod_{i\neq j} R_i(k\tau)\right] \\ &\times\left[\sum_{M\in\mathfrak{M}} \frac{\sum_{j\neq l; j=1}^{J} P(M_k=M|(k-1)\tau<T<k\tau \ \& \ K=j) P((k-1)\tau<T<k\tau \ \& \ K=j)}{\sum_{j\neq l; j=1}^{J} P((k-1)\tau<T<k\tau \ \& \ K=j)}\right. \\ &\times I_l\{M\} I(p_{lM}>\rho)]] \end{aligned} \quad (8)$$

Long-run cost rate

The time from the component installation to its first replacement or the time between two successive replacement of each component is referred to as a renewal cycle. Let L and L_j denote the average long-run maintenance cost per unit of time for the system and component j, respectively. Therefore, based on the renewal reward theorem the expected long-run maintenance cost rate for component j is

$$L_j(\tau, \rho) = \lim_{t \to \infty} \frac{C_j(t)}{t} = \frac{E(C_{rj})}{E(T_{rj})} \tag{9}$$

where $E(C_{rj})$ and $E(T_{rj})$ are total expected cost during a replacement cycle and expected length of the replacement cycle for component j, respectively such that

$$E(C_{rj}) = \sum_{k=1}^{\infty} \left[\left(\frac{k c_{ins}}{J} + c_{jp} \right) P_{pj}(k\tau) + \left(\frac{k c_{ins}}{J} + c_{jc} \right) P_{cj}(k\tau) \right] \tag{10}$$

and

$$E(T_{rj}) = \sum_{k=1}^{\infty} k\tau [P_{pj}(k\tau) + P_{cj}(k\tau)]. \tag{11}$$

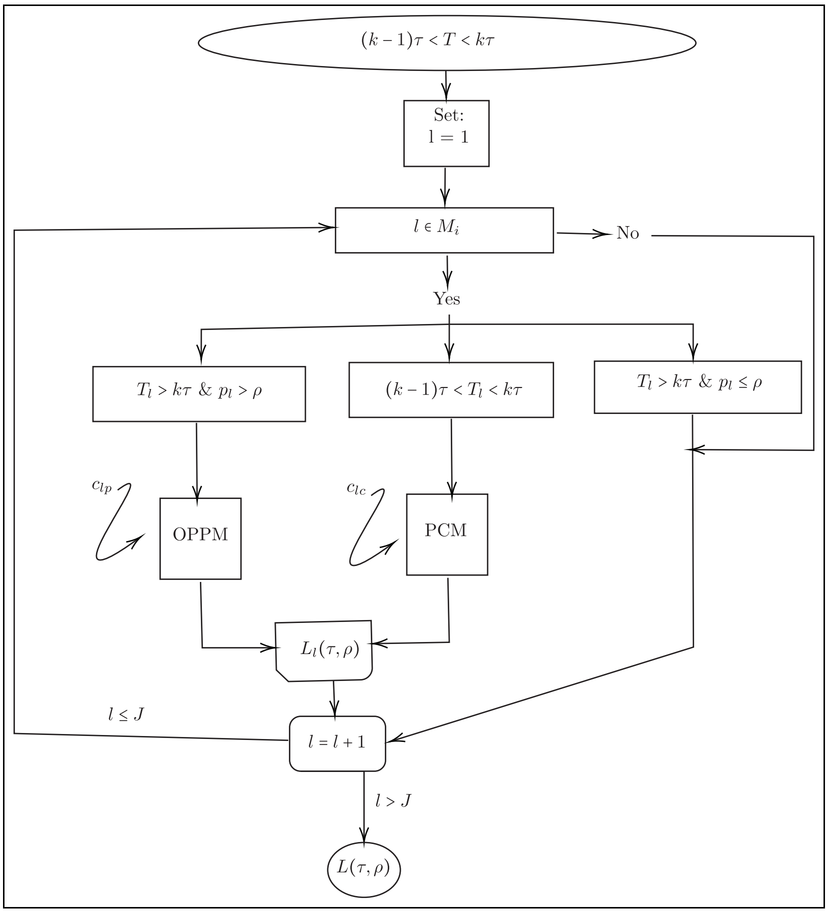
Finally, the total expected long-run maintenance cost rate for the series system until time t is given by[29,30]

$$L(\tau, \rho) = \sum_{j=1}^{J} L_j(\tau, \rho). \tag{12}$$

The maintenance plan is briefly illustrated in the Algorithm 1.

