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# Critique of the cerebralization of notions belonging to the physical sciences and mathematics

Laurent Goffart

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9èmes Journées Complexité - désordre  
Paris  
juin 2023



## Critique of the cerebralization of notions belonging to the physical sciences and mathematics


Laurent Goffart  
[laurent.goffart@cns.fr](mailto:laurent.goffart@cns.fr)

## Confusions

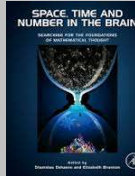
Over the past decades, confusions have arisen in the brain sciences between

- the concepts
- the measurements (and their subsequent numerical transformations) and
- the physiological processes that generate the measured phenomenon.

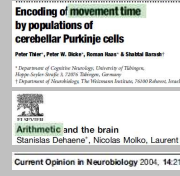
The theoretical notions associated with some measurements led some authors to imagine options such as a "coding" in the brain of space, time, numbers, movement duration, speed, acceleration, geometry and arithmetic.



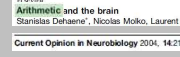
2003



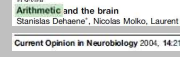
2011



2000



2004



2007

### Compression of visual space before saccades

John Ross<sup>1</sup>, M. Concetta Morrone<sup>1</sup> & David C. Burr<sup>1,2</sup>

Saccadic eye movements cause compression of time as well as space

M Concetta Morrone<sup>1</sup>, John Ross<sup>2</sup> & David Burr<sup>1,4</sup>

*Nature Neuroscience* 2005

### Saccades compress space, time and number

David C. Burr<sup>1,2,3</sup>, John Ross<sup>2</sup>, Paola Binda<sup>4,5</sup> and M. Concetta Morrone<sup>6,7</sup>

*Trends in Cognitive Sciences* 2010

### Recruitment of an Area Involved in Eye Movements During Mental Arithmetic

André Knops<sup>1,2,3</sup>, Bertrand Thirion<sup>2,4</sup>, Edward M. Hubbard<sup>1,2,3</sup>, Vincent Michel<sup>1,2,3,4</sup>, Stanislas Dehaene<sup>1,2,3,5</sup>

*Science* 2009

### Correspondences

#### Eye position predicts what number you have in mind

Tobias Loetscher<sup>1,2</sup>, Christopher J. Boecksch<sup>2,3</sup>, Michael E.R. Nicholls<sup>1</sup> and Peter Brugger<sup>2</sup>

*Current Biology* 2010

## Confusions

Over the past decades, confusions have arisen in the brain sciences between

- the concepts
- the measurements (and their subsequent numerical transformations) and
- the physiological processes that generate the measured phenomenon.

The theoretical notions associated with some measurements led some authors to imagine options such as a "coding" in the brain of space, time, numbers, movement duration, speed, acceleration, geometry and arithmetic.

In the neuroscientific study of movement generation, kinematic parameters were imagined to be "encoded" in the activity of neurons. While they emit spikes, the neurons would "speak a coded language" imbued with notions of classical mechanics.

Yet the movement of a body segment (like the eyeball) is the ultimate measured product, the outcome of multiple processes occurring in parallel in the brain and converging on the groups of neurons which are responsible for muscle contractions.

To the totally ordered structure of numerical values (measured positions of the moving segment), a radically different structure is associated: dynamic, firstly diffuent then confluent, whose elements are spatially distributed and whose activities temporally overlap in the brain.

## What happens in the brain?

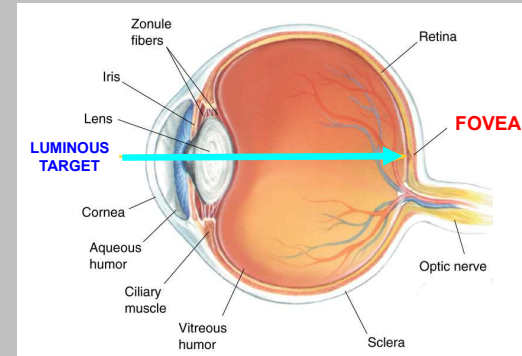
Let's consider the case of a spot of light in the visual field.

At the ends of the optic nerves, the retinal excitation triggers a flow of activity that manifests in the brain as a kind of tree-like explosion (**diffluence**). The evoked activity propagates spatially. It is distributed in multiple groups of neurons whose activities are more or less persistent.

