

# Symmetry breaking organizes the brain's resting state manifold

Jan Fousek, Giovanni Rabuffo, Kashyap Gudibanda, Hiba Sheheitli, Viktor

Jirsa, Spase Petkoski

# ► To cite this version:

Jan Fousek, Giovanni Rabuffo, Kashyap Gudibanda, Hiba Sheheitli, Viktor Jirsa, et al.. Symmetry breaking organizes the brain's resting state manifold. Cold Spring Harbor Laboratory Press, 2023, 10.1101/2022.01.03.474841. hal-04086340

# HAL Id: hal-04086340 https://amu.hal.science/hal-04086340

Submitted on 2 May 2023

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

# Symmetry breaking organizes the brain's resting state manifold

Jan Fousek<sup>1</sup>, Giovanni Rabuffo<sup>1</sup>, Kashyap Gudibanda<sup>1</sup>, Hiba Sheheitli<sup>1</sup>, Viktor Jirsa<sup>1</sup>, and Spase Petkoski<sup>1</sup>

<sup>5</sup> <sup>1</sup>Aix Marseille University, INSERM, INS, Institut de Neurosciences
 <sup>6</sup> des Systèmes, 13005 Marseille, France

March 12, 2023

#### Abstract

Spontaneously fluctuating brain activity patterns that emerge at rest have been linked to brain's health and cognition. Despite detailed descriptions of the spatio-temporal brain patterns, our understanding of their generative mechanism is still incomplete. Using a combination of computational modeling and dynamical systems analysis we provide a mechanistic description of the formation of a resting state manifold via the network connectivity. We demonstrate that the symmetry breaking by the connectivity creates a characteristic flow on the manifold, which produces the major data features across scales and imaging modalities. These include spontaneous high amplitude co-activations, neuronal cascades, spectral cortical gradients, multistability and characteristic functional connectivity dynamics. When aggregated across cortical hierarchies, these match the profiles from empirical data. The understanding of the brain's resting state manifold is fundamental for the construction of task-specific flows and manifolds used in theories of brain function such as predictive coding. In addition, it shifts the focus from the single recordings towards brain's capacity to generate certain dynamics characteristic of health and pathology.

# 27 1 Introduction

3

10

11

12

13

14

15

16

17 18

19

20

21

22

23

24

25

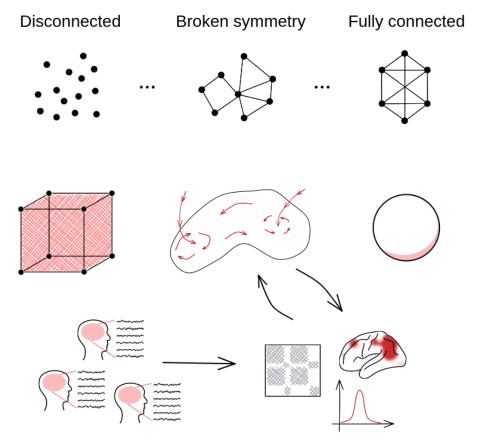
26

The human brain at rest exhibits remarkable richness of neural activity structured both in time and space. Early computational modeling studies explored how these spontaneous fluctuations are constrained and how their organisation is shaped by the anatomic connectivity [1–4] enabling to start disentangling the mechanisms of the resting state dynamics *in silico*. A substantial body of work has related the emergent activity patterns at rest to the brain functional networks involved in task conditions [5, 6], and shown that the spatio-temporal

variability of resting-state activity possesses functional significance [7–9], rel-35 evance to cognitive task performance [10], consciousness levels [11], changes 36 during ageing [12, 13], mental disorders [14], and neurodegenerative diseases 37 (e.g. Alzheimer's dementia; [15]). The structure of the resting state dynam-38 ics changes over time [16] and is characterized by a range of properties such 39 as metastability [17, 18], event-like coactivations [19–21] and traveling waves 40 [22]. However, our understanding of the mechanisms underlying these spatio-41 temporal patterns of the brain activity at rest is still incomplete [23] and whole 42 brain network models have a crucial role to play on that front [24]. 43

There is general agreement that the resting brain operates near critical-44 ity [25]. This is supported by a large range of analyses performed on simu-45 lated and empirical data using network based measures (functional connectivity, 46 functional connectivity dynamics), information theoretical measures (entropy, 47 ignition) and descriptions of spatiotemporal dynamics (avalanches, cascades). 48 Modeling efforts provide further evidence for the close relationship between the 49 empirical data features and the properties of the structural network, local dy-50 namics, coupling strength, neural gain [4, 13, 26–31]. The resting state dynamics 51 can then be understood as noise-driven fluctuations of brain activity, operat-52 ing near criticality and constrained by the brain connectivity [2, 32]. However, 53 none of the above qualifies as a description of a mechanism. Descriptions of 54 mechanisms require formulation in terms of causal activities of their constituent 55 entities and render the end stage, in our context the resting state dynamics. 56 intelligible by showing how it is produced [33]. To explain is thus not merely to 57 redescribe one regularity (e.g. functional connectivity dynamics, or maximiza-58 tion of entropy) as a series of several (such as near-criticality, cascades, ignition). 59 Rather, explanation involves revealing the productive relation between causal 60 activities linked to their constituent entities. 61

In this paper we aim to remedy this situation and provide this explanation 62 using Structured Flows on Manifolds (SFMs) [34–38]. SFMs is a mathematical 63 framework explaining how low dimensional dynamics, reflecting generative sets 64 of rules underlying behavior, emerges in high-dimensional nonlinear systems, 65 specifically dynamical systems on networks modeling macroscale brain dynam-66 ics. When properly linked to the network's constituent entities (functional nodes 67 and connectivity), we will demonstrate how their causal activities lead to the 68 formation of brain's resting SFM, comprising all its dynamic signatures (see 69 Figure 1). If we distill the previous reports of brain resting state data analysis 70 from the dynamical systems point of view, we arrive at the following main em-71 pirical signatures that should be part of the end stage of a successful mechanistic 72 description: bistability of single region activation [39–41], low-dimensionality of 73 the global system dynamics in state space [7, 42, 43], cascade propagation [44], 74 multistability of recurrent coactivation spatial patterns [18, 45] and their non-75 trivial temporal dynamics or intermittency [21, 32, 46]. These signatures will 76 constitute the key features of what we will describe as structured flows on the 77 low dimensional resting state manifold. 78



79

Structured flows on manifolds as focus of resting state Figure 1: 80 characterization. With respect to the structure of the connectivity of the 81 dynamical system, we consider spectrum defined by the two symmetrical limit 82 cases: fully connected and fully disconnected network. Driven by noise, the 83 disconnected system exhibits fully statistical, high-dimensional dynamics - it 84 explores the whole state space in a equidirectional manner. On the other hand, 85 the dynamics of the fully connected system is fully constrained corresponding to 86 a SO(n) hypersphere with zero flow. The dynamics on the sparsely connected 87 system leads to an object in between - a low-dimensional attractive manifold 88 with an associated flow (SFM). It is this object we wish to put in the center of 89 interest and characterize. While the SFM object remains the same, connections 90 are made to data of various modalities with the help of suitable data features. 91

# 92 2 Results

In what follows we employ whole-brain modeling to study the low dimensional
 manifold and the associated structured flows of the spontaneous resting state

dynamics, and how these relate to the structural connectome. We constructed a 95 brain network model (BNM) in the Virtual Brain [47] using the two-dimensional 96 mean-field model of an ensemble of quadratic integrate-and-fire neurons ([48]; 97 MPR) to govern the regional dynamics coupled with a connectome derived from 98 a subject from the Human Connectome Project [49]. We applied Balloon-99 Windkessel model [50] to the simulated neuronal mass activity to generate 100 realistic BOLD signals. From these, we computed the Dynamical Functional 101 Connectivity (dFC) that captures the changes in the system's dynamics on the 102 slow time scale, which we compared with empirical recordings. The fast neu-103 ronal activity is decomposed in a 2N-dimensional state space using Principal 104 Component Analysis (PCA) to unveil the low-dimensional manifold on which 105 the system evolves (see Materials and Methods for more details). 106

When driven by noise, the network of the bistable MPR nodes has the ca-107 pacity to exhibit realistic dFC when the network input is scaled appropriately 108 [44]. The noise together with the network input drives the switching between 109 up- and down-state of the individual nodes, while the network mediates the co-110 ordination reflected in the functional connectivity. In the following sections, we 111 explore how the manifold of the resting state activity arises from the networked 112 interactions, how it shapes the multistability of the functional connectivity in 113 the simulated BOLD, and how it relates to empirical observations. 114

#### <sup>115</sup> 2.1 Symmetry breaking: working point for dFC

To assess the impact of the symmetry breaking by the connectome, we simu-116 lated 10 minutes of spontaneous activity for a range of values of the coupling 117 scaling parameter G and noise variance  $\sigma$ , and applied PCA to the source signal 118  $\Psi(t)$  and dFC to the BOLD (Figure 2). We used the variance accounted for 119 (VAF) of the first two PCA components as an estimate for the dimensionality 120 of the system's dynamics in the state-space (Figure 2D), and the variance of the 121 upper triangle of the dFC matrix as a measure of the *fluidity* of the system's 122 dynamics—that is the propensity to dwell in specific brain states (defined by the 123 functional connectivity) and shift and return between several such states (Fig-124 ure 2A). In addition, using Kolmogorov-Smirnov distance between the centered 125 distributions of the values of the upper triangle of the  $dFC_w$  in the empirical 126 and simulated data, we have verified that the region of the parameter space 127 where dFC is most similar to the one derived from empirical data overlaps with 128 the region with the highest fluidity, Figure 2A. 129

For low values of G, the system exhibits high-dimensional dynamics as re-130 flected in the low variance explained by the first PCA components and low 131 values of the variance of the off-diagonal values of dFC with the mean around 132 0—reflecting the absence of recurrence in the system dynamics (Figure 2B,C). 133 Note, that the explained variance for each PCA component is equal to 1/N134 (in this case N = 84 nodes of the network), and the projection in panel D 135 reflects the independent infrequent switching of two nodes, each captured by 136 one PCA component. Around the value of  $G = G_w = 0.525$  and  $\sigma_w = 0.030$ 137 (working point) the variance explained by the first two components of the PCA 138

139 increases substantially, and so does the fluidity of the dFC as the characteristic

- $_{^{140}}\,$  intervals of FC invariance (on-diagonal nonzero blocks) appear together with
- <sup>141</sup> similarity across time (high off-diagonal correlations). Past the working point
- $_{^{142}}\quad (G>0.6)$  the explained variance in PCA drops as well as the off-diagonal dFC
- <sup>143</sup> correlations, signifying increase in dimensionality of the spontaneous dynamics.