Illustrative example

In this section, a series system with $J = 3$ components is considered. The system is inspected periodically at times $k\tau$; $k = 1, 2, \ldots,$ and M_k; $k = 1, 2, \ldots$ are corresponding masked sets. The inspection cost for the system is c_{ins}. The collected data are $(k\tau, M_k)$; $k = 1, 2, \ldots$. When the system fails, an opportunistic perfect preventive replacement is carried out for components that are in the masked set M_k where $p_{lM_k} > \rho$ and a perfect corrective replacement is made for failed component with costs c_{lp} and c_{lc}, respectively. Two illustrative examples have been constructed to clarify previous sections. In



Algorithm 1. The plan of imperfect corrective maintenance of the system.

Table 1. The optimal inter-inspection interval value (τ) as decision parameter with different cost rates and ρ.

Constant			Decreasing			Increasing		
τ	ρ	Cost rate	τ	ρ	Cost rate	τ	ρ	Cost rate
0.03	0.00	154.38	0.01	0.20	445.45	0.01	0.00	368.44
0.04	0.00	125.23	0.02	0.20	234.48	0.02	0.00	184.31
0.05	0.10	102.16	0.03	0.10	161.29	0.03	0.10	120.26
0.06	0.10	92.90	0.04	0.10	123.22	0.10	0.20	37.42
0.07	0.10	86.11	0.10	0.20	56.15	0.20	0.20	21.17
0.09	0.10	69.64	0.20	0.20	32.80	0.29	0.20	16.37
0.11	0.10	62.42	0.30	0.20	24.18	0.30	0.20	16.01
0.14	0.10	55.72	0.40	0.20	19.50	0.40	0.20	13.37
0.17	0.10	48.73	0.50	0.20	16.30	0.50	0.20	11.76
0.26	0.10	36.90	0.70	0.20	12.72	0.59	0.20	10.69
0.27	0.10	34.14	0.80	0.20	11.42	0.60	0.20	10.58
0.31	0.10	32.96	0.81	0.20	11.31	0.67	0.20	9.99
0.42	0.10	24.21	0.82	0.20	11.20	0.68	0.10	10.04
0.44	0.10	23.16	0.85	0.20	10.88	0.69	0.10	9.95
0.50	0.10	21.13	0.86	0.20	10.77	0.70	0.10	9.87
0.53	0.10	20.10	0.87	0.20	10.68	0.71	0.10	9.80
0.54	0.10	19.90	0.88	0.20	10.58	0.72	0.10	9.72

both of them, it is assumed that the lifetime of components are independent and follow the Weibull distribution with parameter set (α_j, β_j); $j = 1, 2, 3$, as follows:

$$f_{Tj}(t_j) = \frac{\alpha_j}{\beta_j}\left(\frac{t_j}{\beta_j}\right)^{(\alpha_j-1)} exp\left(-\left(\frac{t_j}{\beta_j}\right)^{\alpha_j}\right).$$

We set $c_{ins} = 1$, $(c_{1p}, c_{2p}, c_{3p}) = (1.5, 2.5, 3.5)$, and $(c_{1c}, c_{2c}, c_{3c}) = (2, 4, 5)$.

Completely masked sets

In this subsection, all masked sets are considered as completely masked sets which is similar to missing setup, that is, $M_k = \{1, 2, ..., J\}$; $k = 1, 2, ...,$ thus

$$p_{jM_k} = p(K = j|M_k, u \in ((k-1)\tau, k\tau]) = \frac{\int_{(k-1)\tau}^{k\tau} f_{T,K}(u, j)du}{\int_{(k-1)\tau}^{k\tau} \sum_{l' \in M_k} f_{T,K}(u, l')du}$$

since $\forall j \in \{1, 2, ..., J\}$ and $t \in [0, \infty)$

$$P(M_k = M|K = j, t) = \begin{cases} 1 & \text{if } M = \{1, 2, ..., J\} \\ 0 & \text{if } M \neq \{1, 2, ..., J\} \end{cases}$$

where the inter-inspection interval, τ, is considered as decision parameter and should be optimized through maintenance optimization problem.

One of the most important aspects of the Weibull distribution is comprising three different failure rates according to its shape parameter (α). Weibull distributions with $\alpha < 1$ have a failure rate that decreases with time, also known as infantile or early-life failures, with α close to or equal to 1 have a fairly constant failure rate, indicative of useful life or random failures, with $\alpha > 1$ have a failure rate that increases with time, also known as wear-out failures.

To begin, the decision parameters τ and ρ are optimized by considering three different types of failure rates with different parameter sets as follows.