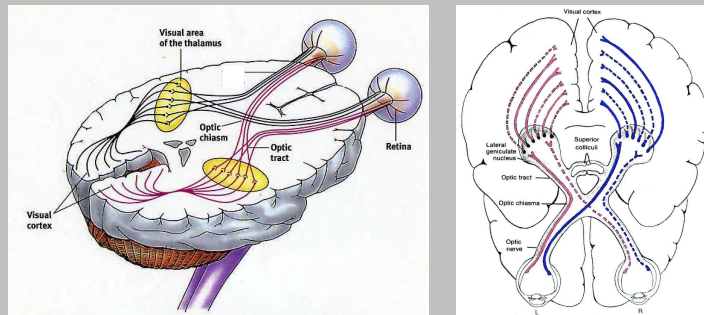
As the activity spreads and joins the groups of cells at the origin of the motor nerves that innervate the muscles, this explosion is transformed into a kind of compression (**confluence**), of which only the duration matters (longer muscle contractions make longer movement) in addition to the recruitment of new groups of muscles (e.g. extraocular + neck muscles → larger gaze shift).

## The eye

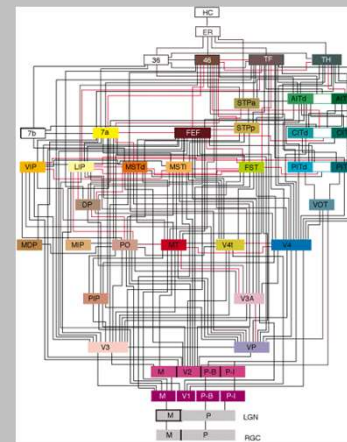
Sagittal view



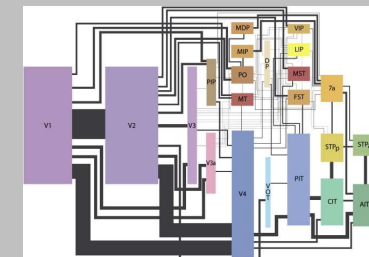
## At the end of the optic nerves



## Explosion of visual signals: spatial and temporal diffluence

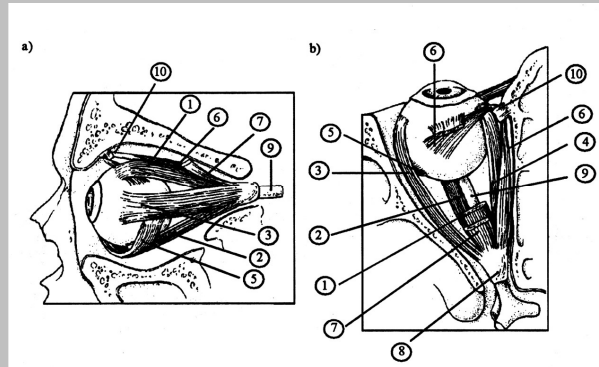


Felleman & Van Essen *Cerebral Cortex* 1991

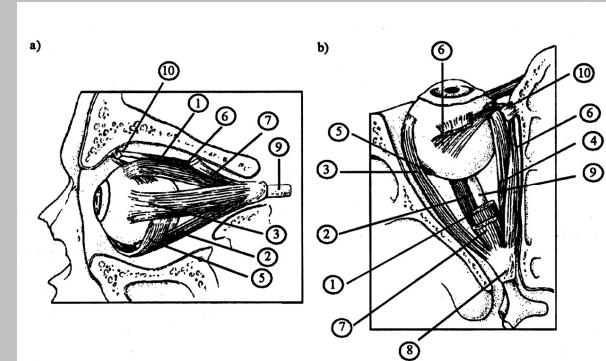


Wallisch & Movshon *Neuron* 2008

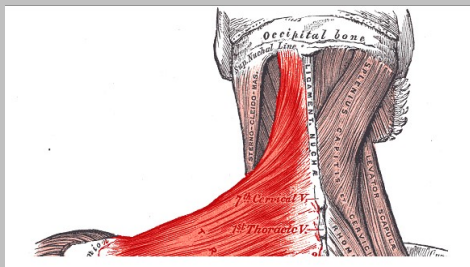
The eye is not only a retina



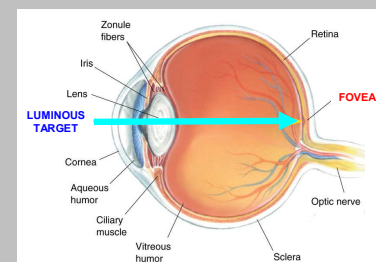
Projection of the target on the fovea  
**IF & only IF** proper contraction of extraocular muscles  
 (direction of gaze)



Projection of the target on the fovea  
**IF & only IF** proper contraction of neck muscles  
 (head direction)



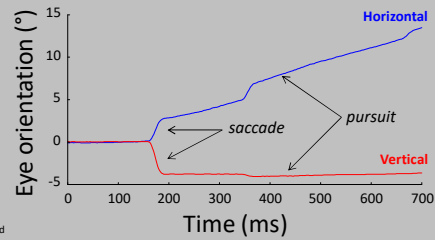
LOOKING AT A TARGET =  
 contracting several muscles