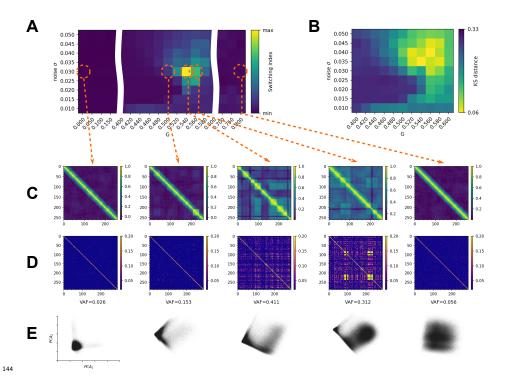


Figure 2: Brain network model and symmetry breaking. The brain 145 network model is simulated for varying levels of global coupling parameter G146 and noise variance  $\sigma$  to produce both time-series of the state space variables 147  $\mathbf{r}(t), \mathbf{V}(t)$ , and the BOLD signal. For each combination of G and  $\sigma$  we compute 148 the sliding window  $dFC_w$  matrix from the simulated BOLD signal, and quantify 149 the "switching index" of the dFC as the variance of the upper triangle (A). 150 Kolmogorov-Smirnov distance of the centered (mean-subtracted) distributions 151 of the values of the upper triangle of the dFC computed from empirical and 152 simulated resting state BOLD time series. The region of parameter space where 153 the distributions are closest overlaps with the region with high fluidity of dFC 154 (B). For selected values of  $(G, \sigma)$  we show the sliding window  $dFC_w$  (C), edge 155 based  $dFC_e$  (D) and the projection of  $\mathbf{r}(t)$  time series in the first two PCA 156 components (E) annotated with corresponding fractional variance accounted 157 for (VAF). In the working point around G = 0.54 and intermediate values of  $\sigma$ 158 the system exhibits recurrence in the large-scale dynamics as captured by non-159 zero switching index, and reduction of dimensionality as captured in the increase 160 in explained variance by the first PCA components and the asymmetry in the 161 respective projection. For values of G below or above the working point, the 162 systems loses the fluidity property as reflected in the absence of the off-diagonal 163 blocks on the dFC, and exhibits high-dimensional dynamics. 164

# <sup>165</sup> 2.2 Network dynamics

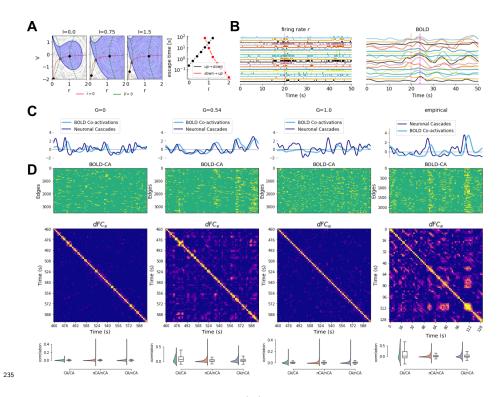
Before we delve into the characterization of the low-dimensional manifold, let 166 us first describe the network dynamics in detail. For the MPR model, the dy-167 namical profile of an isolated node in the bistable parametrization consists of 168 an unstable fixed point (saddle node) and two stable fixed points: down-state 169 stable node and up-state focus (Figure 3A). Considering the uncoupled system, 170 that is, the joint dynamics of the N populations (nodes) in the absence of any 171 inter-population synaptic coupling, the phase flow is represented by  $2^N$  stable 172 fixed points that contain all possible combinations of the populations firing at 173 either their low or high mean firing rates (down or up state, respectively). Start-174 ing from an initial condition, the system will settle into the nearest accessible 175 such fixed point, a stable network state composed of a corresponding combina-176 tion of regions in their up or down state. Thus, the dynamics of the uncoupled 177 system in phase space can be thought of as being driven by a potential energy 178 landscape with multiple stable local minima representing the stable attractor 179 states of the network. In fact, the uncoupled system, as such, is invariant under 180 permutation of the indexes of the populations, such that these latter attractor 181 network states are distinguished only in terms of the respective number of nodes 182 in up and down states. The global dynamics of the system, thus, collapses in 183 finite time onto this stable attractor state composed of a finite set of stable 184 equilibrium points that is invariant under shuffling of indexes of the nodes. 185 The associated global phase flow can be decomposed into N projections onto 186 the identical 2D phase planes of individual populations, depicted in Figure 3A. 187 Viewed from this perspective, the structure of the basins of attraction of the 188  $2^{N}$  stable system equilibrium points redundantly inherits, in higher dimensions, 189 the relative structure of the basins of attraction of the two stable fixed points 190 of an individual population. For an isolated node, varying the external input 191  $I_i$  changes the size of the basins of attraction of the stable fixed points. This 192 modulates the probability of switching between the two states when driven by 193 noise as captured by the mean escape times (Figure 3A, see Methods for more 194 detail). For a connected node, the external input  $I_e$  depends on the state of the 195 neighboring nodes (see Equation 4), fluctuating as they transition between the 196 up- and down-state. On the network level, given right scaling of the network 197 connections, this enables the cascades of up- and down-state switching at the 198 fast time-scale, and the co-fluctuation of the BOLD signal (Figure 3B). 199

To understand better the dynamical underpinning of the increase of fluidity 200 of the dFC we assess the characteristics of the co-fluctuations of the BOLD 201 signal and the cascades in the source signal. For the co-activations, we start 202 from the edge time series which is defined as pairwise dot product of z-scored 203 BOLD signal (an average over the edge time series would correspond to the 204 pearson correlation). The correlation across time-points yield the  $dFC_e$  matrix 205 capturing the recurrence of the edge configurations, and the root sum squared 206 (RSS) over the edges at each time point captures the contribution of that par-207 ticular time point to the overall functional connectivity (see Methods for more 208 details). The time-points crossing the 95-th percentile threshold of the RSS are 209

considered as strong co-activation events. The neuronal cascades [44] are long lasting perturbations of the neuroelectric activity and are measured on a global level as a sum over regions of the binarized firing rate activity (at the threshold of 3 standard deviations). We compared these measures between the working point  $G_w$ , the disconnected system G = 0, the strong network coupling regime  $G >> G_w$ , and the empirical data (Figure 3C,D).

In the working point  $G_w$  the co-activations include large number of edges 216 (Figure 3D) and the RSS follows the number of cascades up to a short delay 217 corresponding to the delay of the BOLD signal. Moreover, some of the strong 218 co-activations re-occur partially in time as reflected in the non-zero elements 219 of the  $dFC_e$  matrix. The same profiles can be observed in the empirical data, 220 namely in the simultaneous EEG and fMRI recordings. On the other hand, 221 the characteristic spatial and temporal structure is lost outside of the working 222 point, that is either for the weakly coupled system  $(G \ll G_w)$ , or for too strong 223 coupling  $(G >> G_w)$ . 224

To quantify how the co-activation events contribute to the characteristic 225 similarity across time, we compare the correlation of the edge vectors during the 226 events, during the non-events, and between events and non-events. As a result 227 we observe an increased similarity of the edge vectors during the events both in 228 the empirical data and in the simulations in the working point  $G_w$ . Again, this 229 property is lost for too weak  $(G \ll G_w)$  or for too strong coupling  $(G \gg G_w)$ . 230 Together, these results show, that the system has a similar dynamical profile 231 in the working point  $G_w$  as observed in the empirical data with respect to the 232 network-carried fluctuations on both the fast and slow timescales (as captured 233 by dFC and cascades respectively). 234



Network dynamics. (A) The network input I modulates the Figure 3: 236 probability of a noise-driven transition between the down- and up-state. (B) 237 Example of a cascade—coordinated increase in activity translating to a delayed 238 correlated peak in the BOLD signal. Below we compare the network dynamics 239 in and outside the working point, and the empirical data. In both empirical 240 data and the working point (G = 0.54), the BOLD co-activations follow the 241 neuronal cascades with a latency (C), and show distinct spatial profiles which 242 are recurrent in time (D): edge time series on the (top panel) captures the 243 spatial profiles of the co-activations, the similarity across time is captured by 244 the  $dFC_e$  matrix (middle panel), and the distributions of correlation between 245 co-activation events (CA) and non-events (nCA) is compared (bottom panel). 246

# 247 2.3 Manifold of the resting state and characteristic sub 248 spaces

Having characterized the dynamics of the system in the working point with an appropriate measure, we proceed with the description of the manifold on which it evolves—that is to show that this behavior is low-dimensional and constrained to a specific subspace. To relate cascades and co-activations to the trajectories of the system in the 2N-dimensional state space, we first select time intervals with similar functional connectivity. Starting from the edge time series for the magnitude of co-fluctuations, we clustered the time points using k-means

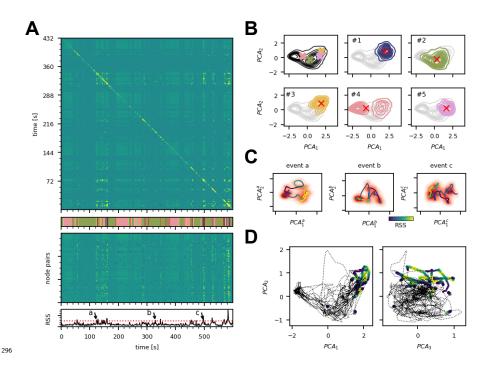
(k=5). This separated the high-activity intervals (majority of the nodes in the up-state), low-activity intervals (majority of the nodes in the down-state), and
the co-fluctuation events (Figure 4A).