For constant failure rate, the parameter sets $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$ and $(\beta_1, \beta_2, \beta_3) = (0.25, 0.5, 1.5)$, for increasing failure rate, the parameter sets $(\alpha_1, \alpha_2, \alpha_3) = (1.25, 2, 2.5)$ and $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$, and for decreasing failure rate, the parameter sets $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.25, 0.7)$ and $(\beta_1, \beta_2, \beta_3) = (1.5, 1.25, 1.75)$ are considered. Corresponding reliability and failure rate functions of the system are drawn in Figures 1(a), (b), 2(a), (b), 3(a), and (b), respectively. Without loss of generality, inspections are scheduled periodically at times $k\tau$; $k = 1, 2, ...$ First, τ is prespecified and then the optimal values of ρ and corresponding cost rates for $\tau = 0.09$, $\tau = 0.03$, and $\tau = 0.04$ are obtained and depicted in Figures 1(c), 2(c), and 3(c) for constant, increasing, and decreasing failure rate modes, respectively. Considering this setup, the optimal values of ρ are 0.1, 0.1, and $[0.1, 0.2]$ for constant, increasing, and decreasing failure rates with cost rates 69.639, 120.261, and 123.221, respectively.

Afterward, besides ρ, τ is also considered as a decision parameter of the maintenance optimization problem. In such a case, the optimal value of τ is a trade-off between rate of cost and ρ. The value of τ should be chosen to minimize cost and maximize ρ, simultaneously. According to the obtained results given in Table 1, the optimal value of τ is obtained as $\tau = 0.54$, 0.67, and 0.88 for constant, increasing, and decreasing failure rates, respectively and corresponding optimal values of ρ are drawn in Figures 1(d), 2(d), and 3(d). Considering this setup, the optimal values of ρ are 0.1, 0.2, and 0.2 for constant, increasing and decreasing

