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# Optimal Recursive Terminal Sliding-Mode Control Using Super-Twisting Algorithm For Improving High Efficiency and Reliability of Pump Systems

Samir Nassiri, Moussa Labbadi, *Member, IEEE*, Chakib Chatri, and Mohamed Cherkaoui

**Abstract**—In this letter, we propose an optimal recursive terminal sliding-mode control (ORTSMC) combined with super-twisting algorithm (STA) for a pump system under uncertainties. The main objective of the approach developed is to ensure rapid convergence of the pumping system with minimal power losses. To calculate the optimal input parameters of the pump system, a quantum particle swarm optimization algorithm (QPSO) is used. Next, we introduce a non-linear sliding variable into the cost function of the linear quadratic regulator (LQR). This proposal, along with the ORTSM manifold, aims to achieve fast convergence, dynamic stability, and minimize energy consumption. Additionally, the STA is employed to enhance performance during the reaching phase and reduce the chattering problem. The stability of the closed-loop control system is guaranteed using the Lyapunov theory. Finally, we conduct a comparative simulation analysis with two existing control schemes to demonstrate the superiority and effectiveness of our proposed control strategy.

**Index Terms**—Sliding mode control, quantum particle swarm optimization algorithm, Linear Quadratic Regulator.

## I. INTRODUCTION

PUMPS powered by asynchronous three-phase electric motors represent a significant part of the industry, of which 45% are centrifugal pumps being used for liquid transfer and delivery. In most cases, the pump operate at off-design conditions where it's operating conditions deviate from the designed conditions. As a result, imbalanced axial and radial forces may occur and lead to overheating, recirculation, and cavitation. Hence, this issue can be thought of as an essential factor that led to a lower service life and a great increase in electric energy consumption, notably during peak times. As the price of energy goes up, industries are we are looking for new solutions to improve the efficiency of the pumping system. Various methods for increasing energy efficiency have been developed. Many of them demonstrate the important role of variable speed drives which are paid particular attention

to reducing energy consumption [1]. This system precisely are needed to meets varying demand and to limit the pumps' operating conditions to a point nearby possible to the design conditions with the purpose of high efficiency and reliability [2]. The optimization technique to achieve maximum efficiency of the pump system using variable-frequency drivers (VFDs) is a challenge due to the constraints of the system. Evolutionary algorithms have been widely considered by researchers to solve the optimization problem such as, Genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization; fuzzy optimization, golden section search (GSS), neural-network-based methods, Multi-objective optimization, convex problems, and multidisciplinary optimization. A review of different efficiency improvement schemes in this regard is presented in [3]. Other researchers have suggested alternative ways to improve energy efficiency. Optimal control are required to achieve electrical energy savings for hydraulic systems [3]. In recent years, researchers have been exploring the optimal control, especially the linear quadratic regulator (LQR) which is one of the most important issue of modern Control Theory [4]. In an industrial process control, the LQR technique are becoming increasingly popular, which easy implementation using simple mathematical processes without prior knowledge can guarantee robustness, best response [5]. Besides, some researchers use of advanced methods in determining the weight matrices in the Riccati equation which has a major influence on the performance of the LQR controller [6]. In most recent studies, the genetic algorithm is added to the LQR for selecting the optimal Q and R matrices, since it is a practical alternative to obtain the control parameters, and provide a powerful technique for tuning optimized parameter method [7]. On the other hand, in industrial production, the output flow of centrifugal pumps changes according to frequently changing working conditions due to uncertainties of the external environments of the pump system. While when designing the controller, uncertainties and external disturbances with parameter variation need more consideration, because they have an important influence on the system [8]. In the literature, sliding mode control (SMC) has been previously proposed and investigated to control of motor drives due to its robustness against parameter uncertainties and disturbances [8]. However, the SMC technique will

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lead to the well-known chattering problem [9]. To overcome the chattering problem and to achieve good performance of robustness, stability, and precision. An appropriate control strategies are necessary to ensure effective suppression of chattering, minimize instability, guarantee a fast response, and increase disturbance rejection performance. Many techniques have been designed, such as intelligent fuzzy logic control, Hybrid control techniques, neuro-fuzzy technique, discrete-time neural inverse optimal control, model predictive torque control, super-twisting algorithm (STA), and fractional order terminal SMC [10], [11]. Currently, a super-twisting algorithm (STA) has significant attention to reduce chattering in a vector control system. Terminal sliding mode (TSM) control has been widely applied by many researchers, giving rise to a robust control with tunable finite-time convergence delivering high precision, fast response, and strong robustness compared with the conventional sliding-mode control [12], [13]. However, the controller design of the TSMC systems has singularity and slow convergence speed in the case of the system state being at a distance from the equilibrium [14], [15], [16]. Non-singular terminal sliding mode (NTSM) control has been developed to offer the singularity avoidance and fast convergence under environmental variations [17], [18], [19], [20]. Recently, with the development of fractional controls, the most prominent method used is fractional order sliding mode control (FSMC) which is introduced to enhance the robustness and the performance of the control systems [21], [22], [23]. Inspired by above-mentioned works, we propose a novel optimal strategy for the pump systems that is composed of a recursive integral terminal sliding function and a fast nonsingular terminal sliding surface related to the cost function of the LQR. The model is applied to guarantee not only fast convergence, dynamic stability and transient response, but also to minimise the input energy. Finally, STA is added to increase the robustness and reduce the chattering problem of the control system. The stability of the closed-loop control system is proved based on the Lyapunov theory. The foremost contributions of the paper are summarized as follows:

- **Proposing new ORTSM-STA manifold which can guarantee better performance;**
- **Offering a suitable operating mode by using quantum particle swarm optimization algorithm;**
- **Applied Lyapunov theorem to study and establish the closed-loop system's stability under the presence of external disturbances, and uncertainties.**

## II. MATHEMATICAL MODELLING OF SYSTEM

### A. Mathematical Modelling of Induction Motor

The mathematical modelling of a squirrel-cage induction motor can be formulated by using  $(d - q)$  rotating reference frame synchronously with the rotor flux space vector [1], [2], [3], [4], and [9]. According to [3], the total power losses of the induction machine are divided into winding losses and core losses. On the basis of [3], and [9], we applied the condition for the field orientation  $\psi_{sd} = \psi_s, \psi_{sq} = 0$ , the total motor losses can be described in terms of d-and q-axis

current components as follows:

$$P_{Loss} = \frac{3}{2} \left( R_s + R_r \frac{L_m^2}{L_r^2} \right) (i_{ds}^2 + i_{qs}^2) - 3R_r \frac{L_m}{L_r^2} \psi_{rd} i_{ds} + \frac{3}{2} \left( \frac{R_r}{L_r^2} + n_p^2 \omega_M^2 \frac{L_m}{R_m L_r^2} \right) \psi_{rd}^2 \quad (1)$$

where  $c_{Fe}$  and  $c_{str}$  are the stray and iron loss coefficients, respectively.

### B. Mathematical Modelling of Pump

The polynomial approximation of a centrifugal pump, able to predict the pressure differential  $H_i$  created by impeller rotating speed [1], [2], and [4].

$$H_i = a_1 Q_i^2 + a_2 Q_i + a_3, \quad (2)$$

where  $Q$ ,  $a_1$ ,  $a_2$ , and  $a_3$  are output flow rate and polynomial coefficients which can be approximated with plotting and fitting (2) [2], [10], [11], [12], [13]. The centrifugal pump resistive torque can be calculated by means

$$T_p = k_p Q_i \omega_r, \quad (3)$$

where  $k_p$  is the centrifugal pump constant, which depends on pump nominal data obtained by using the affinity laws.

## III. LOSS MODEL TECHNIQUE

### A. Loss Minimization in Steady State

In steady state, it is possible to reduce the loss of machine in the field-oriented d,q frame, by minimizing the value of the electric losses while maintaining the desired torque, The optimal steady-state value of the flux is found by solving [3], [9], and [14].

$$\frac{\partial P_{Loss}}{\partial \psi_r} |_{\omega_e = const.} = 0, \quad (4)$$

From the loss model, we obtain the optimal steady-state value of the flux which is a function of torque, speed, and the machine parameters.

$$\psi_{rd_{opt}} = \sqrt{T_e^{ref}} \sqrt{\frac{2 L_r}{3 n_p} \sqrt{\frac{R_s L_r^2 + R_r L_m^2}{R_s L_r^2 + L_m^4 \frac{n_p^2 \omega_r^2}{R_m}}}}, \quad (5)$$

### B. Equilibrium operating point

The dynamic nonlinear model is written as follows :

$$J \frac{d\omega_r}{dt} = T_e - k_p Q_i \omega_r - b \omega_r. \quad (6)$$

The equilibrium operating point of the motor-pump system is obtained when

$$\frac{d\omega_r}{dt} = 0, \quad (7)$$

we obtain

$$T_e^{ref} = (k_p Q_i^{ref} + b) \omega_r. \quad (8)$$

With this, the optimal steady-state value of the flux can be rewritten as

$$\psi_{rd_{opt}} = \sqrt{(k_p Q_i^{ref} + b) \omega_r} \sqrt{\frac{2 L_r}{3 n_p} \sqrt{\frac{R_s L_r^2 + R_r L_m^2}{R_s L_r^2 + L_m^4 \frac{n_p^2 \omega_r^2}{R_m}}}}, \quad (9)$$