Next, we identified the trajectories of the system underlying each cluster 259 in the low dimensional projection of state space. For each cluster, we have 260 selected the corresponding time points in the state space of the system, and 261 projected them into the first two principal components of the PCA computed 262 on the complete time series. We have observed that while the corresponding 263 subspaces overlap partially in the projection (Figure 4B, colors correspond to 264 the clusters), the activity within the clusters concentrates to different subspaces. 265 This concentration in different subspaces is reflected in the distance between the 266 centroids of the cluster time points in the PCA projection. 267

While the cluster activity overlaps in the projection in the components of 268 the PCA computed from the whole time series, the co-activation trajectories be-269 come clearer by choosing different basis to span the low-dimensional space, that 270 is, to compute the PCA from the time points corresponding to the individual 271 co-activation events. To project the trajectories of the events observed at the 272 slow time scale of the BOLD on the manifold, we have shifted the BOLD signal 273 by the characteristic lag, and for each BOLD time point belonging to the cluster 274 we selected the corresponding time points in  $\mathbf{r}(t)$ , and convolved the resulting 275 data with a Gaussian kernel to smoothen out the noisy fluctuations (see Meth-276 ods for details). We then spanned the subspace corresponding to the first two 277 PCA components of the co-fluctuation trajectory and overlayed the smoothened 278 trajectory over the density plot of the full  $\mathbf{r}(t)$  time series. The density plots 279 (shades of red, Figure 4C) of the example events show a separation of the event 280 subspace marked by the peak in the RSS (shown in yellow in Figure 4C) on 281 the smoothened trajectory from the rest of the manifold. This suggests that 282 the event subspace is relatively stable, allowing the system to dwell in it long 283 enough to cause the significant peaks in the slow BOLD signal, and that the 284 intermediate states are less stable than the event subspace or the rest of the 285 manifold and visited only transiently. 286

Although the linear embedding of the whole time-series does not separate the event trajectories well when applied to the  $\mathbf{r}(t)$  time series, the event trajectories concentrate in the high-activity subspace spanned by the first two PCA components of the BOLD signal (Figure 4D).

Together, these results chart the low-dimensional manifold of the system in the working point regime, associating the subspaces with specific flows. The fluid dynamics as characterized in the previous section then arise from the slow transitions between the low- and high-activity subspaces, where the latter supports the strong co-activation events which are reflected in the dFC.



Manifold subspaces and characteristic dynamics. (A) The Figure 4: 297 edge-based dynamic functional connectivity  $dFC_e$  of a simulation of the model 298 in the working point (top) shows the off-diagonal structure of similarity of the 299 system's activity across time. The edge time series (middle) shows the time evo-300 lution of the functional connectivity of the simulated BOLD signal between each 301 of the node pairs, and exhibits the characteristic co-activation events defined as 302 time points with the root sum squared (RSS, bottom) crossing the thershold of 303 95th percentile. Dividing the edge-time series into 5 clusters (k-means, shown 304 in the colorbar under the dFC) has separated the event and non-event time 305 points, and also differentiated the events based on their respective similarity. 306 (B) The time intervals in  $\mathbf{r}(t)$  corresponding to the 5 clusters were selected; in 307 the first panel the centroids of the time points of the individual clusters are 308 marked with a cross in the projection to the first two principal components of 309 the whole time series, following panels show the projection of the  $\mathbf{r}(t)$  intervals of 310 particular clusters. Cluster #2 captures the high-activity subspace, cluster #4311 corresponds to the low-activity state, and the clusters #1, #3, and #5 capture 312 the co-activation events. (C) Local trajectories in the manifold subspaces: the 313 time series of the three example events (a,b,c, marked in the panel A bottom) 314 was projected to the first two components of PCA applied to each time seg-315 ment individually. The smoothened trajectory marks the advance of the system 316 through the event and out of it, and is colored by the value of RSS (vellow at the 317 peak of the event blue at the beginning and the end). (D) The event trajectories 318 on the manifold. The trajectory of the simulated BOLD signal is projected in 319 the space defined by the first three PCA components with the events colored 320 by the RSS value (vellow at the peak of the event). 321

# <sup>322</sup> 2.4 Fixed point skeleton and structured flow

To understand how the resting state manifold arises, we start by considering the 323 uncoupled system, that is, the joint dynamics of the N populations (nodes) in 324 the absence of any inter-population synaptic coupling. This uncoupled system's 325 phase flow is dominated by  $2^N$  stable fixed points that represent all possible 326 combinations of the populations firing at either their low or high mean firing 327 rates (down or up state, respectively). Starting from an initial condition and 328 in the absence of noise, the BNM will settle into the nearest accessible such 329 fixed point, a stable network state composed of a corresponding combination of 330 regions in their up or down state. 331

The dynamical effects of the symmetry breaking in the BNM are delineated 332 by the topology of the connectome. The heterogeneity of the in-degree (to-333 tal connectivity) of individual nodes of the network drives a variation in the 334 relative positioning of the separatrices between the basins of attraction of the 335 equilibrium points, mirrored in the variation of the corresponding projections 336 onto the 2D phase planes of corresponding nodes (see Figure 3A). In conjunc-337 tion, connectivity strength and topology give rise to gradients in the relative 338 attractiveness of the system's equilibrium states. This attractiveness (or sta-339 bility) can be quantified by the largest negative real eigenvalues obtained from 340 the linearization of the system about the respective equilibrium state (linear 341 stability analysis). 342

To map the complete manifold outside the simulated trajectories we sampled 343 the stable fixed points for varying coupling scaling parameter G from the  $2^N$ 344 combinations of up- and down-states, and evaluated their stability (see Methods 345 for more details). We found that the number of stable fixed points in the sample 346 decreases with increasing G. This decrease is due to the loss of states with mixed 347 composition of up- and down-state due to the bifurcation of the down state in 348 nodes with high input (Figure 5A). Projecting the  $\mathbf{r}$  component of the fixed-349 points in the first two eigenvectors of the Laplacian confirms this thinning of the 350 intermediate compositions biased towards those with higher number of nodes in 351 the up-state (corresponds to the first Laplacian eigenvector  $\lambda_1$ ). Additionally, 352 the stability of the fixed points was inversely proportional to the number of 353 nodes in the up-state, that is in the direction of  $E_1$  the first eigenvector of the 354 Laplacian (Figure 5B). 355

To put this into the context of the simulated trajectories, we have next identified the fixed points around which the simulated trajectory evolved by taking initial conditions from the simulated trajectory, and integrating the system without noise to the equilibrium. We have confirmed that in all instances the system reached a stable fixed point composed of combination of up- and down-states, and that the stability of these fixed points follows the same gradient in terms of the composition (Figure 5C).

Furthermore, the nodes of the network exhibit a frequency gradient of the oscillations in the up-state (Figure 5D). This gradient reflects variability of the characteristic frequency in the up-state across nodes in the network. In the fixed-point state, if the nodes are treated as isolated systems with an input

<sup>367</sup> current term based on the existing network state, then

$$r_i^* = r^* + \delta_i^r$$

$$v_i^* = v^* + \delta_i^v$$
(1)

where  $(r^*, v^*)$  are the symmetric fixed-points of the network and  $(\delta_i^r, \delta_i^v)$  are the excursions from the symmetric fixed-point and change according to the existing network state. These excursions depend directly on the in-strength of the *i*th node and the local states of its first neighbours.

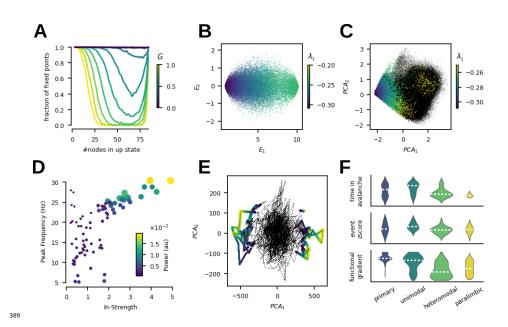
Following linear stability analysis of the *i*th system around the fixed-point (see Methods), the eigenvalues of the Jacobian matrix are given by

$$\lambda_{1,2} = 2v_i^* \pm \sqrt{Jr_i^* - 4\pi^2 r_i^{*^2}}$$
  
=  $2v_i^* \pm \sqrt{2Jr_i^* - 4\pi^2 r_i^{*^2} + 2J\delta_i^r - 4\pi^2 \delta_i^{r^2} - 8\pi^2 r^* \delta_i^r}$  (2)

From the above equation, we see that the frequency of oscillations in the upstate of the *i*th node increases proportional to  $\delta_i^r$  and therefore proportional to the in-strength of the node, which we also observe in the simulation (Figure 5D).