## Two types of eye movement in response to the motion of a visual target

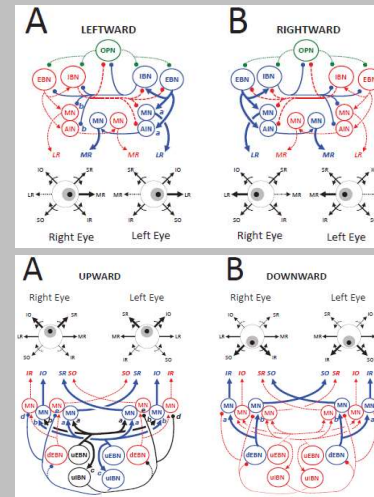
rapid : saccade  
slow : pursuit

same muscles and motor nuclei BUT largely distinct central circuits

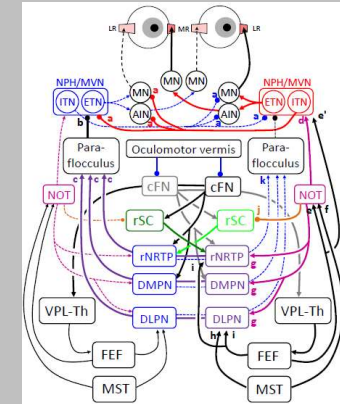


values > 0 : rightward / upward  
value = 0 : straight ahead  
values < 0 : leftward / downward

## SACCADE NETWORK



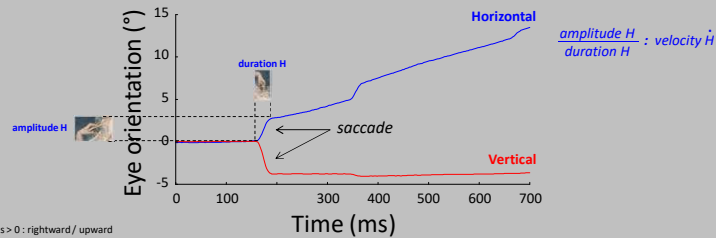
## SLOW PURSUIT NETWORK (Horizontal)



Goffart, Quinet & Boreilly *Encyclopedia Human Brain* submitted

## Two types of eye movement in response to the motion of a visual target

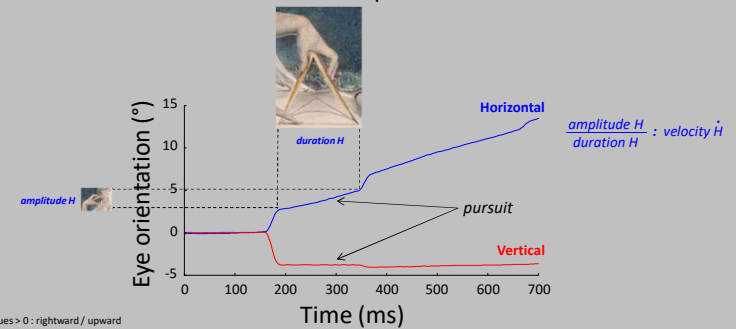
rapid : saccade



values > 0 : rightward / upward  
value = 0 : straight ahead  
values < 0 : leftward / downward

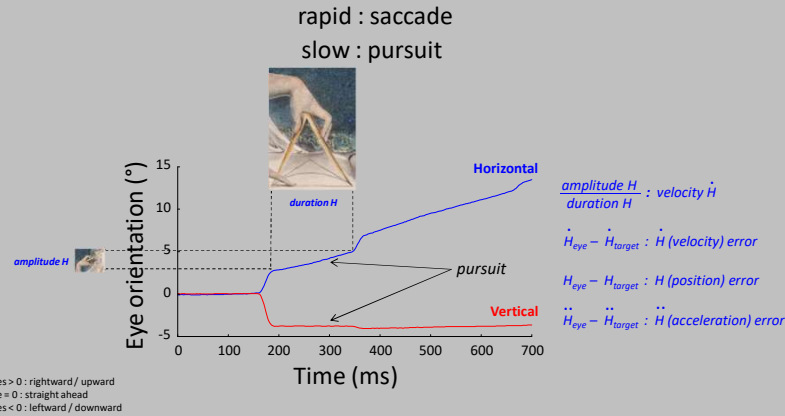
## Two types of eye movement in response to the motion of a visual target

rapid : saccade  
slow : pursuit

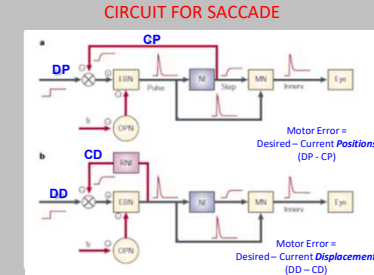
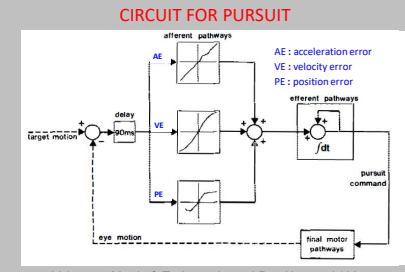