### C. Optimal Operations

The QPSO is chosen as the optimization algorithm in this paper which adjusts speed ratio to compute the best valve positions for the best pressure loss across the valve according to the flow rate set point aiming at adjusting the pipeline resistance [15]. Thus, to optimize operating point of the pump located near the Best Efficiency Point (BEP), the objective function can be described by

$$\min F = P \quad (10)$$

where  $P$  is fitted under the different rotational speeds (Ulanicki et al. 2008) as follows:

$$P_i = d_0 S^3 + d_1 Q_i S^2 + d_2 Q_i^2 S + d_3 Q_i^3 \quad (11)$$

The similarity laws are shown as follows:

$$\frac{Q_i}{Q_0} = S, \quad \frac{H_i}{H_0} = S^2, \quad \frac{P_i}{P_0} = S^3 \quad (12)$$

where  $S = \frac{\omega_r}{\omega_{rn}}$  is speed ration,  $Q_0, H_0$  and  $P_0$  are characteristics at the rate speed. In this paper, we have two constrains, the constrains for obtaining the best pressure loss across the valve and for fixing the pumps operating region to a region near the BEP can be described as

$$W_H = \begin{cases} (H_i - H)^2, & (H_i < H) \\ 0, & (H_i \geq H) \end{cases} \quad (13)$$

$$W_\delta = \begin{cases} 0, & |\delta| \leq \delta_{max} \\ \delta_{max} - \left| \frac{Q_i - SQ_r^{BEP}}{SQ_r^{BEP}} \right|, & |\delta| > \delta_{max} \end{cases} \quad (14)$$

where  $\delta = \frac{Q_i - SQ_r^{BEP}}{SQ_r^{BEP}}$  is the actual flow deviation in the vicinity of the BEP. The range of  $\delta$  is chosen between  $-20\%$  and  $+20\%$ .  $Q_r^{BEP}$  is the pump's BEP flow rate.  $S$  is a positive parameter satisfying  $0.5 < S < 1$ .

Finally, by combining the objective function with the penalty function, the objective function can be rewritten as

$$\min F = P + \gamma_1 W_H + \gamma_2 W_\delta \quad (15)$$

where  $\gamma_1$  and  $\gamma_2$  are penalty coefficients for operating region penalty and head penalty. The QPSO can solve minimum valve of the fitness function and find the optimum of the problem by adjusting the speed ratio  $S$ . Once the optimal flow rate and pump speed ratio are obtained by the above assumptions, the optimal steady-state value of the flux has a new optimal expression as

$$\psi_{rdopt} = \sqrt{(\delta_{opt} + 1) S_{opt} k_p Q_r^{BEP} + b} \quad (16)$$

$$X = \frac{2 L_r}{3 n_p} \sqrt{\frac{R_s L_r^2 + R_r L_m^2}{\frac{R_s L_r^2}{S_{opt}^2 \omega_{rn}^2} + \frac{n_p^2 L_m^4}{R_m}}}$$

It was found in (14) that the optimal flux in steady state is directly proportional to the speed ratio and the deviation of the actual flow rate. This way, in steady state, a desired flow rate is obtained with the optimized flux and a corresponding torque-generating stator current  $i_{sq}$  with the lowest possible losses.

### IV. PROPOSED CONTROL SCHEME

#### A. Construction of the ORTSM-STA Controller

1) **ORTSM Controller:** In order to construct the control scheme based on ORTSM-STA technique, an integral sliding mode manifold is firstly presented as:

$$s = G \left[ e(t) - e(0) - \int (A - BK) e \right]. \quad (17)$$

where  $e(t) = x(t) - x(t)_{ref}$  is the tracking error between the four actual states of the system  $x = [i_{sq} \ i_{sd} \ \psi_{rd} \ \omega_r]^T$  and the desired references,  $e(0)$  is the initial condition of the system,  $G$  is determined by using the left inverse of the input matrix  $B$ , as expressed as  $G = (B^T B)^{-1} B^T$ ,  $A$  and  $B$  represent the state matrix and control matrix of the dynamic model of pumping system around the equilibrium operating point in state space  $\dot{e}(t) = Ae(t) + Bu + E$  with  $E = \Delta A + \Delta B u + d$  represent unknown lumped uncertainties possesses in upper bound, and  $K$  is the optimal gain matrix determined by using  $LQR$  which can be considered a powerful optimal control among various linear controllers [16], and [17]. Secondly, a fast nonsingular terminal sliding mode surface is added to guarantee finite time convergence of  $s(t)$

$$\sigma = \dot{s} + k_1 s + \gamma \text{sig}(s)^\alpha. \quad (18)$$

where  $\text{sig}(x)^\alpha = |x|^\alpha \text{sgn}(x)$ ,  $k_1 > 0$ ,  $\gamma > 0$  and  $\alpha > 0$ . Then, the recursive terminal sliding surface is designed as

$$\begin{aligned} \theta &= \sigma + \lambda \sigma_I. \\ \dot{\sigma}_I &= \text{sig}(\sigma)^\beta. \end{aligned} \quad (19)$$

where  $\lambda > 0$ ,  $\beta \in (0, 1)$  with  $\sigma_I(0) = -\lambda^{-1} \sigma(0)$  is initial value of the integral term to reduce the convergence time [19], [20], [21], [22], and [23]. Further, the derivative of the recursive terminal sliding mode control (RTSMC) is calculated as

$$\dot{\theta} = \ddot{e} - (A - BK) \dot{e} + k_1 \dot{s} + \gamma \alpha |s|^{\alpha-1} \dot{s} + \lambda \dot{\sigma}_I. \quad (20)$$