Furthermore, applying the PCA projection on the empirical BOLD timeseries, we have identified a similar separation of the event trajectories in the global embedding as observed in the simulations (Figure 5E). However in the case of the empirical data the system exhibits both the co-activations and codeactivations as seen on the separation through the first PCA component.

Lastly, symmetry breaking by the connectivity alone results in a spatial organization of the above described flow which is aligned with empirically observed trends (Figure 5F). In particular, per region the time spent in avalanches and the cumulative z-scored BOLD signal within events both decrease across the cortical hierarchy from the primary to paralimbic regions. The established principal functional gradient extracted from the empirical fMRI data is also aligned along this axis.



Mechanistic structure of the manifold. (A) Composition of Figure 5: 390 the sampled stable fixed points in terms of number of nodes in the up-state as 391 a function of G, normalized to G = 0. (B) Projection of the stable fixed points 392 into the first two leading eigenmodes of the network Laplacian  $E_1, E_2$ , color 393 coded with the value of the largest eigenvalue in the linear stability analysis. 394 (C) Fixed points (colored) derived by noise-free integration to equilibrium from 395 the trace of a simulation (black) in the working point  $G_w$ , color coded by the 396 value of the largest eigenvalue  $\lambda_1$ . (D) Frequency peak in the simulated source 397 activity of each of the regions plotted against the node structural connectivity 398 in-strength. (E) Empirical BOLD time series projected into the first two PCA 399 components with the events colored by the RSS value (yellow in the peak). (F) 400 Across the cortical hierarchy, the time spent in avalanches of the  $\mathbf{r}(t)$  time-401 series (top) decreases, as does the cumulative z-scored simulated BOLD from 402 the event time-segments (middle). The spatial distribution of the principal 403 functional gradient extracted from empirical fMRI is also aligned along the 404 cortical hierarchy (bottom). 405

# 406 3 Discussion

<sup>407</sup> Using a combination of computational modeling and dynamical systems analy-<sup>408</sup> sis we have provided a complete mechanistic description in terms of constituent <sup>409</sup> entities and their causal activities leading to spontaneous co-activations and <sup>410</sup> neuronal cascades in the brain's resting state [44]. We showed how the breaking <sup>411</sup> of the symmetry of the BNM's connectivity gives rise to the structured low-<sup>412</sup> dimensional dynamics in the phase space and recurrent fluctuations of the func-

tional connectivity (Figure 2). These fluctuations arise from network-mediated 413 cascades of up- and down-state switching and capture well the empirically found 414 relationship between the strong co-activation events and the recurrence struc-415 ture reflected by the functional connectivity dynamics (Figure 3). The subspace 416 accessible to the brain in this regime was charted and partitioned according to 417 the characteristic flow associated with each of the partition (Figure 4). Finally, 418 this subspace and its associated flows arise from the rich fixed point structure 419 of the system and the differential stability of the nodes in these fixed points is 420 not only reflected in the propensity to state switching that reflects the corti-421 cal hierarchy, but also influences the dominant oscillation frequency (Figure 5). 422 In summary, these results support our hypothesis that the recurrent functional 423 connectivity states of the resting state correspond to distinct subspaces on a 424 low-dimensional manifold associated with distinct structured flows. 425

The central result of our work is that the symmetry breaking via the struc-426 tural connectivity carves out an attractive subspace of all the possible states 427 of the brain, and that the flow on this manifold governs the characteristic dy-428 namics of the brain (that is discarding the transient towards the manifold from 429 arbitrary initial conditions). In this regime, the model captures the multistabil-430 ity and noise-driven exploration of the dynamic repertoire explored previously 431 in computational studies [2, 31, 32, 51, 52]. The data features extracted from 432 the time series provides a link between the empirical data and the model. Here, 433 the functional structure in the brain is carried by the rare high amplitude co-434 fluctuation events as it was previously demonstrated in empirical fMRI data [19, 435 21, 53], and in simultaneous EEG and fMRI measurements [44]. Similarly, re-436 cent modeling study has shown the role of structural modules of the network in 437 shaping the co-fluctuation events [54], which is aligned with the brain network 438 as the symmetry breaking gives rise to the low-dimensional dynamics. 439

The slow time scale fluctuations of the dynamical functional connectivity 440 reflect the movement of the brain activity between the low- and high-activity 441 subspaces of the manifold. The flow in the high-activity subspace supports 442 the cascades, which in turn are reflected in the high activity coactivations. This 443 movement points to the multistable rather then metastable interpretation of the 444 resting state dynamics [55], and reflects the observation of switching between a 445 low-amplitude incoherent and high-amplitude coherent states in empirical data 446 [56]. Furthermore, the slow transitions between the high- and low-activity sub-447 spaces is compatible with the reports on the spontaneous infra-slow brain ac-448 tivity [57, 58] and the detailed reports on its spatio-temporal structure. For 449 example, the slow traveling waves [22] propagating along the principal gradi-450 ent of cortical organization [59, 60] would provide a refined description of the 451 trajectory through the manifold subspaces. 452

The attractive subspace of the low-dimensional manifold and the associated structured flow arise in the presented system from the changes in the fixedpoint structure due to the irregular connectivity. In particular, the network input mediates the modulation of the escape times of the noise-induced transitions. These chain into domino-like sequences [61, 62], which in turn constitute the neuronal cascades. On a network level, our results elaborate the previous

analytical results of increased entropy of the attractors in an Ising-spin network 459 model for intermediate values of coupling strength [63]. The relationship to 460 the dimensionality of the exhibited dynamics is such that for the low values of 461 coupling strength G, where the Ising model is in the trivial state with all spins 462 equal to 0, the model presented here is also driven by noise to the all-down 463 state due to the significantly larger basin of attraction of the down-state, and 464 the nodes make uncoordinated noise-driven excursions to the up-state reflected 465 in the high-dimensionality of the dynamics. For high values of G the situation is 466 opposite, and for intermediate values of G the Ising model exhibits high entropy 467 of attractors, which is in our case reflected in the available states organized in 468 the low-dimensional manifold with the structured flow governed by the stability 469 of these states. 470

Overall, the movement of the system through the subspaces of a low-dimensional 471 manifold is in accordance with empirical and modeling results on recurrence and 472 state clustering of resting state fMRI BOLD recordings. Using clustering algo-473 rithms to partition the BOLD time series yields statistically similar and tem-474 porally recurrent whole brain spatial coactivation patterns [18, 45] associated 475 with specific dwell times and transition probabilities. However, compared to the 476 clustering approaches applied to the BOLD time-series, the SFMs allows us to 477 refine the partitioning of the state-space in two aspects: we unfold the subspaces 478 based on the similarity of the coactivations on the level of the BOLD signal, and 479 we provide a detailed description of the flow of the system in these subspaces 480 e.g. in terms of the cascades. The former is in line with the recent advances 481 regarding the low-dimensional representation of meso- [64, 65] and macroscopic 482 [66, 67] brain dynamics, but the latter describes the origin of those subspaces 483 as constrained by the connectome. Interestingly, the clustering of phase-locking 484 BOLD states [43] leads to very similar low-dimensional representation of the 485 resting state dynamics to our approach, with a single dominant global phase 486 locked state and a number of transient partially phase-locked states related to 487 functional networks. Similarly, by embedding the resting state data onto the 488 task manifold extracted with the help of diffusion maps, [68] found that rest-489 ing state time-points concentrate in the task-fixation and transition subspaces, 490 and only a minority of time-points reach to the cognitive subspaces of the task 491 manifold. 492

The description of the structured flow addresses also the fast time-scale by in-493 cluding the cascades, which we previously showed to relate to the co-activations 494 observed in the BOLD signal [44]. In EEG literature, the spatio-temporal struc-495 ture of the resting state dynamics is characterized with the help of microstates-496 sensor-level transient patterns lasting on average for 60-150 ms [69]. Attempts 497 have been made to relate the microstates to BOLD activation clusters [70, 71], 498 but identifying the sources generating the microstates with clustering or regres-499 sion analysis has been challenging so far due to unclear relationship between the 500 broadband EEG activity and the BOLD signal fluctuations [72]. To advance 501 we propose to reframe the question as a search for a shared manifold of the 502 neuronal activity with specific slow and fast time-scale characteristics which in 503 turn are reflected in the EEG and the BOLD observables. 504

The manifold we describe is conceptually reminiscent of energy landscapes 505 described in previous works [40, 41]. However, previous energy landscape mod-506 els, such as in [41], implicitly assume energy minimization and thus, by con-507 struction, encode the hypothesis that the activation of two brain regions that 508 are connected via a direct structural connection is more energetically favorable 509 than that of two regions that are not directly connected. We make no such 510 assumption here and, instead, the effective energy landscape emerges, in the 511 form of a low dimensional manifold, out of the interplay of the non-linearity in 512 the local neural mass model and the connectome, thus fully embracing the the 513 network impact, beyond the pair-wise interactions. In addition, previous en-514 ergy landscape analysis [40] assumed that the network changes only gradually 515 by flipping one region at a time, and did not account for transitions in which 516 several regions flip simultaneously. Treating the brain as a whole, the BNM 517 that we presented here instead allows for such latter transitions of the system 518 in state space, which may very well be due to strongly connected regions that 519 are able to simultaneously influence their nearest neighbors during coactivation 520 events. 521