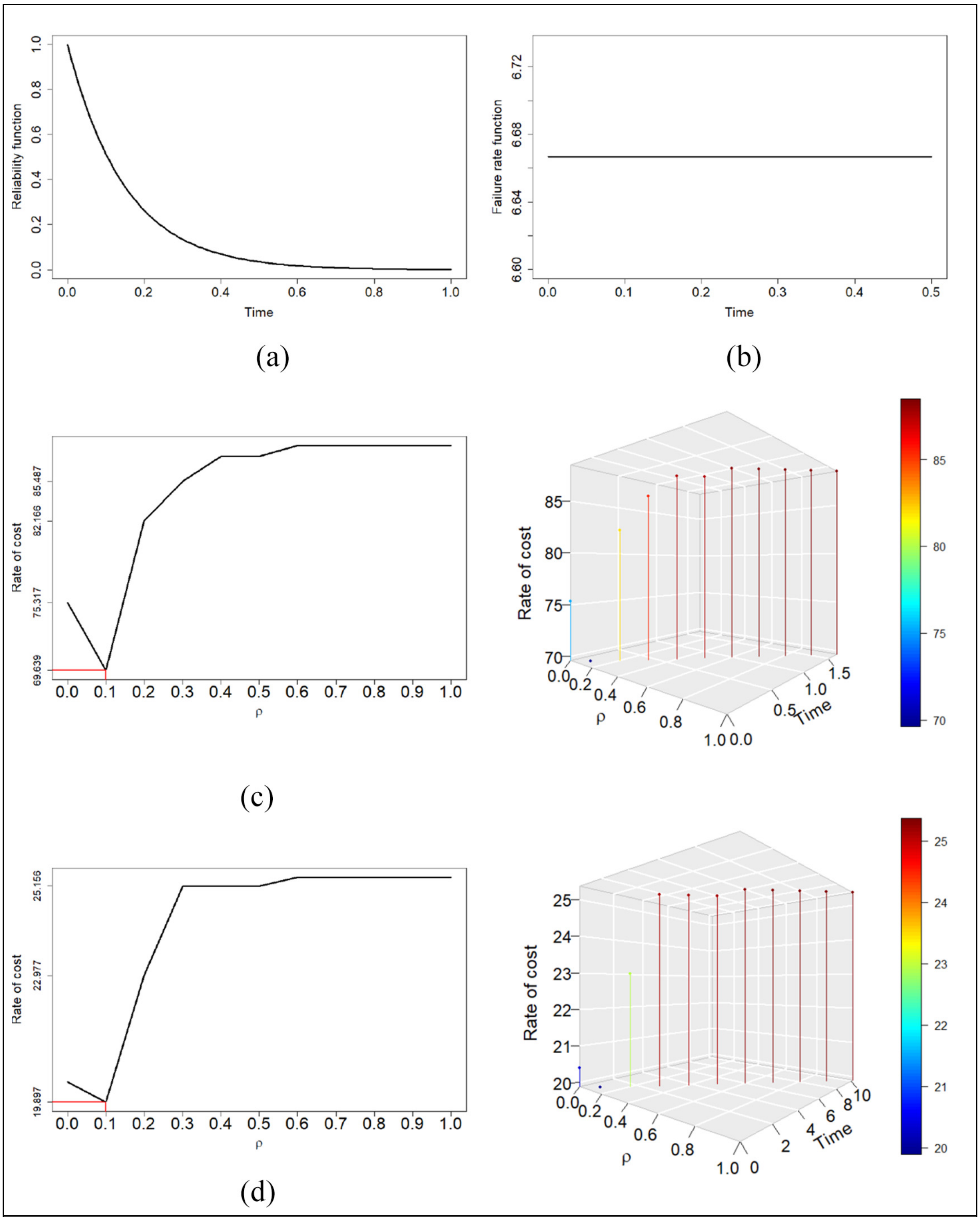


Figure 1. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$ and $(\beta_1, \beta_2, \beta_3) = (0.25, 0.5, 1.5)$: (a) reliability function of the system, (b) failure rate of the system (constant), (c) $\tau = 0.09$, and (d) $\tau = 0.54$ (the optimal value).

failure rates with cost rates 19.897, 9.988, and 10.577, respectively. Based on the results, an optimal value of ρ decreases rate of cost in three different types of failure rates of the Weibull distribution. Moreover, the optimal value of ρ, while the inter-inspection interval is also optimized, results in a remarkable decreasing in rate of cost in all types of failure rates such that, in the case of constant failure rate, the rate of cost decreases from 69.639 to 19.897, in the increasing failure rate case, the rate of cost decreases from 120.261 to 9.988 and in the decreasing failure rate case, the rate of cost decreases from 123.221 to 10.577.

Moreover, three different types of failure rates are considered in order to study how the uncertainty about the shape parameter affects on the obtained results. This can be done by varying the values of α but