Replacing the tracking error dynamics  $\ddot{e}(t)$  with its expression without taking into account the disturbances  $\ddot{e}(t) = A\dot{e}(t) + B\dot{u}_{eq}$ , and using the condition  $\dot{s}(t) = 0$  at which the state of the pump system reaches the sliding surface, we obtain

$$\dot{\theta} = GB\dot{u}_{eq} + GBK\dot{e} + k_1 \dot{s} + \gamma \alpha |s|^{\alpha-1} \dot{s} + \lambda \dot{\sigma}_I. \quad (21)$$

Then the equivalent control part is given by the following expression:

$$u_{eq} = -(GB)^{-1} \int \left[ GBK\dot{e} + k_1 \dot{s} + \gamma \alpha |s|^{\alpha-1} \dot{s} + \lambda \dot{\sigma}_I \right]. \quad (22)$$

2) **Supertwisting SMC:** The discontinuous part is proposed based on the super-twisting algorithm as [19], [20], [21], [22].

$$\begin{cases} \dot{\theta} = -k_2 \sqrt{|\theta|} \text{sgn}(\theta) + f \\ \dot{f} = -k_3 \text{sgn}(\theta) + \dot{E} \end{cases} \quad (23)$$

where  $k_2$ , and  $k_3$  are diagonal matrices with positive constant elements. Thus, the proposed control scheme is designed as

$$\begin{aligned} u &= -(GB)^{-1} \int \left( GBK\dot{e} + k_1 \dot{s} + \gamma \alpha |s|^{\alpha-1} \dot{s} + \lambda \dot{\sigma}_I \right) \\ &\quad - (GB)^{-1} \left[ -k_2 \sqrt{|\theta|} \text{sgn}(\theta) - \int \left( k_3 \text{sgn}(\theta) + \dot{E} \right) \right]. \end{aligned} \quad (24)$$

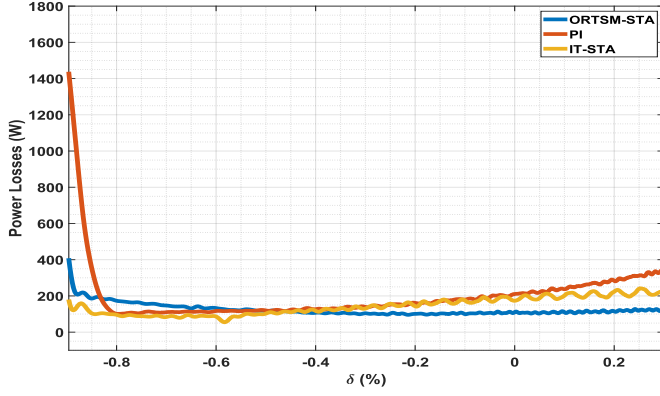


Fig. 1. Electric loss of the motor in different degrees of deviation  $\delta$ .

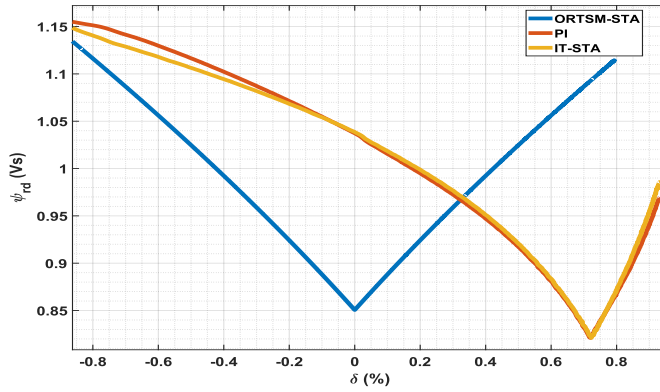


Fig. 2. Evaluation of steady-state response of the optimization flux at nominal speed.