It is worth pointing that our framework covers only one part of the mecha-522 nisms that shape the brain's manifold and the flow on it, that is the connectome. 523 We have assumed identical parameters for each region, ignoring the known struc-524 tural hierarchies [73], which have been shown to improve the predictive value of 525 the BNMs [56, 74, 75]. While we observed differential functional properties of 526 the nodes across the cortical hierarchy [76], we didn't recover the exact spatial 527 correspondence to the established functional gradients [59]. Neuromodulation 528 and the subcortical drives [77] are another missing aspect that similarly improve 529 the performance of BNMs [78]. However, both of these elements are not yet es-530 tablished in the framework of BNMs, as is the impact of the connectome [24]. 531 Thus our goal here is not to generate in silico observables that are as close as 532 possible to the empirical one, which nevertheless differ a lot depending on the 533 preprocessing, e.g. see [79], but to focus on the generative mechanisms for the 534 key data features across time-scales and neuroimaging modalities that render 535 functional activity identifiable across subjects [12, 13, 80]. 536

A natural next step will be to extend the analysis to include the impact 537 of the data-informed regional variance [81] which is now reachable by TVB 538 through Ebrains [82]. Similarly intriguing direction for the extension of the 539 framework presented here is in more refined inclusion of the subcortical struc-540 tures, especially their impact through the neuromodulation. Notably, recent 541 works [77, 83, 84] exploring the role of thalamus, locus coeruleus, and basal nu-542 cleus of Meynert in shaping of the dynamical landscape of the cortical activity 543 are already formulated in the dynamical systems language while incorporating 544 carefully the detailed anatomical and cytoarchitectural knowledge. Integrating 545 these advances in the SFM framework is a natural next step towards the original motivation of SFM, that is to link the mesoscopic neuronal activity to the 547 behaviour, as the intricate interactions between cortex and the subcortical areas 548 are one of the organizing principles of the underlying the biological mechanisms 549 supporting behaviour [85]. 550

Parcellation-induced variation of empirical and simulated brain connectomes
at group and subject levels is another issue that needs to be considered [86].
Nevertheless, we focus on general mechanisms without going on regional level
specificities, so the choice of parcelation should not play such a role.

In conclusion, our results show how the low-dimensional dynamics arises from breaking the symmetry in the brain on the level of the connectome. Describing these dynamics as structured flows on manifolds allows us to bridge the gap between the observational measures and the state-space trajectories of the system. As such, this object is well suited for comparison across different models, scales, and neuroimaging modalities, and provides means for integration of the diverse descriptions of the resting state dynamics.

# <sup>562</sup> 4 Materials and methods

#### 563 4.1 Brain network model

<sup>564</sup> Computational brain network model [87] is used to simulate resting state activity <sup>565</sup> under varying values of network coupling scaling parameter G. The dynamics <sup>566</sup> of each of the network nodes were governed by the neural mass model (NMM) <sup>567</sup> derived analytically as the limit of infinitely all-to-all coupled  $\theta$ -neurons [48], <sup>568</sup> namely for *i*-th node for the firing rate  $r_i$  and membrane potential  $v_i$  as:

$$\tau_c \dot{r}_i = \frac{\Delta}{\pi \tau_c} + 2r_i v_i,$$
  

$$\tau_c \dot{v}_i = v_i^2 + \eta - (\tau_c \pi r_i)^2 + J \tau_c r_i + I_i,$$
(3)

where  $I_i$  is the input current,  $\eta$  is the average neuronal excitability, J is the synaptic weight,  $\Delta$  is the spread of heterogeneous noise distribution, and  $\tau_c$  is the characteristic time.

The N nodes are then coupled with a connectome derived from empirical data as  $^{572}$ 

$$I_{i}(t) = G \sum_{j} W_{ij} r_{j}(t - D_{ij}),$$
(4)

where G is the network scaling parameter,  $W_{ij}$  is the connection weight,  $D_{ij} = L_{ij}/S$  is the delay caused by propagation of the signal on a tract of length  $L_{ij}$  with finite speed S. We picked the speed S = 2m/s from the biologically plausible range [88], and a connectivity matrix of a subject from the Human Connectome Project [49] in the Desikan-Killiany parcellation [89] with 84 regions including subcortical structures.

The equations 3 and 4 comprise the drift  $a(\Psi, t)$  in the stochastic delay differential equation formulation with linear additive noise reading:

$$d\Psi(t) = a(\Psi(t))dt + b(\Psi(t))dW(t), \tag{5}$$

where  $\Psi$  is the state vector  $[\psi_1, ..., \psi_n]$  with  $\psi_n = [r_n, V_n], dW(t)$  is a differential

of a Wiener process with Gaussian increment with variance  $\sigma^2$ , and  $b(\Psi, t) = 1$ 

is the diffusion coefficient—here constant yielding the noise term additive.

The model was implemented in The Virtual Brain [47] and equipped with BOLD forward solution comprising the Balloon-Windkessel model applied to the firing rate  $\mathbf{r}(t)$  [50].

The model parameters  $\eta = -5.0$ , J = 15.0,  $\tau_c = 1.0$ , and  $\Delta = 1.0$  were selected to set the nodes in the bi-stable regime in the absence of coupling [48]. We then varied the global coupling parameter G and the noise variance  $\sigma$ , and simulated 10 minutes of resting state BOLD activity with TR = 720ms after discarding 10 seconds of the initial transient from random initial conditions.

# <sup>593</sup> 4.2 Functional connectivity dynamics

In order to track the time-dependent changes in the functional connectivity, we compute the windowed dynamical functional connectivity  $dFC_w$  [32, 90] and edge dynamical functional connectivity  $dFC_e$  [44]. Starting from the regional BOLD time-series  $B_n(t)$  for each node n, we compute functional connectivity matrices FC(w) for each time window w = 1...W defined as  $B_n(t)|_{t_w}^{t_w+\tau}$  with window length  $\tau = 60s$  and window step size  $t_{(w+1)} - t_w = 2s$ . Next we compute the  $dFC_w$  matrix of order W as

$$dFC_w(i,j) = corr(FC(w_i)^{\Delta}, FC(w_j)^{\Delta}), \tag{6}$$

where  $FC(w)^{\Delta}$  is the vectorized upper part of the FC matrix.

For the window-less  $dFC_e$  [44] we start from the edge time-series [21] defined as  $E_{nm}(t) = z_n(t)z_m(t)$  for n, m = 1...N where  $z_n(t) = \frac{B_n - \mu_n}{\sigma_n}$  is the z-scored BOLD time-series of a node n. The edge dynamical functional connectivity is then computed as correlation between the edge vectors at each pair of time points  $t_1, t_2$ :

$$dFC_e(t_1, t_2) = corr(E_{nm}(t_1), E_{nm}(t_2)).$$
(7)

The co-fluctuation events (CF) are defined as time points in the edge timeseries  $E_{nm}(t)$  during which the root sum squared  $RSS = \sqrt{\sum_{nm} E_{nm}^2(t)}$  crosses a given threshold, here chosen as 95th percentile. Time points where RSS is below the threshold are then labeled as non-events (nCF).

The avalanches were computed on the binary mask  $\mathbf{a}(t)$  on the  $\mathbf{r}(t)$  such that  $a_i(t) = 1 \iff z(r_i(t)) > 3$  where  $z(r_i(t))$  is the z-score of firing rate r of a node i.

#### <sup>614</sup> 4.3 Manifold subspaces

As a first step in the analysis of the local dynamics specific to a particular attractive subspace, we have extracted the time-points belonging to these subspaces with k-means clustering applied to the edge time series  $E_{nm}(t)$ . We varied the number of clusters k and selected k = 5 at which the co-fluctuation events separated to distinct cluster.

To extract the segments of  $\mathbf{r}(t)$  corresponding to the  $E_{nm}(t)$  time points we first estimated the BOLD signal lag  $l = 2500 \,\mathrm{ms}$  as optimal peak-to-peak

alignment with  $\mathbf{r}(t)$  smoothened by a Gaussian filter with same effective width ( $\sigma = 700$ ). Then for all BOLD time points in a given cluster c we selected the 2000 corresponding time points in  $\mathbf{r}(t)$  and concatenate these to get the fast time-scale activity  $\mathbf{r}_c(t)$  in the subspace corresponding to cluster c. Each of the  $\mathbf{r}_c(t)$  was then projected to space spanned by the first two PCA components of the whole  $\mathbf{r}(t)$  time-series to evaluate how much of the overall state-space is covered by individual clusters.

The local trajectory for a given event e was computed by selecting interval  $\mathbf{r}_{e}(t)$  corresponding to BOLD timepoints above the RSS threshold and three timepoints before and after the event. Local  $PCA^{e}$  of was then computed from  $\mathbf{r}_{e}(t)$ , and the smoothened trajectory was computed by convolving  $\mathbf{r}_{e}(t)$  with a Gaussian filter ( $\sigma = 100$ ).

#### <sup>634</sup> 4.4 Manifold sampling

To identify the fixed point scaffold of the manifold as traced by the trajectory resulting from integrating the Equation 5, we sample the segments from the simulated trajectory  $(r_i(t), v_i(t))|_{t_s}^{t_s + \tau_{max}}$ , and use them as initial conditions for integration of the deterministic interpretation of Equation 5, i.e.  $d\Psi(t) = a(\Psi(t))dt$ . From each such an initial condition, we integrated the system to steady state equilibrium corresponding to a fixed point  $(\mathbf{r}^*, \mathbf{v}^*)$ .