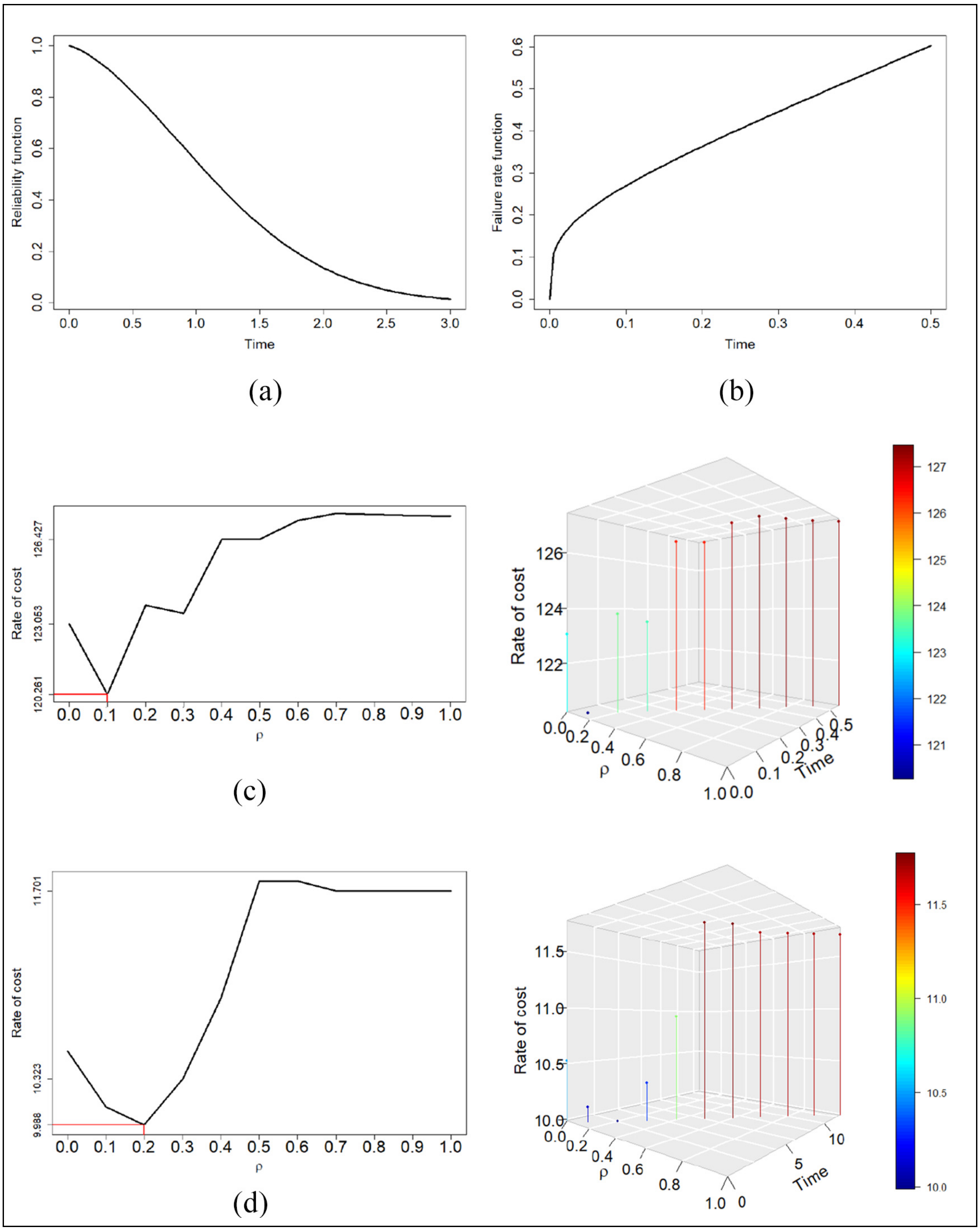


Figure 2. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (1.25, 2, 2.5)$ and $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$: (a) reliability function of the system, (b) failure rate of the system (increasing), (c) $\tau = 0.03$, and (d) $\tau = 0.67$ (the optimal value).

keeping β values as the same. We set $(\beta_1, \beta_2, \beta_3) = (0.25, 0.5, 1.5)$ for three types of parameter sets $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1), (1.25, 2, 2.5), (0.5, 0.25, 0.7)$ for constant, increasing, and decreasing failure rates, respectively. As the results show in Figure 4, different values for the shape parameter lead to different optimal values since the operation mechanism varies for different shape values. Such that, for $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$ the optimal values are $(\tau, \rho) = (0.54, 0.1)$, for $(\alpha_1, \alpha_2, \alpha_3) = (1.25, 2, 2.5)$ the optimal values are $(\tau, \rho) = (0.11, 0.2)$ and for $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.25, 0.7))$ the optimal values are $(\tau, \rho) = (1.11, [0.2, 0.3])$ with cost rates 19.897, 60.530, and 9.709, respectively.