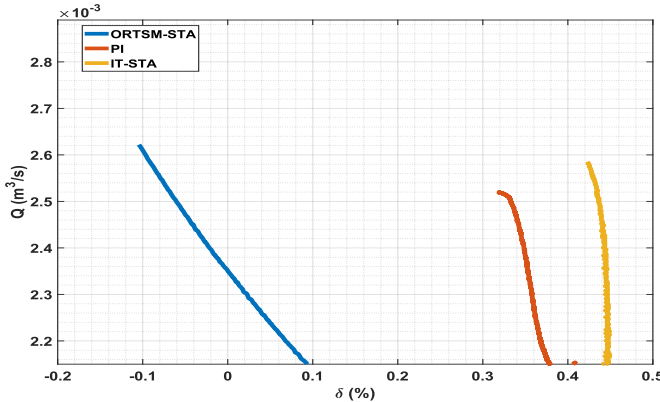


Fig. 3. Actual flow rates in different degrees of deviation  $\delta$ .

## B. Stability Analysis

The results of this letter is given in the following Theorem.

*Theorem 1:* Consider the pump system model with designed controller in (24) and sliding variable in (19), the super-twisting control algorithm ensures the convergence of the sliding surface and its derivative to zero in a finite-time

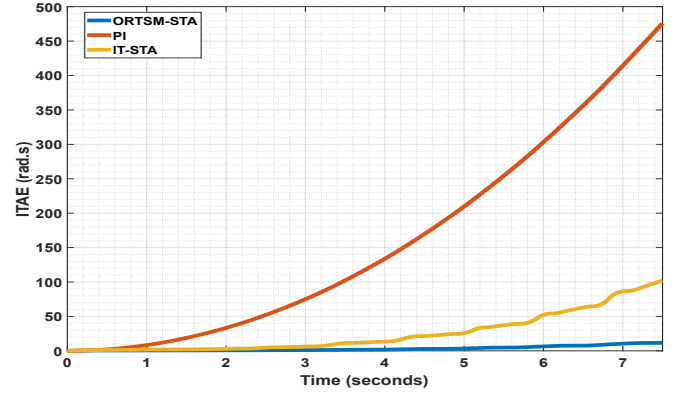


Fig. 4. Performance indices integral of time multiplied by absolute error (ITAE) using different control schemes and taking into account the disturbances.

TABLE I  
SPECIFICATIONS OF THE COMPONENTS.

Component	Specifications
Electrical Motor	Max. power of the motor: 1.1 KW. Nominal speed 2800 rpm; $L_r = 0.029$ ; $R_r = 5.313$ ; $L_s = 0.029$ ; $R_s = 6.959$ ; $L_m = 0.6786$ ; $J = 0.6786$ ; $b = 0.00114$ .
Centrifugal Pump	Nominal speed 1770 rpm, pump's parameters all defined in MATLAB by approximating polynomial
Pipeline	Nominal diameter: 0.0381 m (1.5 in). Length: 10 m
Control Valve	Proportional Valve Nominal diameter: 0.0381 m

smaller than:

$$T = \frac{2\sqrt{\lambda_{\min}\{M\}}}{\lambda_{\min}\{Y\}} V^{\frac{1}{2}} (\Theta(0)) \quad (25)$$

*Proof:* Fatherly, follow the literature [20]–[22], we prove that the improved Super-Twisting control law converges stably on the sliding mode surface.

$$\begin{aligned} V_1(x) &= 2k_3 |\theta| + \frac{1}{2}\theta^2 + \frac{1}{2}f^2 \\ &= 2k_3 |\theta| + \frac{1}{2}f^2 + \frac{1}{2} \left( -k_2\sqrt{|\theta|}\text{sgn}(\theta) + f \right)^2 \end{aligned} \quad (26)$$

The Lyapunov candidate (27) can be re-structured as:

$$V_1 = \Theta^T M \Theta \quad (27)$$

where

$$M = \frac{1}{2} \begin{pmatrix} k_2^2 + 4k_3 & -k_2 \\ -k_2 & 2 \end{pmatrix}; \Theta = \begin{pmatrix} \sqrt{|\theta|}\text{sgn}(\theta) \\ f \end{pmatrix} \quad (28)$$

with

$$\lambda_{\min}\{M\} \|\Theta\|_2^2 \leq V \leq \lambda_{\max}\{M\} \|\Theta\|_2^2 \quad (29)$$

$\lambda_{\min}\{M\}$  and  $\lambda_{\max}\{M\}$  are the minimum and maximum eigenvalues of  $M$  respectively and  $\|\Theta\|_2^2$  is the Euclidean norm of  $\Theta$ .