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## Two types of eye movement in response to the motion of a visual target

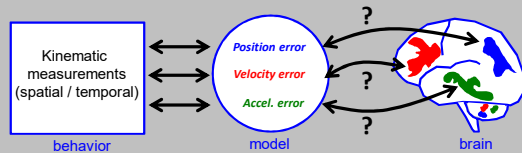


## Dividers, physical quantities and arithmetic in the brain ?



## Dividers, physical quantities and arithmetic in the brain ?

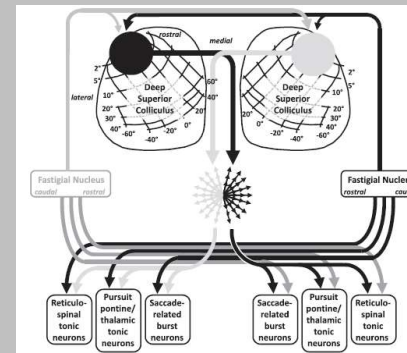
### QUESTIONING THE BRAIN-MODEL ISOMORPHISM



Measuring and modelling the movement of the eyeball (a rigid body) with kinematic notions **does not imply** equivalence in the brain, and notably that the firing of neurons ought to "encode" (i.e., **emit signals related to**) amplitude, velocity or acceleration of the eye movement.

Thinking otherwise does not lead to a dead end.

## New theory



No movement = (poly)-equilibrium  
Movement = restoration of equilibrium

Laurent Goffart, Clary Beaudry, and Jean-Charles Ostner  
The Neurophysiology of Visually Guided Eye Movements: Critical Review and Alternative Viewpoint  
Journal of Neurophysiology 120: 3234-3245, 2018

Kinematics and the neurophysiological study of visually-guided eye movements

Laurent Goffart  
Progress in Brain Research, vol. 249, 375-384, 2019

Aix-Marseille Université  
Ecole Doctorale Sciences de la Vie et de la Santé  
Mention de recherche en Neurosciences  
Laurent Goffart  
pour l'attribution d'une habilitation à diriger les recherches.  
Mars 2020

Aix-Marseille Université  
Ecole Doctorale Sciences de la Vie et de la Santé  
Mention de recherche en Neurosciences  
Laurent Goffart  
Le cerveau est l'empaquetage des neurones et leur fonctionnement dans le fonctionnement cérébral de neurones qui lui sont étrangers

<https://cnrs.hal.science/tel-02968891>

<https://cnrs.hal.science/tel-04070171>

## Brain and Mathematics

**Alain Berthoz (2005):**

Mathematics is "the expression of laws and mechanisms which are in operation in the brain" ... "a language of the brain on the brain itself, that is to say, since the brain is part of the physical world, [a language] about the Universe and its laws"

**Helen De Cruz (2006):**

"The philosophy of mathematics has not paid much attention to the biological context of mathematics, although it goes without saying that mathematicians are biological organisms and that they produce mathematical theories and practices with their brains."

**Jean-Pierre Changeux (1989):**

"Mathematics is in the mathematician's brain, especially in his long-term memory."

**Stanislas Dehaene (2010):**

"Numbers, like all mathematical objects, are mental constructions whose roots lie ... in the adaptation of the human brain to the regularities of the universe."

**Stanislas Dehaene (2012):**

"All human brains receive in sharing, from birth, sophisticated mental representations, independent of language that can be described as 'proto-mathematical'. During its evolution, our brain has been endowed with elementary representations of space, time and number, which we share with many other animal species, and which are the basis of our mathematical intuitions."

## Culture and Mathematics

By reducing the mind to the brain and the intellectual development to brain maturation, many authors have confused an individual's brain functioning with the historical and collective evolution of scientific discourse, which is memorized in books (and stored in libraries) and obeying Boolean logic.

**Fernand Doridot & Marco Panza 2004b**

"the men [and women] who ... managed to constitute abstract objects, and thereby constituted mathematical theories, are ... men [and women] in the plural, concrete men [and women] acting in historically determined conditions, and not sorts of excrescences of a 'cognitive subject'."