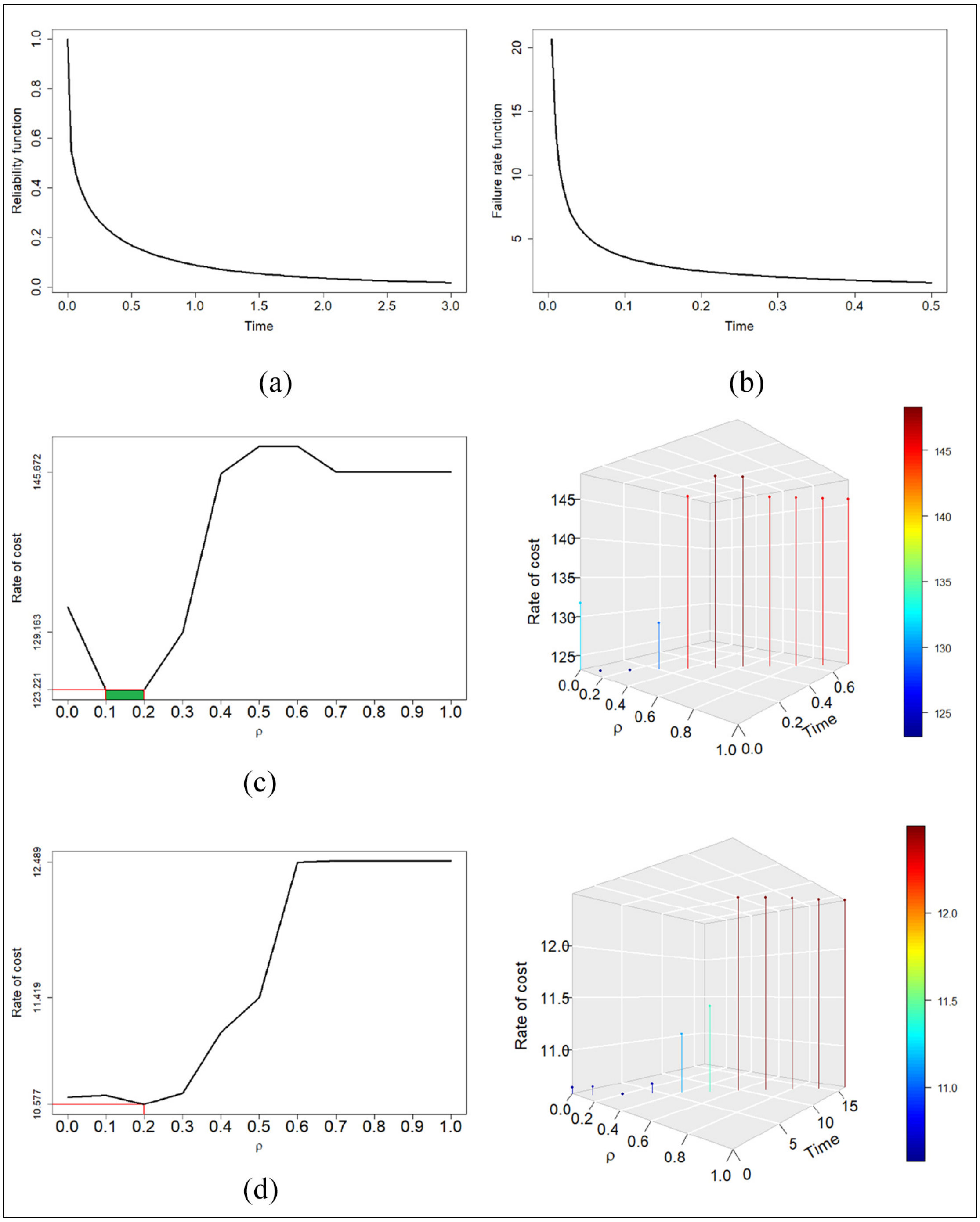


Figure 3. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.25, 0.7)$ and $(\beta_1, \beta_2, \beta_3) = (1.5, 1.25, 1.75)$: (a) reliability function of the system, (b) failure rate of the system (decreasing), (c) $\tau = 0.04$, and (d) $\tau = 0.88$ (the optimal value).

Not-completely masked sets

In this subsection, the data are masked randomly using multinomial distribution with probability vector (p_1, p_2, p_3), where $p_i = p(|M_k| = i)$ that is, p_i is the probability that cardinality of M_k is i; $i = 1, 2, 3$. We set $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$ and $(p_1, p_2, p_3) = (0.5, 0.3, 0.2)$ for three types of failure rates. Considering $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$ as the constant failure rate leads to optimal values $(\tau, \rho) = (0.201, [0.6, 0.9])$ presented in Figure 5, $(\alpha_1, \alpha_2, \alpha_3) = (1.25, 2, 2.5)$ as the increasing failure rate leads to optimal values $(\tau, \rho) = (1.2, [0.7, 0.9])$ presented in Figure 6 and $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.25, 0.7)$ as the decreasing failure rate leads to optimal values $(\tau, \rho) = (0.101, 0.3)$ presented in Figure 7 with cost rates 26.415, 7.268, and 58.726, respectively.