Take the derivative of  $\Theta$

$$\dot{\Theta} = -\frac{1}{\sqrt{|\theta|}} \left( N_1 \Theta + N_2 \dot{E} \sqrt{|\theta|} \right) \quad (30)$$

where

$$N_1 = \begin{pmatrix} -\frac{1}{2}k_2 & \frac{1}{2} \\ -k_3 & 0 \end{pmatrix}; N_2 = \begin{pmatrix} 0 \\ 2\dot{E} \end{pmatrix} \quad (31)$$

The derivative of  $V_1$  is:

$$\begin{aligned}\dot{V}_1 &= \dot{\Theta}^T M \Theta + \Theta^T M \dot{\Theta} \\ &= -\frac{1}{\sqrt{|\theta|}} \Theta^T \left( N_1^T M + M N_1 \right) \Theta \\ &\quad + \frac{1}{\sqrt{|\theta|}} \sqrt{|\theta|} N_2^T M \Theta\end{aligned}\quad (32)$$

Then, the following expression can be stated:

$$\sqrt{|\theta|} N_2^T M \Theta = \Theta^T M N_2 N_2^T M \Theta \quad (33)$$

substituting (33) into (32) leads to:

$$\begin{aligned}\dot{V}_1 &= -\frac{1}{\sqrt{|\theta|}} \Theta^T \left( N_1^T M + M N_1 \right) \Theta + \frac{1}{\sqrt{|\theta|}} \Theta^T M N_2 N_2^T M \Theta \\ &= -\frac{1}{\sqrt{|\theta|}} \Theta^T \left( N_1^T M + M N_1 - M N_2 N_2^T M \right) \Theta \\ &= -\frac{1}{\sqrt{|\theta|}} \Theta^T Y \Theta\end{aligned}\quad (34)$$

where

$$\begin{aligned}Y &= N_1^T M + M N_1 - M N_2 N_2^T M \\ &= \begin{pmatrix} \frac{1}{2} k_2^3 - \frac{1}{4} k_2^2 \dot{E} + k_2 k_3 & -\frac{1}{2} (k_2^2 - k_2 \dot{E}) \\ -\frac{1}{2} (k_2^2 - k_2 \dot{E}) & \frac{1}{2} k_2 - \dot{E} \end{pmatrix};\end{aligned}\quad (35)$$

Using the gains:

$$k_2 > 2\dot{E}; k_3 > \frac{\dot{E}^2 k_2}{8(k_2 - 2\dot{E})} \quad (36)$$

the following condition are held.

$$\det(Y) > 0 \quad (37)$$

Therefore,  $\dot{V}$  is negative definite. Then, we have:

$$\dot{V} \leq -\frac{1}{\sqrt{|\theta|}} \lambda_{\min}\{Y\} \|\Theta\|_2^2 \quad (38)$$

where  $\lambda_{\min}\{Y\}$  is the minimum eigenvalue of  $Y$ . This means that by using Lyapunov theory, the convergence of the sliding surface to the origin is proven. The estimation of convergence time of system is

$$T = \frac{2\sqrt{\lambda_{\min}\{M\}}}{\lambda_{\min}\{Y\}} V^{\frac{1}{2}}(\Theta(0)) \quad (39)$$

using (30) gives:

$$\frac{\sqrt{V}}{\sqrt{\lambda_{\max}\{M\}}} \leq \|\Theta\|_2 \leq \frac{\sqrt{V}}{\sqrt{\lambda_{\min}\{M\}}} \quad (40)$$

In the other hand,  $\sqrt{|\theta|} \leq \|\Theta\|_2$  is obvious. Then we have

$$\dot{V} \leq -\frac{1}{\|\Theta\|_2} \lambda_{\min}\{Y\} \|\Theta\|_2^2 \leq -\frac{\lambda_{\min}\{Y\}}{\sqrt{\lambda_{\max}\{M\}}} \sqrt{V} \quad (41)$$

It can be seen from the equation above, the maximal convergence time of the sliding surface can be set. This concludes the proof. ■

TABLE II  
ORTSM-STA

$S$	0.9953	0.9583	0.9166
System Efficiency $\eta$ (%)	69.84	68.71	67.07
Power losses (W)	103.1	100.3	100.5
Q ( $10^{-4}$ m <sup>3</sup> /s)	18.12	17.61	19.26
H (m)	34.78	31.8	27.41
Valve Opening (%)	57.26	58.31	70.58
flux	0.88272	0.9233	0.96147
$\delta$	0.1985	0.2270	0.3173

TABLE III  
PI

$S$	0.9953	0.9583	0.9166
System Efficiency $\eta$ (%)	62.52	62.35	62.27
Power losses (W)	162.4	165	150.1
Q ( $10^{-4}$ m <sup>3</sup> /s)	13.81	15.14	16.14
H (m)	37.26	33.16	29.4
Valve Opening (%)	41.2	48.55	55.89
flux	0.99502	0.99812	1.102
$\delta$	0.4088	0.4162	0.4540