**Fernand Doridot & Marco Panza 2004a**

"The constitution of a mathematical theory is a historical fact ... it is the business of a few mathematicians working under determined conditions rather than the realization of a cognitive process specific to the species. And if, ... it is so, then only history can account for this constitution and explain it ... The constitution of a mathematical theory seems to us difficult to conceive other than as a historical process, at least historically situated and determined."

**Sylvain Aurox (1998)**

Cognitive structures are external to the individual. They "depend on external instruments (books, libraries, calculation and observation instruments, etc.) as well as on social structures for the production and accumulation of knowledge (encyclopedias, learned societies, etc.) ... knowledge is a whole process at the same time material, social and collective, never limited, locked up, preserved or produced by individual skills which concern only moments and fragments of it"

**Trends in Cognitive Sciences**  
Review  
The Evolutionary History of Brains for Numbers  
Andreas Nieder<sup>1,\*</sup>  
*Trends in Cognitive Sciences* 2021

**The Adaptive Value of Numerical Competence**  
Andreas Nieder<sup>1,\*</sup>  
*Trends in Ecology and Evolution* 2020

**Evolution of cognitive and neural solutions enabling numerosity judgements: lessons from primates and corvids**  
Andreas Nieder  
*Philosophical Trans. Royal Soc.* 2017

**Representation of Number in the Brain**  
Andreas Nieder<sup>1</sup> and Stanislas Dehaene<sup>2</sup>  
*Annual Review Neuroscience* 2009

**Neuroethology of number sense across the animal kingdom**  
Andreas Nieder  
*Journal of Experimental Biology* 2021

**Neural constraints on human number concepts**  
Andreas Nieder  
*Current Opinion Neurobiology* 2020

**The neuronal code for number**  
Andreas Nieder  
*Nature Reviews Neuroscience* 2016

**Coding of abstract quantity by 'number neurons' of the primate brain**  
Andreas Nieder  
*Journal of Comparative Physiology* 2013

**A parieto-frontal network for visual numerical information in the monkey**  
Andreas Nieder<sup>\*</sup> and Earl K. Miller  
*PNAS* 2004

**Letter**  
Number Faculty Is Rooted in Our Biological Heritage  
Andreas Nieder<sup>1,\*</sup>  
*Trends in Cognitive Sciences* 2017

### Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex

Oana Tudusciuc and Andreas Nieder<sup>\*</sup>  
*PNAS* 2007

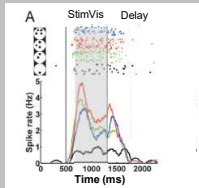
**A** StimVis Delay  
Spike rate (Hz) vs Time (ms)  
sequences of spikes (1 line = 1 trial)

spike  
Time  
synapse  
neuron A (pre-synaptic) axon neuron B (post-synaptic)

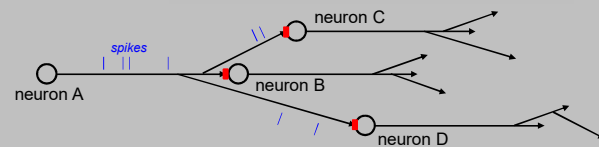
### Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex

Oana Tudusciuc and Andreas Nieder\*

PNAS 2007



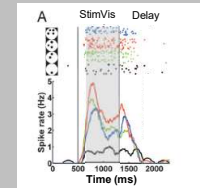
1 neuron (A) contacts several post-synaptic neurons (B, C and D)



### Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex

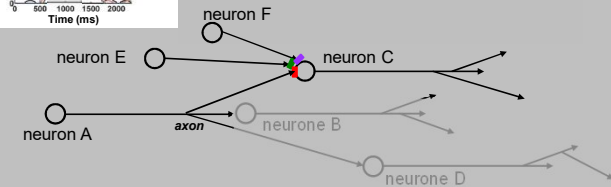
Oana Tudusciuc and Andreas Nieder\*

PNAS 2007



1 neuron (A) contacts several post-synaptic neurons (B, C and D)

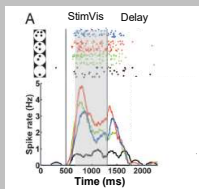
1 neuron (C) is contacted by several pre-synaptic neurons (A, E and F)



### Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex

Oana Tudusciuc and Andreas Nieder\*

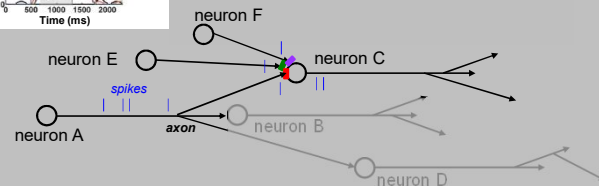
PNAS 2007



1 neuron (A) contacts several post-synaptic neurons (B, C and D)