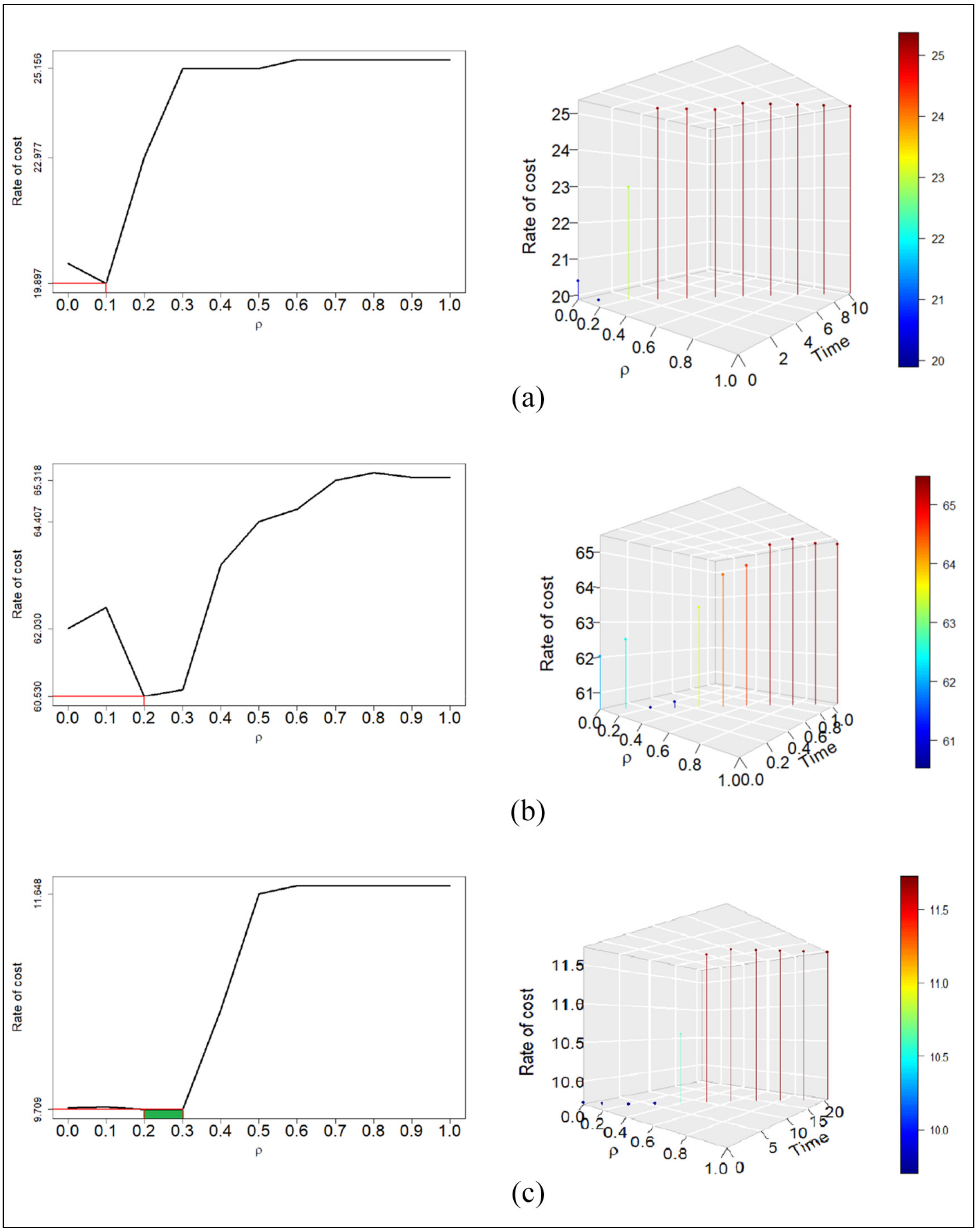


Figure 4. The optimal value of ρ as decision parameter considering $(\beta_1, \beta_2, \beta_3) = (0.25, 0.5, 1.5)$ under different failure rates: (a) $\tau = 0.54$ (the optimal value) and $\alpha = (1, 1, 1)$, (b) $\tau = 0.11$ (the optimal value) and $\alpha = (1.25, 2, 2.5)$, and (c) $\tau = 1.11$ (the optimal value) and $\alpha = (0.5, 0.25, 0.7)$.

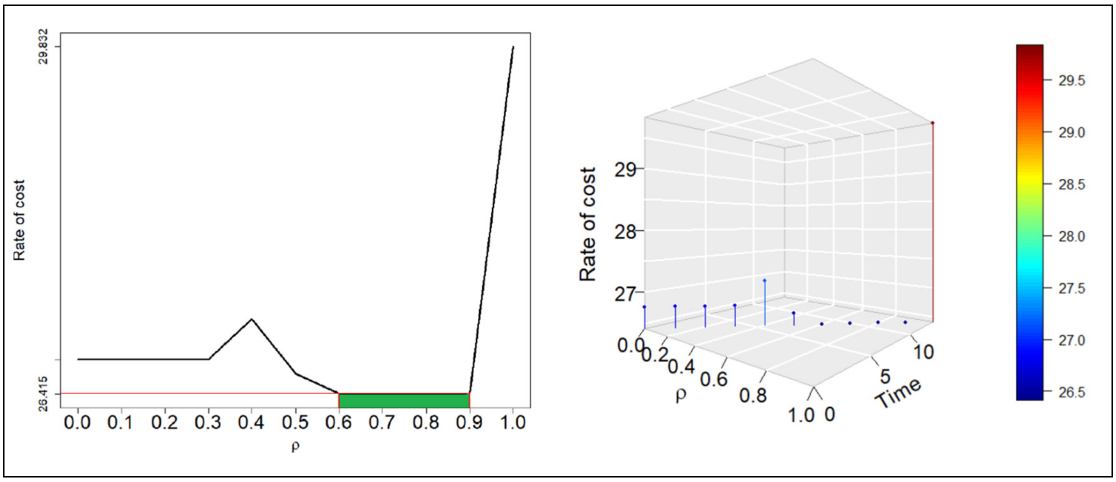


Figure 5. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1)$ and $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$ and $\tau = 0.201$ as the optimal value.