The number of stable fixed points  $(\mathbf{r}^*, \mathbf{v}^*)$  of system with G = 0 is  $2^N$ reflecting all the combinations of up- and down-states of the N nodes. To sample the stable fixed points of the system with G > 0 we solve repeatedly the system of equations:

$$0 = \frac{\Delta}{\pi \tau_c} + 2r_i^* v_i^*,$$

$$0 = v_i^{*2} + \eta - (\tau_c \pi r_i^*)^2 + J \tau_c r_i^* + I_i$$
(8)

using Newton-Raphson method with the initial conditions chosen randomly as a vector of up- and down-state fixed points of the isolated nodes, i.e.  $(r_i^{*0}, v_i^{*0}) \in$  $\{(r_{\uparrow}^*, v_{\uparrow}^*), (r_{\downarrow}^*, v_{\downarrow}^*)\}, \forall i$  where  $(r_{\uparrow}^*, v_{\uparrow}^*)$  and  $(r_{\downarrow}^*, v_{\downarrow}^*)$  are the up- and down-state fixed points for the isolated node. For each initial condition  $(\mathbf{r}^{*0}, \mathbf{v}^{*0})$  we then check if the corresponding solution of Equation 8 is equivalent up to the composition in terms of up- and down-states. If not, it is discarded, otherwise we evaluate the stability of the found fixed point using linear stability analysis.

As a low-dimensional projection of the sampled manifold we have used the two slowest eigenmodes of the structural connectivity. These are computed as eigendecomposition of the graph Laplacian  $\mathbf{L} = \mathbf{W} - \mathbf{I}$ , that is  $\mathbf{L}\mathbf{U} = \mathbf{U}\mathbf{\Lambda}$ , where eigenvalues  $\lambda_k$  can be interpreted as structural frequencies and the eigenmodes  $\mathbf{u}_k$  as structural connectome harmonics [91].

# 657 4.5 Linear stability analysis

We perform a linear stability analysis to identify the fixed-points obtained from the NR method. If each fixed-point  $(\mathbf{r}^*, \mathbf{v}^*)$  is perturbed by  $(\epsilon^{\mathbf{r}}, \epsilon^{\mathbf{v}})$ , then the

evolution of the perturbations depend on the Jacobian matrix (J) and are given by:

$$\begin{bmatrix} \dot{\epsilon}_{1}^{r} \\ \dot{\epsilon}_{2}^{r} \\ \vdots \\ \dot{\epsilon}_{N}^{r} \\ \dot{\epsilon}_{V}^{v} \\ \dot{\epsilon}_{V}^{v} \\ \dot{\epsilon}_{V}^{v} \\ \dot{\epsilon}_{N}^{v} \\ \dot{\epsilon}_{N}^{v}$$

The stability of a fixed-point depends on the eigenvalues of the Jacobian evaluated at the fixed-point. The fixed-point is stable iff all the eigen-values of J are negative. Therefore, we numerically evaluate the largest eigenvalue of Jacobian for each fixed-point and label the point as stable if its real-part is negative.

## 4.6 Fixed point sampling from simulated trajectory

From a given trajectory of the system given as 10 minutes of  $\psi(t)$  we have selected a restart point t' each 50 ms (12000 starting points altogether). For each of the restart point t' we extracted the segment  $\Psi(t)|_{t'}^{t'-\tau_{max}}$  where  $\tau_{max}$ is the length of the longest delay, and used as initial condition to a equivalent system to Equation 5 with b = 0:

$$d\Psi(t) = a(\Psi(t))dt. \tag{10}$$

Integrating this system to equilibrium yielded then for each restart point  $t_r$  a fixed point  $\Psi^* = (\mathbf{r}^*, \mathbf{v}^*)$ . The stability of each of the fixed points  $\Psi^*$  was then evaluated using the linear stability analysis as the largest eigenvalue of the respective Jacobian matrix.

# <sup>676</sup> 4.7 Escape time analysis

The switching behaviour of a single node is driven by the stability of the up-677 and down-state fixed points in the presence of noise. We employed escape time 678 analysis [92] to measure the stability of these fixed points for range of values of 679 external input I. In detail, for a single node of the system given by Equation 3 we found the up- and down-state stable fixed points  $(r^*, v^*)^{\uparrow}$  and  $(r^*, v^*)^{\downarrow}$ , and 681 the unstable saddle node  $(r^*, v^*)^{\times}$ . Next we computed the separatrix between 682 the two basins of attraction by integration of the model backwards in time 683 resulting in an closed curve  $\omega$ . To find the characteristic escape time for a 684 fixed point  $(r^*, v^*)$  we have integrated the system from the initial condition 685  $(r_0, v_0) = (r^*, v^*)$  for a given value of I 100 times, measuring the time  $t_E$  at 686 which the trajectory crosses  $\omega$  for the first time. The values of I were drawn 687

from the range given by  $[0, I_{\text{max}}]$  where  $I_{\text{max}} = \max\{I_i(t), \forall i\}$  is the largest value of  $I_i$  encountered in the integration of the full system in the working point.

# <sup>690</sup> 4.8 Empirical data and spatial analysis

The functional gradient on empirical data was computed from the group connectivity matrix of the Human Connectome Project dataset using the brainspace toolbox [93]. For a simulated resting state session with  $G_w$ , the time in avalanche was computed for each node as total time for which the  $\mathbf{r}_i(t)$  was above the threshold of 3 standard deviations, and the event z-score as a sum of z-scored BOLD signal in time-points marked as events. The nodes were then groupped according to the cortical hieararchy [76] projected to the Desikan-Killiany parcellation.

A parcellation-based BOLD signals of a resting-state session from a subject from the Human Connectome Project [94] were used to validate the separation of the events in the low-dimensional embedding. The data consisted of 1200 time points sampled at 720 ms in the Desikan-Killiany parcellation [89] with 70 cortical regions.

# 704 **References**

- Honey, C. J., Kötter, R., Breakspear, M. & Sporns, O. Network structure of cerebral cortex shapes functional connectivity on multiple time scales.
   *Proc. Natl. Acad. Sci. U. S. A.* **104**, 10240–10245. ISSN: 0027-8424 (June 2007).
- Ghosh, A., Rho, Y, McIntosh, A. R., Kötter, R & Jirsa, V. K. Noise during rest enables the exploration of the brain's dynamic repertoire. en. *PLoS Comput. Biol.* 4, e1000196. ISSN: 1553-734X, 1553-7358 (Oct. 2008).
- Deco, G., Jirsa, V. K. & McIntosh, A. R. Resting brains never rest: computational insights into potential cognitive architectures. en. *Trends Neurosci.* 36, 268–274. ISSN: 0166-2236, 1878-108X (May 2013).
- Cabral, J., Kringelbach, M. L. & Deco, G. Functional connectivity dynamically evolves on multiple time-scales over a static structural connectome: Models and mechanisms. en. *Neuroimage* 160, 84–96. ISSN: 1053-8119, 1095-9572 (Oct. 2017).
- 5. Gusnard, D. A., Raichle, M. E. & Raichle, M. E. Searching for a baseline:
  functional imaging and the resting human brain. en. *Nat. Rev. Neurosci.*2, 685–694. ISSN: 1471-003X (Oct. 2001).
- Damoiseaux, J. S. *et al.* Consistent resting-state networks across healthy
   subjects. en. *Proc. Natl. Acad. Sci. U. S. A.* 103, 13848–13853. ISSN: 0027 8424 (Sept. 2006).
- 7. Deco, G., Jirsa, V. K. & McIntosh, A. R. Emerging concepts for the dynamical organization of resting-state activity in the brain. en. *Nat. Rev. Neurosci.* 12, 43–56. ISSN: 1471-003X, 1471-0048 (Jan. 2011).

- 8. Hutchison, R. M. *et al.* Dynamic functional connectivity: promise, issues, and interpretations. en. *Neuroimage* 80, 360–378. ISSN: 1053-8119, 1095-9572 (Oct. 2013).
- Preti, M. G., Bolton, T. A. & Van De Ville, D. The dynamic functional connectome: State-of-the-art and perspectives. en. *Neuroimage* 160, 41– 54. ISSN: 1053-8119, 1095-9572 (Oct. 2017).
- Shine, J. M. *et al.* The Dynamics of Functional Brain Networks: Integrated
  Network States during Cognitive Task Performance. en. *Neuron* 92, 544–
  554. ISSN: 0896-6273, 1097-4199 (Oct. 2016).
- Cavanna, F., Vilas, M. G., Palmucci, M. & Tagliazucchi, E. Dynamic functional connectivity and brain metastability during altered states of consciousness. en. *Neuroimage* 180, 383–395. ISSN: 1053-8119, 1095-9572 (Oct. 2018).
- Battaglia, D. *et al.* Dynamic Functional Connectivity between order and
   randomness and its evolution across the human adult lifespan. en. *Neuroimage* 222, 117156. ISSN: 1053-8119, 1095-9572 (Nov. 2020).
- Petkoski, S., Ritter, P. & Jirsa, V. K. White-matter degradation and dy namical compensation support age-related functional alterations in human
   brain. Cerebral Cortex bhac500, 1–16 (2023).
- <sup>747</sup> 14. Braun, U. *et al.* From Maps to Multi-dimensional Network Mechanisms of Mental Disorders. en. *Neuron* 97, 14–31. ISSN: 0896-6273, 1097-4199 (Jan. 2018).
- Jones, D. T. *et al.* Non-stationarity in the "resting brain's" modular architecture. en. *PLoS One* 7, e39731. ISSN: 1932-6203 (June 2012).
- <sup>752</sup> 16. Zalesky, A., Fornito, A., Cocchi, L., Gollo, L. L. & Breakspear, M. Time <sup>753</sup> resolved resting-state brain networks. en. *Proc. Natl. Acad. Sci. U. S. A.* <sup>754</sup> **111**, 10341–10346. ISSN: 0027-8424, 1091-6490 (July 2014).
- <sup>755</sup> 17. Baker, A. P. *et al.* Fast transient networks in spontaneous human brain <sup>756</sup> activity. en. *Elife* **3**, e01867. ISSN: 2050-084X (Mar. 2014).
- Beim Graben, P. et al. Metastable Resting State Brain Dynamics. Frontiers
   in Computational Neuroscience 13 (2019).
- Tagliazucchi, E., Balenzuela, P., Fraiman, D. & Chialvo, D. R. Criticality in large-scale brain FMRI dynamics unveiled by a novel point process
  analysis. en. *Front. Physiol.* 3, 15. ISSN: 1664-042X (Feb. 2012).
- Liu, X., Zhang, N., Chang, C. & Duyn, J. H. Co-activation patterns in resting-state fMRI signals. en. *Neuroimage* 180, 485–494. ISSN: 1053-8119, 1095-9572 (Oct. 2018).
- Esfahlani, F. Z. *et al.* High-amplitude cofluctuations in cortical activity
  drive functional connectivity. en. *Proc. Natl. Acad. Sci. U. S. A.* 117,
  28393–28401. ISSN: 0027-8424, 1091-6490 (Nov. 2020).