## V. SIMULATION RESULTS AND DISCUSSION

The whole control system and the performance of the proposed controller ORTSM-STA is evaluated through Matlab/simulink software. The parameters of the system (asynchronous electric motors, centrifugal pump, pipeline, and valve) are mentioned in Table I. The numerical simulations with a comparative analysis with the PI, IT-STA (Integral Sliding-Supertwisting Mode Control) techniques are conducted to better illustrate the performance and the effectiveness of the proposed controller was tested at different range of speeds and openings of throttling valve and also tested in the presence of external disturbance, uncertainties and parameter system variation. Figure 1 shows the electrical losses curves as a function of the degree of deviation  $\delta$  at a reference speed of  $\omega_0 = 240 \text{ rad/s}$  with the initial 0% opening of the valve. As can be seen, the comparison of results between the PI and proposed controller shows that the proposed technique gives an advantage over other controller adjusting the stroke of the throttle valve. Deviation has a significant impact on power losses. As shown in the figure, the minimum electric losses concentrate in the area of  $\pm 20\%$ . The simulation results of the water pumping system in Tables II–III show the comparative performance of the proposed controller with the conventional PI controller in terms of the overall efficiency, power losses, flow rate, and head. As can be seen, when the controls are set at  $\omega_0 = 240 \text{ rad/s}$ , the proposed controller had a considerable reduction in power losses, reaching a 36.515% reduction compared to PI. This superiority can be justified by the effect of the LQR controller. Consequently, the efficiency of the motor is increased by 11.583% compared to PI. This demonstrated that the optimized controller finds new optimal operating point at which the minimum electric losses of the induction motor is attained. It is noticed that the Table II indicate, when the stroke of the throttle valve at 57.26% with speed  $\omega_0 = 240 \text{ rad/s}$ , the maximum possible efficiency of the whole motor-pump system is approximately 69.84%. According to the findings, adjusting both the speed and the valve has

a significant impact on power losses and the operating region, which can be expanded. Based on these results and as shown in Figure 2, the proposed algorithm takes into account the minimization of the controllable power losses, which provides the optimum level of flux for the loss minimization. Figure 3 illustrates the flow rates in different degree of deviation  $\delta$ . As shown in the figure, the pump operating conditions of the proposed controller concentrate in the area of  $\pm 20\%$  and deviate far from the design point of conventional PI method. According to the findings, the pump perform at a higher efficiency and reliability which the operating point of the pumps is closer to the BEP. The flow tracking error and performance indices integral of time multiplied by absolute error (ITAE) are presented in Figure 4. The comparison shows the proposed technique is the best candidate since it renders the flattest error profile and the lowest error and can effectively improve the rejection performance of the disturbance when compared with the other controller. Consequently, the pump-system can be balanced robustly using the proposed technique.

## VI. CONCLUSION

This study proposed ORTSM-STA for high efficiency and high dynamic performance of the whole pumping system. Firstly, the LQR is implemented on fast nonsingular terminal sliding-Mode control to design an optimal controller with low energy consumption. Then, optimal recursive terminal sliding-mode control strategy is introduced that the system could be robust to uncertainties and external disturbances, to prove the dynamic stability and transient response of the motor-pump system in the presence of a perturbation in a relatively short time. The Super-Twisting control is added to provide good performance in the reaching phase. To validate the performance and effectiveness of the proposed control scheme, a comparative analysis based on the overall efficiency, the control robustness, and the performance indices has been made between the PI, IT-STA controllers and the proposed control strategy ORTSM-STA based on the simulated results. The numerical simulation results confirm that the proposed controller is more beneficial for energy efficiency, robustness achievement, and chattering reduction. Future work will include experimental validation. Furthermore, the effectiveness of control paradigms might be further developed by other variants such as recurrent neural network fractional order SMC and fractional-order model predictive control.

## REFERENCES

- [1] S. Nassiri, M. Labbadi, M. Cherkaoui "Optimal Integral Super-Twisting Sliding-Mode Control for high efficiency of Pumping Systems", 14th IFAC Workshop on Adaptive and Learning Control Systems ALCOS 2022.
- [2] C. F. J. Saavedra, S. R. Prada, J. G. Mázaró, "Modeling And Optimal Control of A Variable -Speed Centrifugal Pump With A Pipeline", IMECE2016-67992. 2016, Phoenix, Arizona, USA. November. 11-17.
- [3] J-F. Stumper, A Dotlinger, and R. Kennel, "Loss Minimization of Induction Machines in Dynamic Operation", IEEE Transactions on Energy Conversion, Volume: 28, Issue: 3, September 2013.
- [4] F. Malrait, A. K. Jebai, K. Ejjabraoui, "Power Conversion Optimization for Hydraulic Systems Controlled by Variable Speed Drives", Elsevier Journal of Process Control. Volume 74, February 2019, Pages 133-146, 10.1016/j.jprocont.2017.11.002.