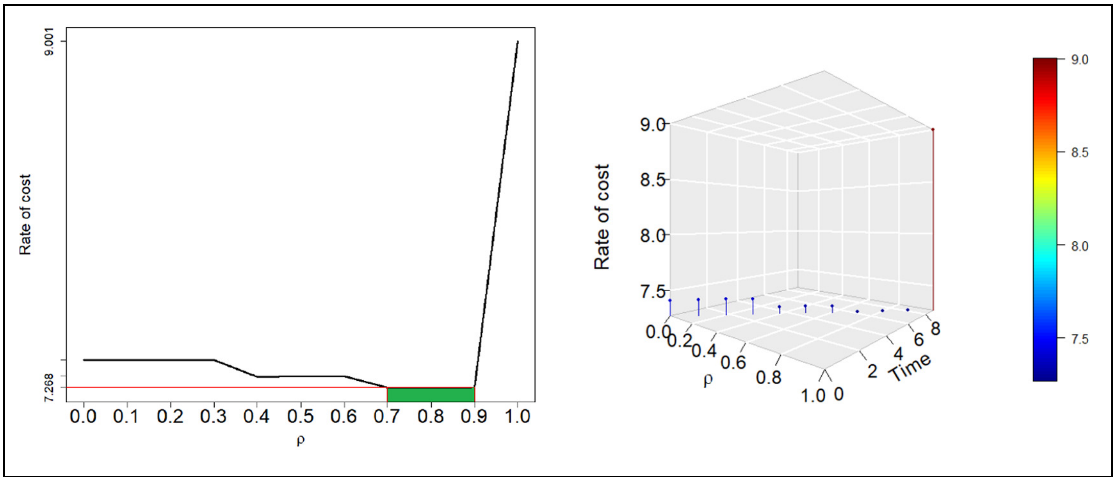


Figure 6. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (1.5, 2, 2.5)$ and $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$ and $\tau = 1.2$ as the optimal value.

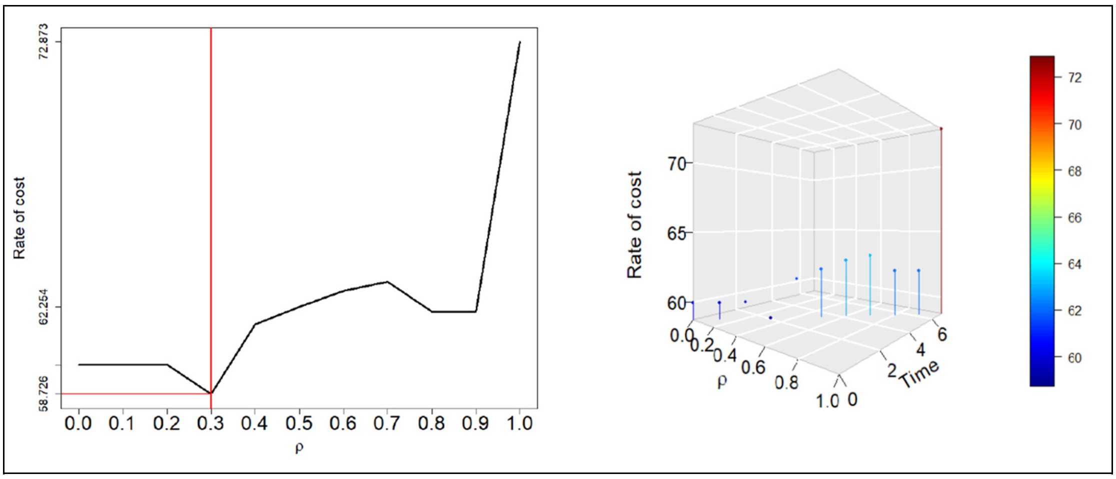


Figure 7. The optimal value of ρ as decision parameter considering $(\alpha_1, \alpha_2, \alpha_3) = (0.5, 0.25, 0.7)$ and $(\beta_1, \beta_2, \beta_3) = (2.5, 2.25, 2.75)$ and $\tau = 0.101$ as the optimal value.

Conclusion

In this work, a maintenance policy is proposed for a multi-component system in which the exact cause of system failure might be masked. The system is periodically inspected and the inter-inspection interval is considered as a maintenance variable. At each inspection time, if the system fails masked set is specified and all components in the masked set are checked. The failed component is replaced by a new one as a corrective replacement action and the conditional probability of cause for other components is compared with a preventive threshold. If it exceeds the preventive threshold the component is replaced by a new one as opportunistic perfect preventive replacement action. Maintenance variables, inter-inspection interval and preventive threshold, are chosen to minimize the average long-run cost rate. The simulation results show that optimal values of maintenance variables significantly reduce the average long-run cost rate. The maintenance problem for multi-component systems with masked causes of failure in a dynamic environment will be considered in the future studies.

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