- <sup>768</sup> 22. Gu, Y. *et al.* Brain Activity Fluctuations Propagate as Waves Traversing
   <sup>769</sup> the Cortical Hierarchy. en. *Cereb. Cortex* **31**, 3986–4005. ISSN: 1047-3211,
   <sup>770</sup> 1460-2199 (July 2021).
- Mišić, B. *et al.* Network-Level Structure-Function Relationships in Human
   Neocortex. en. *Cereb. Cortex* 26, 3285–3296. ISSN: 1047-3211, 1460-2199
   (July 2016).
- Breakspear, M. Dynamic models of large-scale brain activity. en. Nat. Neurosci. 20, 340–352. ISSN: 1097-6256, 1546-1726 (Feb. 2017).
- O'Byrne, J. & Jerbi, K. How critical is brain criticality? Trends in Neuro sciences (2022).
- Deco, G., Jirsa, V., McIntosh, A. R., Sporns, O. & Kötter, R. Key role
   of coupling, delay, and noise in resting brain fluctuations. en. *Proc. Natl. Acad. Sci. U. S. A.* **106**, 10302–10307. ISSN: 0027-8424, 1091-6490 (June 2009).
- <sup>782</sup> 27. Deco, G. *et al.* Dynamical consequences of regional heterogeneity in the
  <sup>783</sup> brain's transcriptional landscape. en. *Sci. Adv.* 7, eabf4752. ISSN: 2375<sup>784</sup> 2548 (July 2021).
- <sup>785</sup> 28. Melozzi, F. *et al.* Individual structural features constrain the mouse functional connectome. en. *Proc. Natl. Acad. Sci. U. S. A.* ISSN: 0027-8424, 1091-6490 (Dec. 2019).
- Shine, J. M. et al. The dynamic basis of cognition: an integrative core under the control of the ascending neuromodulatory system en. May 2018.
- Roberts, J. A. *et al.* Metastable brain waves. en. *Nat. Commun.* 10, 1056.
   ISSN: 2041-1723 (Mar. 2019).
- Courtiol, J., Guye, M., Bartolomei, F., Petkoski, S. & Jirsa, V. K. Dy namical mechanisms of interictal resting-state functional connectivity in
   epilepsy. *Journal of Neuroscience* 40, 5572–5588. ISSN: 15292401 (2020).
- Hansen, E. C. A., Battaglia, D., Spiegler, A., Deco, G. & Jirsa, V. K.
  Functional connectivity dynamics: modeling the switching behavior of the
  resting state. en. *Neuroimage* 105, 525–535. ISSN: 1053-8119, 1095-9572
  (Jan. 2015).
- Machamer, P., Darden, L. & Craver, C. F. Thinking about mechanisms.
   *Philosophy of science* 67, 1–25 (2000).
- <sup>801</sup> 34. Jirsa, V. in Selbstorganisation-ein Paradigma f
  ür die Humanwissenschaften
  <sup>802</sup> 89–102 (Springer, 2020).
- <sup>803</sup> 35. Pillai, A. S. & Jirsa, V. K. Symmetry Breaking in Space-Time Hierarchies
  <sup>804</sup> Shapes Brain Dynamics and Behavior. *Neuron* 94, 1010–1026 (2017).
- Huys, R., Perdikis, D. & Jirsa, V. K. Functional architectures and structured flows on manifolds: a dynamical framework for motor behavior. en. *Psychol. Rev.* 121, 302–336. ISSN: 0033-295X, 1939-1471 (July 2014).

- McIntosh, A. R. & Jirsa, V. K. The hidden repertoire of brain dynamics
  and dysfunction. en. Netw Neurosci 3, 994–1008. ISSN: 2472-1751 (Sept. 2019).
- <sup>811</sup> 38. Woodman, M. M. & Jirsa, V. K. Emergent dynamics from spiking neuron networks through symmetry breaking of connectivity. *PloS one* 8, e64339 (2013).
- Watanabe, T. *et al.* A pairwise maximum entropy model accurately describes resting-state human brain networks. *Nature communications* 4, 1–10 (2013).
- 40. Watanabe, T. *et al.* Energy landscapes of resting-state brain networks. en. *Front. Neuroinform.* 8, 12. ISSN: 1662-5196 (Feb. 2014).
- <sup>819</sup> 41. Gu, S. *et al.* The Energy Landscape of Neurophysiological Activity Implicit
  <sup>820</sup> in Brain Network Structure. en. *Sci. Rep.* 8, 2507. ISSN: 2045-2322 (Feb.
  <sup>821</sup> 2018).
- Ashourvan, A., Gu, S., Mattar, M. G., Vettel, J. M. & Bassett, D. S.
  The energy landscape underpinning module dynamics in the human brain
  connectome. *Neuroimage* 157, 364–380 (2017).
- Vohryzek, J., Deco, G., Cessac, B., Kringelbach, M. L. & Cabral, J. Ghost
  Attractors in Spontaneous Brain Activity: Recurrent Excursions Into FunctionallyRelevant BOLD Phase-Locking States. en. *Front. Syst. Neurosci.* 14, 20.
  ISSN: 1662-5137 (Apr. 2020).
- Rabuffo, G., Fousek, J., Bernard, C. & Jirsa, V. Neuronal Cascades Shape
  Whole-Brain Functional Dynamics at Rest. en. *eNeuro* 8. ISSN: 2373-2822
  (Sept. 2021).
- 45. Cornblath, E. J. *et al.* Temporal sequences of brain activity at rest are constrained by white matter structure and modulated by cognitive demands.
  en. *Commun Biol* 3, 261. ISSN: 2399-3642 (May 2020).
- 46. Wong, K.-F. & Wang, X.-J. A recurrent network mechanism of time integration in perceptual decisions. *Journal of Neuroscience* 26, 1314–1328
  (2006).
- 47. Sanz-Leon, P. *et al.* The Virtual Brain: a simulator of primate brain net work dynamics. *Frontiers in neuroinformatics* 7 (2013).
- 48. Montbrió, E., Pazó, D. & Roxin, A. Macroscopic description for networks
  of spiking neurons. *Physical Review X* 5, 021028 (2015).
- <sup>842</sup> 49. Van Essen, D. C. *et al.* The WU-Minn human connectome project: an
  <sup>843</sup> overview. *Neuroimage* 80, 62–79 (2013).
- Stephan, K. E., Weiskopf, N., Drysdale, P. M., Robinson, P. A. & Friston,
  K. J. Comparing hemodynamic models with DCM. en. *Neuroimage* 38, 387–401. ISSN: 1053-8119 (Nov. 2007).
- <sup>847</sup> 51. Deco, G. & Jirsa, V. K. Ongoing cortical activity at rest: criticality, multi <sup>848</sup> stability, and ghost attractors. en. J. Neurosci. **32**, 3366–3375. ISSN: 0270 <sup>849</sup> 6474, 1529-2401 (Mar. 2012).