1 neuron (C) is contacted by several pre-synaptic neurons (A, E and F)

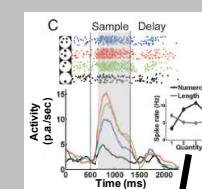
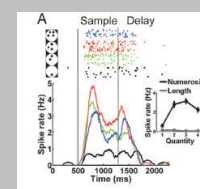
The probability that a post-synaptic neuron (C) emits spikes depends on the quasi-simultaneity of pre-synaptic spikes (when the influence is excitatory). **There exist also inhibitory influences.**



### Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex

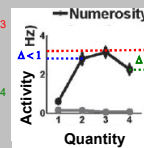
Oana Tudusciuc and Andreas Nieder\*

PNAS 2007

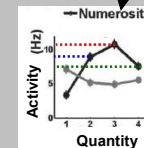


Between Q=2 and Q=3  
 $\Delta(2,3) < 1$  spike/s  
 0.1 spike/100 ms  
 0.6 spike/600 ms

Between Q=3 and Q=4  
 $\Delta(3,4) < 2$  spikes/s  
 0.2 spike/100 ms  
 1.2 spike/600 ms



Out of the numerous spikes they receive from other neurons during 600 milliseconds, how do the post-synaptic neurons manage to distinguish the spikes emitted by this neuron and enable the monkey to produce a different response according to whether 2, 3 or 4 stimuli were presented in its visual field?



Between Q=2 and Q=3  
 $\Delta(2,3) = 2$  spikes/s  
 0.2 spike/100 ms  
 1.2 spike/600 ms

Between Q=3 and Q=4  
 $\Delta(3,4) = 4$  spikes/s  
 0.4 spike/100 ms  
 2.4 spike/600 ms



OPEN ACCESS Freely available online

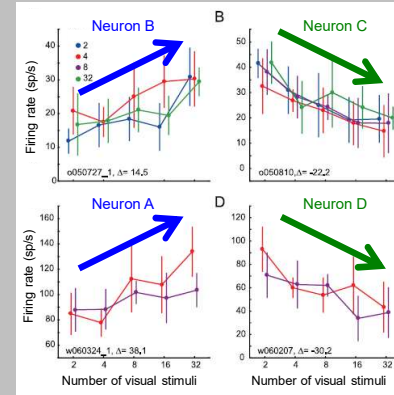
PLOS BIOLOGY

# Monotonic Coding of Numerosity in Macaque Lateral Intraparietal Area 2007

Jamie D. Roitman<sup>1\*</sup>, Elizabeth M. Brannon<sup>2,3</sup>, Michael L. Platt<sup>1,3</sup>

<sup>1</sup> Department of Neurobiology, Duke University, Durham, North Carolina, United States of America, <sup>2</sup> Department of Psychology and Neuroscience, Duke University, Durham, North Carolina, United States of America, <sup>3</sup> Center for Cognitive Neuroscience, Duke University, Durham, North Carolina, United States of America

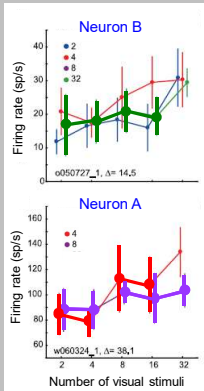
**“accumulator” neurons**      **“decumulator” neurons**



modified from Roitman, Brannon & Platt PLOS Biol 2007



How do post-synaptic neurons “distinguish” whether 4, 8 or 16 stimuli were presented in the response field of the recorded neuron ?



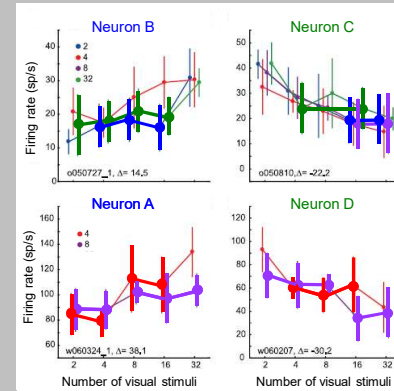
modified from Roitman, Brannon & Platt PLOS Biol 2007

**Neuron B :**  
 During task 32, the post-synaptic neurons cannot “distinguish” whether 2-4-8 or 16 stimuli were presented in the response field of the recorded neuron.  
 During task 2, they receive no signal enabling them to “distinguish” whether 4, 8 or 16 stimuli were presented.