- [5] L. Gevorkov, A. Rassólkin, A. Kallaste and T. Vaimann, "Simulation Study of Mixed Pressure and Flow Control Systems for Optimal Operation of Centrifugal Pumping Plants", Electrical, Control and Communication Engineering 2018, vol. 14, no. 1, pp. 89–94, 10.2478/ecce-2018-0010.
- [6] L. Gevorkov, A. Rassólkin, A. Kallaste and T. Vaimann, "Simulation Study of A Centrifugal Pumping Plant's Power Consumption At Throttling And Speed Control", IEEE 58th International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON), 12-13 October 2017.
- [7] M. I. Jahmeerbacus, "Flow Rate Regulation of A Variable Speed Driven Pumping System Using Fuzzy Logic ", 4th International Conference on Electric Power and Energy Conversion Systems (EPECS), 24-26 November 2015.
- [8] X. Guo, Y. Wu, G. Li, C. Lu, "Dynamic Simulation of an Induction-Motor Centrifugal-Pump System under Variable Speed Conditions", ElktrotehniŠki Vestnik 84(3): 125-132, (2017).
- [9] S. M. Yang, and F. C. Lin, "Loss-Minimization Control of Vector-Controlled Induction Motor Drives", 4th IEEE International Conference on Power Electronics and Drive Systems. IEEE PEDS 2001 - Indonesia. Proceedings (Cat. No. 01TH8594), 25-25 October 2001.
- [10] F. Alkarrami, T. Iqbal, K. Pope, and G. Rideout, "Dynamic Modelling of Submersible Pump Based Solar Water-Pumping System with Three-Phase Induction Motor Using MATLAB", Journal of Power and Energy Engineering  $\zeta$ , Vol.8 No.2, February 2020.
- [11] J. Ghafouri and H. Khayatzadeh, "Dynamic Modeling of Variable Speed Centrifugal Pump Utilizing MATLAB/Simulink", International Journal of Science and Engineering Investigations, vol. 1, no. 5, pp. 1-7, Jun. (2012).
- [12] M. H. A. Khalifah and G. K. Mcmillan, "Control Valve Versus Variable-Speed Drive for Flow Control", International Society of Automation (2013).
- [13] Cristian Patrascioui, "Control Valves - Modeling and Simulation", WSEAS international conference of dynamical systems and control, Romania (2013).
- [14] R. Leidhold, G. Garcia, and M. I. Valla, "Field-oriented controlled induction generator with loss minimization", IEEE Trans. Ind. Electron., vol. 49, no. 1, pp. 147–156, February 2002.
- [15] Z. LAI, Q. LI, A. ZHAO, W. ZHOU, H. XU, and D WU , "Improving Reliability of Pumps in Parallel Pump Systems Using Particle Swarm Optimization Approach", IEEE Access ( Volume: 8), pp. 58427 - 58434, March 2020.
- [16] A. Betka , A. Attali, "Optimization of a photovoltaic pumping system based on the optimal control theory", Elsevier. Solar Energy Volume 84, Issue 7, July 2010, Pages 1273-1283.
- [17] M. L. Shanoob, K. Iqbal and A. Al-Maliki, "Wind turbine transient response and fault ride-through improvements with optimal control", Wiley. Int Trans Electr Eng Syst. Volume27, Issue11, November 2017.
- [18] K. Benlarbi, L. Mokrani and M.S. Nait-Said , " A fuzzy global efficiency optimization of a photovoltaic water pumping system ", Elsevier Ltd, Solar Energy Volume 77, Issue 2, 2004, Pages 203-216.
- [19] Y. Wang, F. Yan, F. Ju, B. Chen, and H. Wu, "Optimal Nonsingular Terminal Sliding Mode Control of Cable-Driven Manipulators Using Super-Twisting Algorithm and Time-Delay Estimation", IEEE Access ( Volume: 6).2018.2875685, DOI 10.1109.
- [20] K. Rsetam, Z. Cao, Z. Man and M. Mitrevska, "Optimal Second Order Integral Sliding Mode Control for a Flexible Joint Robot Manipulator ", IECON 2017 - 43rd Annual Conference of the IEEE Industrial Electronics Society, December 2017.
- [21] K Rsetam, Z Cao and Z Man, "Super-Twisting Based Integral Sliding Mode Control Applied to a Rotary Flexible Joint Robot Manipulator ", 11th Asian Control Conference (ASCC) Gold Coast Convention Centre, Australia December 17-20, 2017.
- [22] I. Sami, S. Ullah, A. Basit, N. Ullah and J-S Ro, "Integral Super Twisting Sliding Mode Based Sensorless Predictive Torque Control of Induction Motor", IEEE Access ( Volume: 8). Power Syst., vol. 8, Electronic ISSN: 2169-3536, pp. 186740 - 186755, October 2020.
- [23] C. JIA , D. KONG , and L. DU "Recursive Terminal Sliding-Mode Control Method for Nonlinear System Based on Double Hidden Layer Fuzzy Emotional Recurrent Neural Network ", IEEE Access ( Volume: 10), November 2022.