- Bio 52. Haimovici, A., Tagliazucchi, E., Balenzuela, P. & Chialvo, D. R. Brain organization into resting state networks emerges at criticality on a model of the human connectome. en. *Phys. Rev. Lett.* **110**, 178101. ISSN: 0031-9007, 1079-7114 (Apr. 2013).
- Sporns, O., Faskowitz, J., Teixeira, A. S., Cutts, S. A. & Betzel, R. F. Dynamic expression of brain functional systems disclosed by fine-scale analysis of edge time series. en. *Netw. Neurosci.* 5, 405–433. ISSN: 2472-1751 (Apr. 2021).
- <sup>858</sup> 54. Pope, M., Fukushima, M., Betzel, R. F. & Sporns, O. Modular origins of high-amplitude cofluctuations in fine-scale functional connectivity dynamics. en. *Proc. Natl. Acad. Sci. U. S. A.* **118.** ISSN: 0027-8424, 1091-6490
  <sup>861</sup> (Nov. 2021).
- <sup>862</sup> 55. Cocchi, L., Gollo, L. L., Zalesky, A. & Breakspear, M. Criticality in the
  <sup>863</sup> brain: A synthesis of neurobiology, models and cognition. en. *Prog. Neurobiol.* 158, 132–152. ISSN: 0301-0082, 1873-5118 (Nov. 2017).
- Kong, X. *et al.* Sensory-motor cortices shape functional connectivity dynamics in the human brain. en. *Nat. Commun.* **12**, 6373. ISSN: 2041-1723
  (Nov. 2021).
- Fox, M. D. & Raichle, M. E. Spontaneous fluctuations in brain activity
  observed with functional magnetic resonance imaging. en. Nat. Rev. Neurosci. 8, 700–711. ISSN: 1471-003X (Sept. 2007).
- 58. Mitra, A. *et al.* Spontaneous Infra-slow Brain Activity Has Unique Spatiotemporal Dynamics and Laminar Structure. en. *Neuron* 98, 297–305.e6.
  ISSN: 0896-6273, 1097-4199 (Apr. 2018).
- Margulies, D. S. *et al.* Situating the default-mode network along a principal gradient of macroscale cortical organization. en. *Proc. Natl. Acad. Sci. U. S. A.* **113**, 12574–12579. ISSN: 0027-8424, 1091-6490 (Nov. 2016).
- Huntenburg, J. M., Bazin, P.-L. & Margulies, D. S. Large-Scale Gradients
  in Human Cortical Organization. en. *Trends Cogn. Sci.* 22, 21–31. ISSN:
  1364-6613, 1879-307X (Jan. 2018).
- Ashwin, P., Creaser, J. & Tsaneva-Atanasova, K. Fast and slow domino
   regimes in transient network dynamics. en. *Phys Rev E* 96, 052309. ISSN:
   2470-0053, 2470-0045. arXiv: 1701.06148 [math.DS] (Nov. 2017).
- Ashwin, P., Creaser, J. & Tsaneva-Atanasova, K. Sequential escapes: onset of slow domino regime via a saddle connection. *Eur. Phys. J. Spec. Top.* 227, 1091–1100. ISSN: 1951-6355, 1951-6401 (Nov. 2018).
- 63. Deco, G., Senden, M. & Jirsa, V. How anatomy shapes dynamics: a semianalytical study of the brain at rest by a simple spin model. en. *Front. Comput. Neurosci.* 6, 68. ISSN: 1662-5188 (Sept. 2012).
- Kim, J., Joshi, A., Frank, L. & Ganguly, K. Cortical-hippocampal coupling during manifold exploration in motor cortex. *Nature*. ISSN: 14764687 (2022).

- <sup>892</sup> 65. Chaudhuri, R., Gerçek, B., Pandey, B., Peyrache, A. & Fiete, I. The intrinsic attractor manifold and population dynamics of a canonical cognitive circuit across waking and sleep. *Nature Neuroscience* 22, 1512–1520. ISSN: 15461726 (2019).
- <sup>896</sup> 66. Favaretto, C. *et al.* Subcortical-cortical dynamical states of the human
  <sup>897</sup> brain and their breakdown in stroke. *Nature Communications* 13. ISSN:
  <sup>898</sup> 20411723 (2022).
- <sup>899</sup> 67. Rué-Queralt, J. *et al.* Decoding brain states on the intrinsic manifold of
  <sup>900</sup> human brain dynamics across wakefulness and sleep. *Communications Bi-*<sup>901</sup> ology 4, 1–11. ISSN: 23993642 (2021).
- Gao, S., Mishne, G. & Scheinost, D. Nonlinear manifold learning in functional magnetic resonance imaging uncovers a low-dimensional space of brain dynamics. en. *Hum. Brain Mapp.* 42, 4510–4524. ISSN: 1065-9471, 1097-0193 (Oct. 2021).
- Michel, C. M. & Koenig, T. EEG microstates as a tool for studying the
   temporal dynamics of whole-brain neuronal networks: A review. en. Neuroimage 180, 577–593. ISSN: 1053-8119, 1095-9572 (Oct. 2018).
- <sup>909</sup> 70. Britz, J., Van De Ville, D. & Michel, C. M. BOLD correlates of EEG
  <sup>910</sup> topography reveal rapid resting-state network dynamics. *Neuroimage* 52,
  <sup>911</sup> 1162–1170 (2010).
- <sup>912</sup> 71. Bréchet, L. *et al.* Capturing the spatiotemporal dynamics of self-generated,
  <sup>913</sup> task-initiated thoughts with EEG and fMRI. en. *Neuroimage* 194, 82–92.
  <sup>914</sup> ISSN: 1053-8119, 1095-9572 (July 2019).
- 72. Drew, P. J., Mateo, C., Turner, K. L., Yu, X. & Kleinfeld, D. Ultra-slow oscillations in fMRI and resting-state connectivity: neuronal and vascular contributions and technical confounds. *Neuron* **107**, 782–804 (2020).
- <sup>918</sup> 73. Wang, X.-J. Macroscopic gradients of synaptic excitation and inhibition in the neocortex. *Nature Reviews Neuroscience* **21**, 169–178 (2020).
- 74. Chaudhuri, R., Knoblauch, K., Gariel, M. A., Kennedy, H. & Wang, X. J.
  A Large-Scale Circuit Mechanism for Hierarchical Dynamical Processing in the Primate Cortex. *Neuron* 88, 419–431. ISSN: 10974199 (2015).
- <sup>923</sup> 75. Wang, P. *et al.* Inversion of a large-scale circuit model reveals a cortical hi<sup>924</sup> erarchy in the dynamic resting human brain. *Science advances* 5, eaat7854
  <sup>925</sup> (2019).
- 76. Mesulam, M. M. From sensation to cognition. en. Brain 121 ( Pt 6),
   1013-1052. ISSN: 0006-8950 (June 1998).
- <sup>928</sup> 77. Shine, J. M. The thalamus integrates the macrosystems of the brain to <sup>929</sup> facilitate complex, adaptive brain network dynamics 2020.

<sup>930</sup> 78. Kringelbach, M. L. *et al.* Dynamic coupling of whole-brain neuronal and
<sup>931</sup> neurotransmitter systems. *Proceedings of the National Academy of Sciences*<sup>932</sup> 117, 9566–9576 (2020).

- <sup>933</sup> 79. Aquino, K. M. *et al.* On the intersection between data quality and dynamical modelling of large-scale fMRI signals. *NeuroImage* 256, 119051. ISSN: 10959572 (2022).
- <sup>936</sup> 80. Van De Ville, D., Farouj, Y., Preti, M. G., Liégeois, R. & Amico, E. When
  <sup>937</sup> makes you unique: temporality of the human brain fingerprint. *Science*<sup>938</sup> advances 7, eabj0751 (2021).
- Amunts, K., Mohlberg, H., Bludau, S. & Zilles, K. Julich-Brain: A 3D probabilistic atlas of the human brain's cytoarchitecture. *Science* 369, 988–992. ISSN: 10959203 (2020).
- Schirner, M. et al. Brain simulation as a cloud service: The Virtual Brain
  on EBRAINS. NeuroImage 251, 118973. ISSN: 10959572 (2022).
- <sup>944</sup> 83. Müller, E. J., Munn, B. R. & Shine, J. M. Diffuse neural coupling mediates
  <sup>945</sup> complex network dynamics through the formation of quasi-critical brain
  <sup>946</sup> states. *Nat. Commun.* **11.** ISSN: 2041-1723 (Dec. 2020).
- <sup>947</sup> 84. Munn, B. R., Müller, E. J., Wainstein, G. & Shine, J. M. The ascending
  <sup>948</sup> arousal system shapes neural dynamics to mediate awareness of cognitive
  <sup>949</sup> states. en. Nat. Commun. 12, 6016. ISSN: 2041-1723 (Oct. 2021).
- 85. Cisek, P. Resynthesizing behavior through phylogenetic refinement. en. Atten. Percept. Psychophys. 81, 2265–2287. ISSN: 1943-3921, 1943-393X
  (Oct. 2019).
- <sup>953</sup> 86. Domhof, J. W. M., Jung, K., Eickhoff, S. B. & Popovych, O. V. Parcellation<sup>954</sup> induced variation of empirical and simulated brain connectomes at group
  <sup>955</sup> and subject levels. *Network Neuroscience* 5, 798–830 (2021).
- 87. Sanz-Leon, P., Knock, S. A., Spiegler, A. & Jirsa, V. K. Mathematical
  framework for large-scale brain network modeling in The Virtual Brain. *NeuroImage* 111, 385–430 (2015).
- 88. Trebaul, L. *et al.* Probabilistic functional tractography of the human cortex revisited. en. *Neuroimage* **181**, 414–429. ISSN: 1053-8119, 1095-9572 (Nov. 2018).
- <sup>962</sup> 89. Desikan, R. S. *et al.* An automated labeling system for subdividing the
  <sup>963</sup> human cerebral cortex on MRI scans into gyral based regions of interest.
  <sup>964</sup> en. *Neuroimage* **31**, 968–980. ISSN: 1053-8119 (July 2006).
- 90. Allen, E. A. *et al.* Tracking whole-brain connectivity dynamics in the resting state. en. *Cerebral Cortex* 24, 663–676. ISSN: 1047-3211, 1460-2199
   967 (Mar. 2014).
- 968 91. Preti, M. G. & Van De Ville, D. Decoupling of brain function from structure reveals regional behavioral specialization in humans. en. *Nat. Commun.*970 10, 4747. ISSN: 2041-1723 (Oct. 2019).
- 971 92. Yamapi, R, Filatrella, G & Aziz-Alaoui, M. A. Global stability analysis
  972 of birhythmicity in a self-sustained oscillator. en. *Chaos* 20, 013114. ISSN:
  973 1054-1500, 1089-7682 (Mar. 2010).

- 974 93. Vos de Wael, R. *et al.* BrainSpace: a toolbox for the analysis of macroscale gradients in neuroimaging and connectomics datasets. en. *Commun Biol*976 3, 103. ISSN: 2399-3642 (Mar. 2020).
  977 94. Domhof, J. W. M., Jung, K., Eickhoff, S. B. & Popovych, O. V. *Parcellation*-
- based resting-state blood-oxygen-level-dependent (BOLD) signals of a healthy
   cohort (v1.0) 2022.