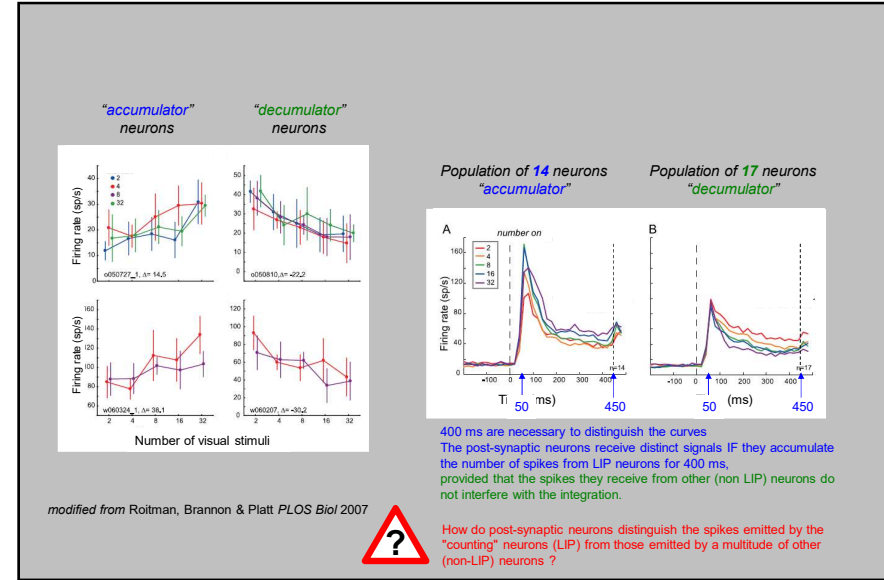
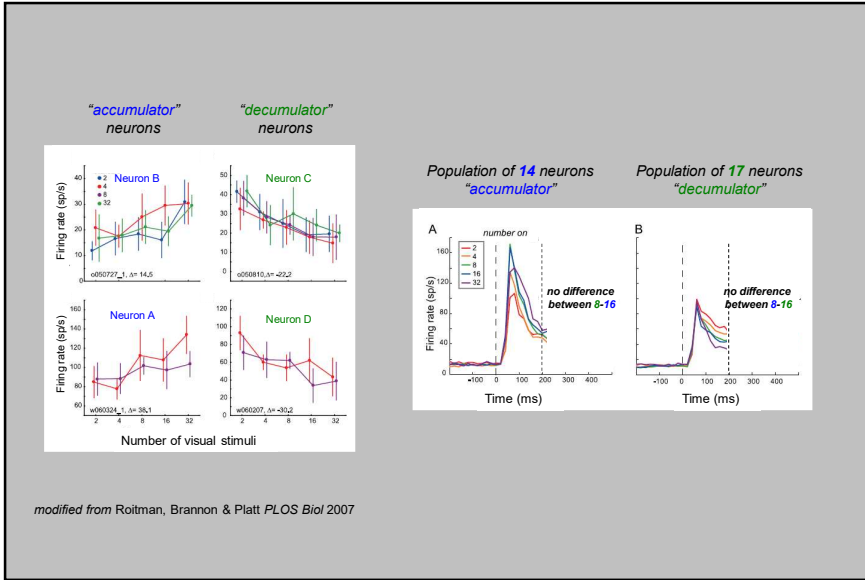
**Neuron A :**  
 During task 8, the post-synaptic neurons (targeted by neuron A) cannot distinguish whether 8-16 or 32 stimuli were presented in the response field of the recorded neuron. They receive cannot distinguish either whether 2 or 4 stimuli were presented.  
 During task 4, the post-synaptic neurons cannot distinguish whether 8 versus 16 (or 2 versus 4) stimuli were presented.



How do the post-synaptic neurons distinguish whether 8, 16 or 32 stimuli were presented in the response field of the recorded neuron ?  
 How do they distinguish whether 2 or 4 stimuli were presented?



modified from Roitman, Brannon & Platt PLOS Biol 2007

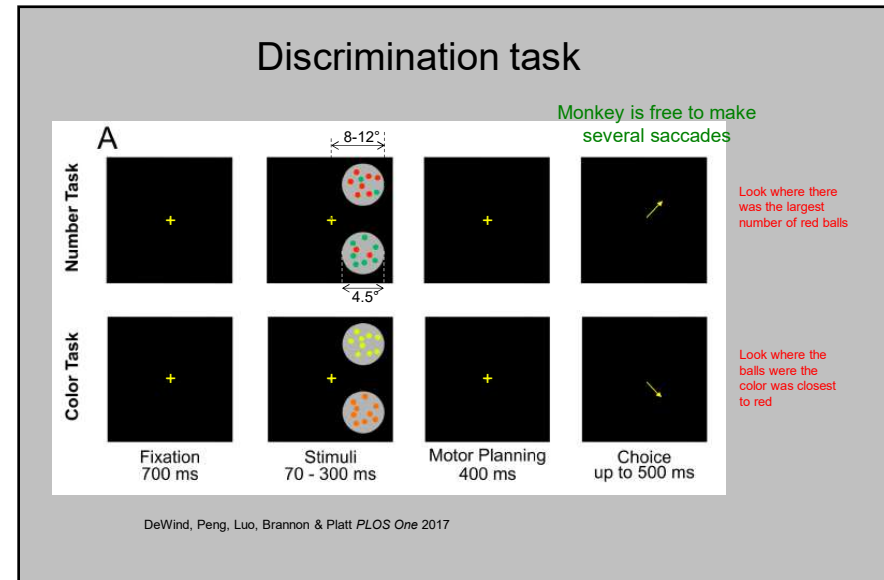


The correlation between the firing rate of neurons and an external event (number of items in the visual field) does not necessarily imply that the spikes contribute to the behavioral response.

Pharmacological inactivation does not support a unique causal role for intraparietal sulcus in the discrimination of visual number

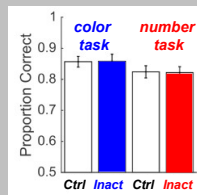
Nicholas K. DeWind<sup>1</sup>\*, Jiyun Peng<sup>2</sup>, Andrew Luo<sup>3</sup>, Elizabeth M. Brannon<sup>1</sup>, Michael L. Platt<sup>1,4,5</sup>

PLOS One 2017



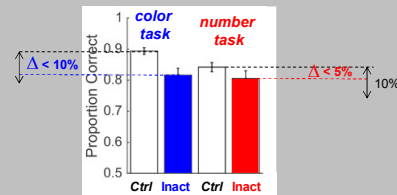
## LIP area does not participate to visual "counting" either

### Inactivation of VIP



DeWind, Peng, Luo, Brannon & Platt PLOS One 2017

### Inactivation of LIP



No effect → refutation of  
Andreas Nieder's  
hypothesis

## Fallacious logic

According to DeWind, Peng, Luo, Brannon & Platt (2017),  
If LIP and VIP areas are specialized in processing the number of visual items,  
THEN inactivation of either of these areas should cause a greater deficit in a numerical  
discrimination task than in a color discrimination task.

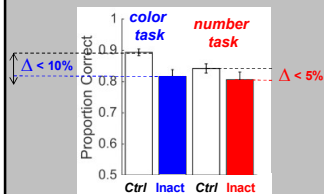
→ We have seen that this is not the case

Instead of rejecting their premises and concluding that LIP and VIP areas play no role in processing  
the number of visual items, they propose another inference :

IF inactivation leads to similar deficits in both tasks (numerical and color discrimination)  
THEN the numerical processing in LIP and VIP areas is the mere signature of a general purpose  
circuit for ordinal comparison, visual attention or perceptual decision making

## Simpler explanation

### Inactivation of LIP



The slight drops of performance  
**8%** (color discrimination) and  
**4%** (quantitative discrimination)  
may result from a mild visual impairment  
(diplopia, ocular divergence, etc.?).

DeWind, Peng, Luo, Brannon & Platt PLOS One 2017

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Neurons in Monkey Parietal Area LIP Are Tuned for Eye-Movement  
Parameters in Three-Dimensional Space

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## Conclusion

There is no evidence of an involvement of the parietal cerebral cortex in numerical "cognition" in monkeys.

In humans, brain imaging studies say nothing about the activity of neurons. The fMRI-BOLD signal reflects  
pre-synaptic activity and not the emission of action potentials (Lippert et al. *Magnetic Resonance Imaging* 2010).

During the last decades, confusion has arisen in brain sciences between  
- the measurements (and their numerical transformations),  
- the physiological processes that generate the measured phenomenon and  
- the concepts

The limitations of measurements and the associated theoretical notions led many cognitive neuroscientists (**authors,  
reviewers, popularizers and teachers**) to promote a nativist and evolutionary option, such as coding in the brain of  
space, time, numbers, Geometry and Arithmetic.

Neuroscience has neglected the fact that a multitude of cognitive structures are external to the individual, i.e.,  
the "external instruments (books, libraries, calculation and observation instruments...) and the social structures  
of production and knowledge accumulation (encyclopedias, learned societies, etc.)" (Sylvain Auroux 1998).

"Mathematics is a collective work of art that derives its objectivity through social interactions" (Yehuda Rav 1988)

"The locus of scientific rationality is not the individual scientist but the scientific community" (Miriam Solomon 1996)